Effect of slowly varying drift forces on the motion characteristics of truss spar platforms

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A B S T R A C T

Spar platform has been regarded as a competitive floating structure for deep and ultra deep water oil and gas production. In this paper, an efficient methodology has been developed to determine the slow motion responses of slender floating offshore structures due to wave forces. Based on this methodology, a MATLAB program named ‘TRSPAR’ was developed to predict the dynamic responses in time domain and it was used in this study to obtain the numerical results of a typical truss spar platform connected to sea bed using nine taut mooring lines. The difference frequency forces were calculated using the principles of the extension of Morison equation for an inclined cylinder and the wave kinematics were predicted using hyperbolic extrapolation. Mooring lines were modelled as nonlinear springs and their stiffness was obtained by conducting the static offset simulation. Because of the lack of detailed calculations in literature, most of the equations used were derived and presented in this paper. The effects of the different sources of the second order difference frequency forces were compared for inertia and drag forces in terms of response spectra. To validate the TRSPAR code, its results were compared to results of a typical truss spar model test.

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1. Introduction

As offshore oil and gas exploration and production are going into ultra deep water, many innovative floating offshore structures are being proposed for cost savings. One of these innovations is spar platform. Although the concept is not new (Rudnick, 1964, 1967), it has been recently the subject of renewed interest. Spar can be more economical in deep water than TLPs, they are insensitive to the deck load (Perryman and Beynet, 1994), and they can be relocated regardless of the number of wells or the water depth. Recently truss spar platforms, which are significantly modified from the conventional classic spar platforms, are becoming more popular. The conventional spar has a long circular cylinder hull, whereas the truss spar consists of an upper circular tank, a middle truss part with some horizontal plates and a lower ballast tank at the keel. Because these two types of spars are quite different in shape, their motion characteristics are also quite different.

The spar is designed to have natural periods of vibration much higher than the dominant wave periods, so that there are hardly any linear forces at the natural frequencies. Due to the nature of nonlinear surface water waves, the difference frequency interactions among ocean wave components may result in low frequency wave excitation forces. Although the nonlinear low frequency wave forces are small in magnitude, the structure, however, may experience large low frequency motions, known as slow drift motions, because the exciting frequency is close to the natural frequency.

The research interest on spars has evolved recently and within a short period several researchers have investigated the dynamic response of spars numerically as well as experimentally. Most of the previous studies on the second order forces were applied to the first generation spar, namely classic spar. A study by a Joint Industry Project (Johnson, 1995) showed that the responses of spar buoy at the wave frequency, even near the spectrum peak frequency, were small but relatively large near its natural frequencies, although elevation measurements showed that the incident waves had insignificant energy at these low frequencies. Several studies demonstrated the importance of these second order forces. Mekha et al. (1995, 1996) studied the behaviour of spar in deep water. In their work, they applied two different methods to calculate the wave forces in time domain considering several second order effects. They also investigated the effect of modelling the mooring lines as nonlinear spring. Diffraction theory and boundary elements were used by Ran et al. (1996) to study the behaviour of spar in time domain using second order difference frequency wave loads. This was further
enhanced by Ran and Kim (1997) and Ran et al. (1998), who executed a nonlinear coupled dynamic analysis of a moored spar in random waves with and without current in both time and frequency domains.

A new methodology was developed by Cao and Zhang (1996) to predict slow drift responses of slender compliant offshore structures due to ocean waves. They used hybrid wave model (Zhang et al., 1995) and the Morison equation to predict the wave kinematics and wave forces respectively for irregular waves. Hybrid wave model is different from the other approaches by decomposition of the observed wave elevation into 'free' waves up to second order accuracy while the conventional methods consider the wave elevations to be only linear combinations of individual sinusoids.

In this paper, a quasi-static cable analysis was conducted in order to obtain the tension-displacement characteristics of the mooring system. This study addresses the effect of the difference frequency forces on the dynamic responses of truss spar using an efficient numerical scheme for predicting the first and second order dynamic responses of typical floating offshore structures. In addition, the paper includes a comparison between numerical and experimental results for validating the numerical scheme.

2. Theoretical formulation and numerical scheme

2.1. Governing equations

One of the most useful theories in calculating the kinematics of a progressive wave (Fig. 1) is linear airy theory (LAT), which is based on the assumption that the wave height ($H$) is small compared to the wave length ($L$) or water depth ($d$). This assumption allows the free surface boundary conditions to be linearized and local gravity wave height terms which are beyond the first order and also to be satisfied at the mean water level (MWL), rather than at the oscillating free surface. For unidirectional regular waves, the first order velocity potential is given by

$$\phi^{(1)} = \frac{ag}{cosh k} \sin \theta$$

where $g$ is the gravity acceleration. $\omega$, $k$ and $a$ are the wave frequency, wave number and wave amplitude, respectively.

$$\theta = kx - \omega t + \beta$$

where $\beta$ is the initial phase angle.

