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DYNAMIC RESPONSE PREDICTION OF MOORED FLOATING STRUCTURES VIA TIME-VARYING TRANSFER FUNCTION

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ABSTRACT

The purpose of this paper is to propose an approach for dynamic response prediction of moored offshore structures via time-varying transfer function. By utilizing the time-varying autoregressive with exogenous input (TVARX) model, time-varying transfer function of an eight-column semi-submersible model can be generated. Input-output pair for transfer function generation is wave height as excitation input and surge motion as system response, obtained experimentally from a scaled 1:100 model of a prototype semi-submersible. Keeping the system the same set-up, a single time-varying transfer function generated from one wave spectrum can be used to predict other surge responses of the model under different peak frequencies of random wave. It is found out that the dynamic response predictions from the proposed approach have shown good agreement with the experimental results either in time or frequency domain.

KEYWORDS

Semi-submersible model, Surge motion response; Time-varying transfer function; TVARX model

INTRODUCTION

In offshore engineering field, model tests of floating structures either small or large scale tests have been constructed to study the dynamic response and taken as a mutual comparison for theoretical or numerical studies. Experimental investigations of such models are usually carried out in the wave tank where they are excited by regular or random wave in a particular frequency range. However, model testing across many wave frequencies would be time consuming and cumbersome. Dynamic response prediction of the model outside of the tested frequency range is much needed without conducting the experiment. It leads to a significant saving in test time and test cost. Moreover, error and repetition of the experiments could be avoided. This effort can be accomplished by acquiring the measured time series of wave height and motion responses in the wave tank. From the acquired data, empirical models could be derived to describe the dynamic behaviour of the system. If the interest is in the amplitude and statistics of the floating structures motion, the empirical models are usually based on nonparametric quantities, such as transfer function (TF) and response amplitude operator (RAOs) [1]. This model is also suitable to the design and stability analysis of motion control systems. The generated TF can be classified as wave-to-motion TF [2] and dynamic response prediction can be carried out. However, the TF must have the ability to capture the nonlinearity or nonstationarity of the system to create reliable dynamic response prediction. It certainly needs an accurate system identification method. Hence, this paper proposes an approach for dynamic response prediction via time-varying transfer function. Such a TF is expected to predict the dynamic response in a wide range of wave frequency. The method will be described in the next section. As study case, a scaled 1:100 model of a prototype semi-submersible as a moored offshore structure is used and dynamic response prediction in surge motion is carried out. This motion is chosen because second-order motion response is dominant in surge direction. However, the proposed method is also applicable for other motion responses.

TVARX MODEL

Relationship between a set of measured time series of wave height and motion response of structure is approached with a model structure via TVARX which is given by:

$$y(k)=\sum_{i=1}^{P} a_i(k)\,y(k-i)+\sum_{\ell=0}^{M} b_\ell(k)\,u(k-\ell)+e(k). \quad (1)$$

Notations $y(k-i)$ and $u(k-\ell)$ are delayed motion response and wave height variables, called regressors in discrete time index k, respectively. Notation P and M are number of respective delayed regressors which are the order of TVARX model. Values $a_i(k)$ and $b_\ell(k)$ are the TVARX coefficients which is time-variant. If the coefficients are time-invariant, then equation (1) is equivalent with the well-known ARX model. Model error is denoted with $e(k)$, which is Gaussian with zero mean and variance σ_e^2. From equation (1), the work is mostly concerned with the identification of coefficients $a_i(k)$ and $b_\ell(k)$. Equation (1) describes that the current motion responses depend on the summation between previous states of motion responses and wave height, together with current states of wave height. In order to estimate the TVARX coefficients, equation (1) can be rewritten in a discrete state-space form and expressed in equation (2a),

$$y(k)=x(k)L(k)+v(k) \quad (2a)$$

$$L(k)=AL(k-1)+w(k). \quad (2b)$$

Vector $x(k)$ contains the regressors, past values of measured motion responses and wave height $[y(k-1),\cdots,y(k-P),u(k),\cdots,u(k-M)]$, while vector $L(k)$ contains the TVARX coefficients. If time evolution of the coefficients is restricted to be linear and stochastic, then $L(k)$ is then expressed in equation (2b). Term A is the state transition matrix which will be restricted as an identity matrix, while $v(k)$ and $w(k)$ are the observation and state noise, respectively. Minimization error between the models's output in equation (1) and the measured data can be accomplished by adopting some numerical methods. Because the floating offshore structures are dynamic system with slow variation, adaptive methods can be utilized. For more detail about the adaptive methods, one may refer to [3].