The wave elevation is

$$H = \frac{Z}{2} \cos \theta$$

The water-particle velocities and accelerations in the $x$ and $z$ directions are

$$u = \frac{H}{2} \omega \frac{\cosh ks}{\sinh kd} \cos \theta$$  \hspace{1cm} (3)

$$v = \frac{H}{2} \omega \frac{\sinh ks}{\sinh kd} \sin \theta$$  \hspace{1cm} (4)

$$\frac{\partial u}{\partial t} = \frac{H}{2} \frac{\cosh ks}{\sinh kd} \sin \theta$$  \hspace{1cm} (5)

$$\frac{\partial v}{\partial t} = \frac{H}{2} \frac{\sinh ks}{\sinh kd} \cos \theta$$  \hspace{1cm} (6)

For random waves, these equations apply to each wave component in order to get the resultant wave kinematics by adding up the individual effects from all wave components.

In this study, hyperbolic extrapolation is used. It is based on the assumption that the wave kinematics between the MWL and free surface follow the same LAT hyperbolic variations with depth as they do up to the MWL.

The wave forces are decomposed into two components; normal and tangential to the structure. The preceding component is calculated using an extension of the Morison equation for an inclined cylinder, which is based upon normal velocities and accelerations as shown in Fig. 2

$$u_x = u - C_D (C_U u + C_V v)$$

$$u_z = v - C_D (C_U u + C_V v)$$  \hspace{1cm} (7)

where $C_U$, $C_V$ are $x$ and $z$ components, respectively, of the unit vector $C$, which is acting along the cylinder axis directed up or down.

Two coordinate systems are illustrated in Fig. 3, the global axis ($xoz$) is fixed at MWL and the local axis ($z'Gt$) is fixed on the centre of gravity of the structure.

The normal wave force per unit length can be written as

$$f = \rho C_m T V w + \frac{1}{2} \rho C_D w |w|^2 V \tau$$

where $\rho$ is the mass density of wave. $C_m$, $C_D$ and $C_m$ are the inertia, drag and added mass coefficients, respectively. $A$ and $D$ are the cross-sectional area and structure diameter, respectively. $w$ is the relative normal velocity.

$$\tau = \frac{\sin \beta}{\cos \beta}$$

where $\beta$ is the pitch angle.
As mentioned earlier, floating structures, like spar, have natural frequencies lower than the incident ocean wave frequencies so that the third term in Eq. (11) is the most important term in studying the effects of the second order forces on spar.

The second order difference frequency forces can be classified into five categories:

1. Structure displacement
2. Axial divergence
3. Free surface fluctuation
4. Convective acceleration
5. Temporal acceleration of the second order incident wave.

The inertia and drag forces are calculated in the structure displaced position and the effect of the free surface fluctuation are also considered. The convective acceleration is added to the first and second order temporal accelerations to obtain the total wave acceleration for the inertia force.

2.2.1. Inertia forces

Because spar platforms have low Keulegan–Carpenter parameter (KC), the contribution of drag force is small compared to the inertia force. Up to the second order, the inertia force can be written as

\[ F_i = \int_{t_0}^{t_0} \rho C_{MA} \frac{\partial (\mathbf{w} \cdot \mathbf{v})}{\partial t} \cdot \mathbf{v} dt + \int_{t_0}^{t_0} \rho C_{MA} \frac{\partial (\mathbf{w} \cdot \mathbf{v})}{\partial t} \cdot \mathbf{n} dt \]

(16)

where the first, second, third and fourth parts of Eq. (16) are for the second order difference frequency inertia forces due to the structural displacement, free surface fluctuation, convective and temporal accelerations respectively. \( t_0 \) is the integration between the bottom and the top of the structure, and \( t \) is the integration between MWL and the instant free surface.

2.2.1.1. Structure displacement

The forces on the structure must be calculated at the displaced position instead of at the mean position. This adds nonlinear force on the spar (Li and Kareem, 1992). The horizontal and vertical wave particle accelerations up to the second order can be written as

\[ \frac{\partial \mathbf{w}}{\partial t} = \sum_i a_i \omega_i^2 \left( \frac{\sinh(ks)}{\sinh(kd)} \right) \sin \theta_i + \sum_i a_i \omega_i^2 \left( \frac{\sinh(ks)}{\sinh(kd)} \right) \sin \theta_i \]

(17)

\[ \frac{\partial \mathbf{w}}{\partial t} = -\sum_i a_i \omega_i^2 \sinh(ks) \left( \frac{\sinh(kd)}{\sinh(kd)} \right) \cos \theta_i + \sum_i a_i \omega_i^2 \sinh(ks) \left( \frac{\sinh(kd)}{\sinh(kd)} \right) \cos \theta_i \]

(18)

where \( \omega_i \) and \( \gamma_i \) are surge and pitch amplitudes, respectively. The first term in Eqs. (17) and (18) is corresponding to the wave horizontal and vertical accelerations at the mean position while the second term is representing the nonlinear structural displacement effect. When \( i=j \) Eq. (18) contributes to the mean force.
2.2.1.2. Axial divergence. Morison equation has been modified by Rainey (1989) by adding a new term to the original formula. This term, which is sometimes known as the Rainey axial divergence correction, represents a second order acceleration in addition to Morison equation. This normal acceleration is given by

\[
\omega^T Vr = \left[ \begin{array}{c}
\left( u - \frac{\partial v}{\partial t} \right) - C_x \left( C_x \left( u - \frac{\partial v}{\partial t} \right) + C_v \left( v - \frac{\partial u}{\partial t} \right) \right) \\
- \left( v - \frac{\partial v}{\partial t} \right) - C_z \left( C_z \left( u - \frac{\partial v}{\partial t} \right) + C_v \left( v - \frac{\partial u}{\partial t} \right) \right)
\end{array} \right] \tau Vr \sin \beta
\]

(19)

where

\[
\left( u - \frac{\partial v}{\partial t} \right) \tau Vr = AD_{hr1} \sin^2 \beta + (AD_{hr2} + AD_{hrMean}) \sin \beta \cos \beta
\]

+ \(AD_{hr3} + AD_{hrMean}) \sin \beta \cos \beta + AD_{hr4} \cos^2 \beta
\]

(20)

\[
AD_{hr1} = \sum \sum \omega_i \omega_j \left\{ a_i a_j \left[ \cosh k_s \cosh k_d \left( k_i - k_j \right) \right] \\
+ a_i k_s (X_Gim + \eta_{ym}) \left[ \left( \cosh k_s \cosh k_d \right) \sinh (\theta_i - \theta_j) \right] \\
- a_i k_s (X_Gim + \eta_{ym}) \left( \cosh k_s \cosh k_d \right) \sin \left( \theta_i + \theta_j \right) \right\}
\]

(21)

\[
AD_{hr2} = \sum \sum \omega_i \omega_j \left\{ a_i a_j \left[ \sinh k_s \cosh k_d \left( k_i + k_j \right) \sinh k_s \cosh k_d \right] \right\}
\]

(22)

\[
AD_{hr3} = AD_{hr2}
\]

(23)

\[
AD_{hr4} = \sum \sum \omega_i \omega_j \left\{ a_i a_j \left[ \cosh k_s \cosh k_d \left( k_i - k_j \right) \right] \\
+ a_i k_s (X_Gim + \eta_{ym}) \left[ \left( \cosh k_s \cosh k_d \right) \sinh (\theta_i - \theta_j) \right] \\
- a_i k_s (X_Gim + \eta_{ym}) \left( \cosh k_s \cosh k_d \right) \sin \left( \theta_i + \theta_j \right) \right\}
\]

(24)

\[
\left( v - \frac{\partial u}{\partial t} \right) \tau Vr = AD_{sr1} + AD_{srMean} \sin^2 \beta
\]

+ \(AD_{sr2} + AD_{srMean}) \sin \beta \cos \beta
\]

+ \(AD_{sr3} + AD_{srMean}) \sin \beta \cos \beta
\]

+ \(AD_{sr4} + AD_{srMean}) \cos^2 \beta
\]

(25)

\[
AD_{sr1} = - \sum \sum \omega_i \omega_j a_i a_j \left[ \sinh k_s \cosh k_d \left( k_i - k_j \right) \sinh k_s \cosh k_d \right] \cos (\theta_i - \theta_j) \right\}
\]

(26)

\[
AD_{sr2} = \sum \sum \omega_i \omega_j \left[ a_i k_s \cosh k_d \left( k_i - k_j \right) \sin \left( \theta_i - \theta_j \right) \right] \\
+ \sum \omega_i a_j \left[ k_s \sinh k_d \left( k_i - k_j \right) \sin \left( \theta_i - \theta_j \right) \right] \left[ \sinh (k_i + k_j) \cosh k_d \right] \times \sin \left( \theta_i + \theta_j \right)
\]

(27)

\[
AD_{sr3} = AD_{sr2}
\]

(28)

\[
AD_{sr4} = \sum \sum \omega_i \omega_j \left[ \left( \cosh k_s \sinh k_d \left( k_i - k_j \right) \sinh k_s \cosh k_d \right) \cos (\theta_i - \theta_j) \right] \\
+ \sum \omega_i \omega_j \left[ a_i k_s \cosh k_d \left( k_i - k_j \right) \sin \left( \theta_i - \theta_j \right) \right] \left[ \sinh (k_i + k_j) \cosh k_d \right] \times \sin \left( \theta_i + \theta_j \right)
\]

(29)

where \(Z_Gim\) is the heave amplitude. Since some of the above equations have a difference frequency cosine terms, there will be a contribution to the mean force. This will occur when \(i=j\).

\[
AD_{hrMean} = \frac{1}{2} \sum \omega_i \omega_j a_i a_j \left[ \sinh k_s \cosh k_d \right] \left[ \cos (\theta_i - \theta_j) \right]
\]

\[
AD_{srMean} = AD_{hrMean}
\]

(30)

(31)

\[
AD_{srMean} = \frac{1}{2} \sum \omega_i \omega_j a_i a_j \left[ \sinh k_s \cosh k_d \right] \left[ \cos (\theta_i - \theta_j) \right]
\]

(32)

(33)

(34)

(35)

2.2.1.3. Free surface fluctuation. The integration of the first order accelerations from the MWL to the wave free surface gives another source of the nonlinear difference frequency forces. The integration of the corresponding second order horizontal and vertical accelerations leads to

\[
\int \frac{\partial v}{\partial t} d \tau = \int \sum \omega i a_i g \frac{\sin (\theta_i - \theta_j)}{\cos (\theta_i - \theta_j)} \sin \beta \cos \beta \]

\[
\int \frac{\partial v}{\partial t} d \tau = - \int \sum \omega i a_i \left( k_i + k_j \right) \frac{\sin (\theta_i - \theta_j)}{\cos (\theta_i - \theta_j)} \sin \beta \cos \beta \]

As in the axial divergence effect, there is also a contribution to the mean force when putting \(i=j\) in Eq. (37).

\[
\int \frac{\partial v}{\partial t} d \tau = - \frac{g}{2 \cos \beta} \sum \omega_i a_i
\]

(36)

(37)

(38)

2.2.1.4. Convective acceleration. The total wave particle acceleration due to the change of the wave particle velocity with time and space. The change with time is known as temporal acceleration while the change with space is known as convective acceleration. The horizontal and vertical wave particle convective accelerations up to the second order can be written as

\[
\left( u - \frac{\partial v}{\partial t} \right) \tau Vr = - \sum \omega i \omega j a_i a_j \left( k_i - k_j \right) \sin \left( \theta_i - \theta_j \right) \right\}
\]

(39)

\[
\left( v - \frac{\partial u}{\partial t} \right) \tau Vr = \sum \omega i \omega j a_i a_j \left( k_i + k_j \right) \sin \left( \theta_i - \theta_j \right)
\]

(40)

The first part of Eq. (40) contributes to the mean force. It is interesting to compare the free surface effect with the convective acceleration, using Eqs. (36), (37), (39), and (40) for this purpose, one can observe that the two forces are working opposite to each other.

2.2.1.5. Temporal acceleration. For the spar, the second order wave particle acceleration divergence from the second order potential at the difference frequency due to the wave components interaction has a major part of the forces at the natural frequencies compared to the first order wave acceleration. The horizontal and vertical wave particle temporal accelerations up to the second order can
be written as

\[
\frac{\partial^2 u}{\partial t^2} = \sum_{k_1} \left[ \frac{(\omega_k \cos \omega_k)/(\omega_k \omega_{k_j}) k_{k_j} [1 + \tanh(k_j d \tanh k_d)] + (k_j^2 \sech^2(k_j d) / \omega_k - k_j^2 \sech^2(k_j d) / \omega_{k_j}) / 2}{g(k_{k_j} - k_{k_j}) \tanh(k_{k_j} - k_{k_j}) \cos(\theta - \theta_j)} \right] \\
\times a \omega_k (\omega_k - \omega_{k_j}) \cos^2(k_{k_j} - k_{k_j}) \frac{\cosh^2(k_{k_j} - k_{k_j})}{\cosh(k_{k_j} - k_{k_j})} \sin(\theta - \theta_j)
\]

(41)

\[
\frac{\partial^2 v}{\partial t^2} = -\sum_{k_1} \left[ \frac{(\omega_k \cos \omega_k)/(\omega_k \omega_{k_j}) k_{k_j} [1 + \tanh(k_j d \tanh k_d)] + (k_j^2 \sech^2(k_j d) / \omega_k - k_j^2 \sech^2(k_j d) / \omega_{k_j}) / 2}{g(k_{k_j} - k_{k_j}) \tanh(k_{k_j} - k_{k_j}) \cos(\theta - \theta_j)} \right] \\
\times a \omega_k (\omega_k - \omega_{k_j}) \cos^2(k_{k_j} - k_{k_j}) \frac{\sinh^2(k_{k_j} - k_{k_j})}{\cosh(k_{k_j} - k_{k_j})} \cos(\theta - \theta_j)
\]

(42)

As in the above equations, \(\partial v / \partial t^2\) will contribute to the mean force when \(i=j\).

2.2.2. Drag force

In the case of inertia force dominant systems, such as spar, the system may be approximated with a reasonable accuracy as a linear system in calculating the drag force and a linearization method may be adopted.

The linear approximation for the drag force per unit length is

\[
f_D(t) = \frac{1}{2} \rho C_D \overline{\bar{v}} W_t
\]

(43)

where \(\sigma_{wR}\) is the variance of the relative normal wave particle velocity. \(w_t\) is the relative normal velocity.

\[
\overline{\bar{v}} = \sqrt{\frac{8}{\pi} \overline{\bar{w}} w_t} = \sqrt{\frac{8}{\pi} \sum \left[ (\omega_k a_k \frac{\cosh k_s}{\sinh k_d} x_{Gim} - n_{y_{im}}) \right]^2 / 2} \\
\times \sum a_k \omega_k \frac{\cosh k_s}{\sinh k_d} \sin \theta_1 - \sum \omega_k (x_{Gim} + n_{y_{im}}) \sin \theta_1
\]

(44)

\[
\overline{\bar{w}} = \sqrt{\frac{8}{\pi} \overline{\bar{w}} w_t} = \sqrt{\frac{8}{\pi} \sum \left[ (\omega_k a_k \frac{\sinh k_s}{\sinh k_d} Z_{Gim} \right]^2 / 2} \\
\times \sum a_k \omega_k \frac{\sinh k_s}{\sinh k_d} \sin \theta_1 - \sum \omega_k Z_{Gim} \sin \theta_1
\]

(45)

In Eqs. (45) and (46) the first part contributes to the excitation force and the second part to the damping of the system. Although Eqs. (45) and (46) represent the linear approximation of the drag force, there are two sources of the second order slow varying difference frequency forces involved. One is due to the integration of the wave forces at the displaced position and the other is due to free surface fluctuation effect.

2.2.2.2. Structure displacement. Just like the inertia force, calculation of the drag force at the displaced position of the structure adds a second order term as follows:

\[
\overline{\bar{v}} = \sqrt{\frac{8}{\pi} \overline{\bar{w}} w_t} = \sqrt{\frac{8}{\pi} \sum \left[ (\omega_k a_k \frac{\sinh k_s}{\sinh k_d} x_{Gim} - n_{y_{im}}) \right]^2 / 2} \\
\times \sum a_k \omega_k \frac{\sinh k_s}{\sinh k_d} \cos \theta_1 - \sum \omega_k x_{Gim} \sin \theta_1
\]

(46)

2.2.2.2. Mean drag force. As for the inertia force, there will be a contribution to the mean force when \(i=j\) in Eqs. (47) and (48). These equations for the mean force can be written as

\[
\overline{\bar{u}} = \sqrt{\frac{8}{\pi} \overline{\bar{w}} w_t} = \sqrt{\frac{8}{\pi} \sum \left[ (\omega_k a_k \frac{\cosh k_s}{\sinh k_d} x_{Gim} - n_{y_{im}}) \right]^2 / 2} \\
\times \sum a_k \omega_k \frac{\cosh k_s}{\sinh k_d} \sin \theta_1 - \sum \omega_k x_{Gim} \sin \theta_1
\]

(47)

2.3. Numerical integration approach

All the above equations are incorporated in a MATLAB program named ‘TRSPAR’ for calculating the wave forces. Newmark-beta integration scheme was adopted to solve the equations of motion

\[
[M] \frac{\partial^2 x}{\partial t^2} + [C] \frac{\partial x}{\partial t} + [K] [x] = [F(t)]
\]

(52)

where \([M]\) is made up of body mass and added mass components as given in Eq. (53) and \([\partial^2 x / \partial t^2]\) is the structural acceleration vector. The resultant force can be defined as

\[
[M] = \begin{bmatrix} m & 0 & 0 & m_{11} & m_{12} & m_{13} \\ 0 & m & 0 & m_{21} & m_{22} & m_{23} \\ 0 & 0 & l & m_{31} & m_{32} & m_{33} \\ \end{bmatrix}
\]

(53)

where \(m\) and \(l\) denote body mass and mass moment of inertia about the y-axis, respectively. The added mass is determined by integrating the added mass from the bottom of the structure/member to the instantaneous surface elevation.
mass forces and moments are as follows:

\[
m_{11} = \int_{n} \rho C_m A \cdot \hat{t} \cos \beta \cos \gamma \\
m_{12} = m_{21} = -\int_{n} \rho C_m A \cdot \hat{t} \sin \beta \cos \gamma \\
m_{13} = m_{31} = \int_{n} \rho C_m A_n \cdot \hat{n} \cos \beta \\
m_{22} = \int_{n} \rho C_m A \cdot \hat{t} \sin \beta \sin \gamma \\
m_{23} = m_{32} = -\int_{n} \rho C_m A_n \cdot \hat{n} \sin \beta \\
m_{33} = \int_{n} \rho C_m A_n^2 \cdot \hat{n}
\]

(54)

\[\{C\} \{\dot{\xi}_C\} = \begin{bmatrix} c_{11} & 0 & 0 \\
0 & c_{22} & 0 \\
0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} \xi_1 \\
\xi_2 \\
\xi_3 \end{bmatrix}
\]

(55)

Damping sources can be identified as structural, radiation, wave drift and mooring lines. The significant contribution comes from the drag force on the truss spar when using the Morison equation as mentioned in Section 2.2.2. The structure damping of the system is small compared to the other forces. That is due to the low natural frequencies of the system in all degrees of freedom. The computations of the structure damping elements are as follows:

\[
c_{11} = 2 \tilde{\zeta}_s \omega_n m \\
c_{22} = 2 \tilde{\zeta}_h \omega_n h m \\
c_{33} = 2 \tilde{\zeta}_p \omega_n p l
\]

(56)

where the subscripts \(s\), \(h\) and \(p\) stand for surge, heave and pitch, respectively, \(\tilde{\zeta}\) is the damping ratio in the specified direction of motion and \(\omega_n\) is the natural frequency of the system in the specified degree of freedom. Wave drift damping can be added to the \(C\) matrix as

\[
\{C\} = \begin{bmatrix} c_{11} + B_{11sd} & 0 & -z_c \times B_{11sd} \\
0 & c_{22} & 0 \\
-z_c \times B_{11sd} & 0 & c_{33} + B_{11sd} \end{bmatrix}
\]

(57)

where \(z_c\) is \(z\)-coordinate of the centre of gravity.

\[
B_{11sd} = 2.6 \rho R_a^2 \omega (kR)^2, \text{ when } (kR) < 1 \\
= 2.6 \rho R_a^2 \omega, \text{ otherwise}
\]

(58)

where \((kR)\) is the diffraction parameter.

In addition to the aforementioned damping, heave plates greatly increase the heave added mass and viscous damping as follows:

\[
F_1 = \frac{1}{2} \rho U |U|^2 C_D + \rho \frac{\partial U}{\partial t} L^3 C_A
\]

(59)

where \(C_D\) and \(C_A\) are drag and added mass coefficients for the heave plates, respectively. \(U\) and \(\partial U/\partial t\) represent the velocity and acceleration, respectively, of the plate perpendicular to its plane.

\[\{K\} \{\dot{x}_C\} = K_{zd} + \begin{bmatrix} k_1 & 0 & k_2 \\
0 & k_1 & 0 \\
k_3 & 0 & k_2 \end{bmatrix} \begin{bmatrix} x_C \\
\dot{x}_C \\
\ddot{x}_C \end{bmatrix}
\]

(60)

where \(k_2 = \pi \omega (D/2)^2\), \(k_3 = \text{buoyancy force \times distance from G to B}\), \(k_4 = \text{horizontal spring stiffness}\), \(h_3 = \text{distance from G to fairlead}\). In general, \(k_4\) is a nonlinear function of the structure displacements. Thus the solution process involves updating the \(K\) matrix for each new displacement.

3. Numerical results and discussions

A numerical simulation for a typical truss spar, as shown in Fig. 4, with nine mooring lines was conducted. The physical characteristics of the structure and the characteristics of the mooring lines are summarized in Tables 1 and 2 respectively.

**Table 1**

<table>
<thead>
<tr>
<th>Physical characteristics of the truss spar.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>389.80 ton</td>
</tr>
<tr>
<td>Vertical centre of gravity (KG)</td>
</tr>
<tr>
<td>126.34 m</td>
</tr>
<tr>
<td>Buoyancy, basic</td>
</tr>
<tr>
<td>389.80 ton</td>
</tr>
<tr>
<td>Vertical centre of buoyancy (KB)</td>
</tr>
<tr>
<td>152.4 m</td>
</tr>
<tr>
<td>Radius of gyration for pitch</td>
</tr>
<tr>
<td>86.2 m</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Characteristics of mooring lines.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Size (m)</td>
</tr>
<tr>
<td>Length (m)</td>
</tr>
<tr>
<td>Wet weight (kg/m)</td>
</tr>
<tr>
<td>Eff. modulus EA (Kn)</td>
</tr>
<tr>
<td>Breaking strength (Kn)</td>
</tr>
</tbody>
</table>
3.1. *Quasi-static simulation*

A separate MATLAB program was developed to perform a quasi-static simulation for mooring line system by applying variable static forces at the fairlead. As a result, mooring stiffness curve was obtained, as shown in Fig. 5. The figure shows that the mooring line system provides nonlinear stiffness to the structure. Moreover, mooring lines restoring force caused by positive

![Fig. 5. Quasi-static simulation: restoring force vs. horizontal excursion.](image1)

![Fig. 6. Surge spectra: mean position (MP) + structure displacement (SD) + drag force (DF).](image2)

![Fig. 7. Surge spectra: MP + SD + DF + axial divergence (AD).](image3)
horizontal excursion, which was in the same direction of the waves, was higher than those due to negative surge motion particularly in relatively high horizontal transient response.

3.2. Time domain simulation

Numerical studies were made using TRSPAR which included almost all the nonlinear effects up to the second order. The wave series considered here was simulated from the 100-year JONSWAP spectrum with a significant wave height of 13 m and a peak wave period of 14 s. The individual effects of the nonlinear difference frequency wave forces were examined and compared in terms of response spectra.

3.2.1. Surge response

The results, which are shown in Figs. 6–10, represent five stages corresponding to the addition of individual nonlinear

![Fig. 8. Surge spectra: MP + SD + DF + AD + free surface fluctuation (FS).](image)

![Fig. 9. Surge spectra: MP + SD + DF + AD + FS + convective acceleration (CA).](image)

![Fig. 10. Surge spectra: MP + SD + DF + AD + FS + CA + temporal acceleration (TA).](image)
forces. It was observed that, in all figures, slowly varying surge responses were dominant. Moreover, the wave frequency surge responses did not change much throughout the addition process and hence the discussions are focused only on the second order responses. These low frequency surge responses were caused by the interactions between the wave components among the random wave.

Fig. 6 represents the first stage of the process, which combined the calculation of the wave forces at the mean position with the effect of the structure displacement and the linearized drag force, which contributes to the damping of the system. In Fig. 7, where the axial divergence was added, the response increased from $1.56 \times 10^5$ to $2.85 \times 10^5$ m$^2$ s. The addition of free surface fluctuation in Fig. 8 shows that the response greatly increased to...
8.45 \times 10^5 \text{ m}^2 \text{ s}. In Fig. 9, by adding the second order convective acceleration, the surge response reduced to 3.385 \times 10^5 \text{ m}^2 \text{ s}. This also agreed with Eqs. (36), (37), (39) and (40), which state that the free surface and convective acceleration effects act opposite to each other. Moreover, the effect of the free surface was more than the effect of the convective acceleration. Finally the second order temporal acceleration was added, as shown in Fig. 10. Also the surge spectra increased and reached 4.5 \times 10^5 \text{ m}^2 \text{ s}.

3.2.2. Pitch response

The results for pitch response are shown in Figs. 11–15 in a way similar to the one used for the study with surge motion. As in surge response, slowly varying pitch responses were dominant. Also, the wave frequency pitch responses did not vary much during the addition process, hence the discussion will focus only on the second order responses.

As shown in Fig. 11, pitch response due to the combined effect of calculation of the wave forces at the mean position with the effect of the structure displacement and the linearized drag force reached 2.1 \times 10^4 \text{ deg}^2 \text{ s}. In Fig. 12, pitch response was increased to 5.2 \times 10^4 \text{ deg}^2 \text{ s} because of adding the axial divergence. Similar to surge motion, adding the free surface fluctuation, as shown in Fig. 13, magnify pitch response which reached 1.98 \times 10^5 \text{ deg}^2 \text{ s}. Convective acceleration in Fig. 14 performs as in surge motion and reduced the pitch response to 1.12 \times 10^5 \text{ deg}^2 \text{ s}. Finally the second order temporal acceleration was added, as shown in Fig. 15. Also the pitch spectra increased and reached 1.588 \times 10^5 \text{ deg}^2 \text{ s}.

4. Comparison with experimental data

To validate the developed numerical scheme, a comparison between numerical results and the corresponding experimental data was conducted for the typical truss spar shown in Fig. 16. The experimental studies, which were carried out at the FORCE Technology basin are related to Kikeh project dry tree unit (Technip Document, 2005). The model scale used was 1–60.

Fig. 17 shows the truss spar model at FORCE Technology basin. The total weight of the prototype was 54 \times 10^3 \text{ ton}. The truss consisted of three bays and a box like soft tank with a length of 10.7 m. There were two heave plates with openings for the risers and strakes with cut-outs for mooring lines and transportation. The prototype mooring system consisted of ten mooring lines in four groups. The mooring coordinate system is shown in Fig. 18 and the fairleads are located 5.5 m above the hard tank bottom. Mooring line characteristics and corresponding prototype pretension are given in Tables 3 and 4, respectively. In model test, four mooring lines consisting of length of chain and springs were used. Each mooring line represents the stiffness of a corresponding mooring group in the prototype.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig14.png}
\caption{Pitch spectra: MP + SD + DF + AD + FS + CA.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig15.png}
\caption{Pitch spectra: MP + SD + DF + AD + FS + CA + TA.}
\end{figure}
Figs. 19–21 show the comparison between the measured Fourier amplitude spectra (FAS) of the model tests and those predicted by TRSPAR, which included all the nonlinear second order slowly varying wave forces. These results are for surge, heave and pitch responses due to random wave with a significant wave height of 6 m and a peak period of 11.7 s. In general, there are good agreement between the predicted and the corresponding measured results in both regions, namely wave frequency and low frequency regions. The fairly small differences between these results might be caused by the strakes and risers contributions which were modelled in the model test, but not in TRSPAR.

5. Conclusions

According to the numerical study performed on the truss spar platform, the following conclusions can be drawn:

1. An efficient methodology was developed to conduct a dynamic analysis for floating offshore structures. This methodology takes care of the inclination of the structure during the dynamic analysis using the extension of the Morison equation for an inclined cylinder for predicting the wave forces and hyperbolic extrapolation for calculating the wave kinematics.
2. Due to the lack of detailed analysis procedure in literature, the equations used for determine the slow varying difference frequency wave forces are derived and presented in this paper.

---

Table 3

<table>
<thead>
<tr>
<th>Spar mooring Platform/anchor chain</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>R4 Studless</td>
</tr>
<tr>
<td>Diameter</td>
<td>12.4 cm</td>
</tr>
<tr>
<td>Wet weight</td>
<td>277 Kg/m</td>
</tr>
<tr>
<td>Wire rope</td>
<td>Spiral strand</td>
</tr>
<tr>
<td>Diameter</td>
<td>10.8 cm</td>
</tr>
<tr>
<td>Wet weight</td>
<td>48.2 Kg/m</td>
</tr>
</tbody>
</table>
3. A MATLAB program namely 'TRSPAR', which based upon the above methodology, was developed to determine the dynamic responses of truss spar platform. This numerical model includes almost all the nonlinear effects up to the second order.

4. A separate MATLAB program using quasi-static analysis was developed to predict the stiffness of mooring lines. From the results, mooring line system shows nonlinear behaviour. It was shown that the restoring force caused by positive horizontal excursion was higher than those due to negative surge motion particularly in relatively high surge motion.

5. Surge and pitch motion results showed that the second order responses were dominant and the wave frequency motions hardly changed throughout the study.

6. Except for the convective acceleration, all other second order effects contributed positively to the surge and pitch motions. It was shown that the magnitude of the convective acceleration is almost equal to the magnitude of the free surface fluctuation.

7. A model test correlation with TRSPAR has been made. The measured results agree with the corresponding predictions. This implies that the developed numerical scheme is capable
of accurately predicting the dynamic responses of truss spar platforms.

References