ESTIMATION OF TRANSFER FUNCTION

After obtaining the coefficients, it can be converted into transfer function, either impulse response function (IRF) or frequency response function (FRF). Transfer function of the system in term of FRF can be estimated using equation (3). Terms in equation (3) are explained as follows: $a_i(k)$ and $b_\ell(k)$ are the i^{th}, ℓ^{th} elements of the estimated TVARX coefficients and ω is the observed frequency, respectively.

$$H\left(k,e^{j\omega}\right)=\frac{\sum_{\ell=0}^{M} b_\ell(k)e^{-j\omega\ell}}{1+\sum_{i=1}^{P} a_i(k)e^{-j\omega i}}. \quad (3)$$

Because term k indicates discrete time index, it leads to the condition that the estimated transfer function is time-varying transfer function (TVTF).

At this step, dynamic response prediction in frequency domain can be obtained by treating the TVTF as a linear filter. The relation is as follows:

$$S_o\left(k,e^{j\omega}\right)=H\left(k,e^{j\omega}\right)\cdot S_i\left(k,e^{j\omega}\right), \quad (4)$$

where $S_o\left(k,e^{j\omega}\right)$ is system's output spectrum (motion responses) and $S_i\left(k,e^{j\omega}\right)$ is system's input spectra (wave height) at particular frequency. Vice versa, response prediction in time domain $y(k)$ can be carried out by converting TVARX coefficients into IRF or simply converting equation (3) into time domain and convoluted with corresponding system input $u(k)$. Both domains will be presented in this paper.

To compare the prediction results with its respective measured time series qualitatively, the normalized mean square error (NMSE) is calculated as a statistical comparison. The NMSE value is expressed in equation (5),

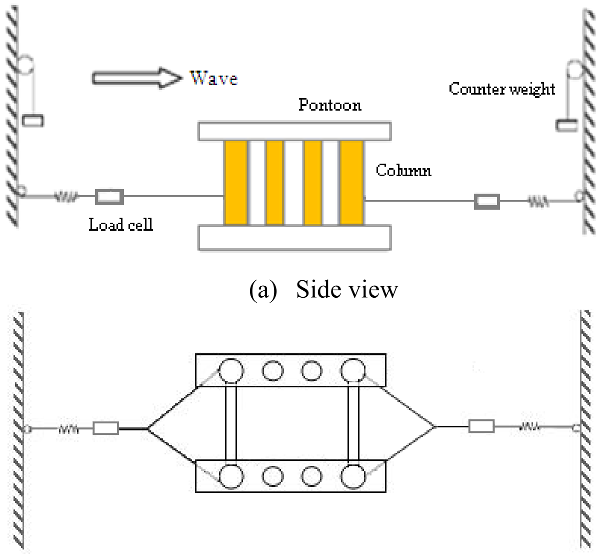
$$NMSE=\sqrt{\frac{\frac{1}{N}\sum_{k=1}^{N}\left\{\left(y_{pred}(k)-\bar{y}_{pred}(k)\right)\right\}-\left(y_{exp}(k)-\bar{y}_{exp}(k)\right)^2}{\frac{1}{N}\sum_{k=1}^{N}\left(y_{exp}(k)-\bar{y}_{exp}(k)\right)^2}} \quad (5)$$

where N is the data length, $y_{pred}(k)$ is the predicted time series, and $y_{exp}(k)$ is the measured time series.

EXPERIMENT

The method is applied to the case of an eight-column semi-submersible model. The experimental layout is shown in Figure 1. The model test was moored with two typical springs that attached with steel wires on fore and aft side of the model. The springs were soft linear springs connected to load cells mounted on the model. This arrangement was set up to measure the second-order motion response. This was made possible by the soft spring wire restraining system attached to the model [4].

The model was then tested in the wave tank of the offshore engineering laboratory, Universiti Teknologi PETRONAS. The wave tank has 22 m length, 10 m width and 1.5 m depth. The JONSWAP spectrum was used to generate the random wave, where the test was conducted for six minutes duration and the model was subjected a unidirectional random wave in head seas orientation. Wave probes were used to measure the wave height, while the motion responses of the semi-submersible model in all six degrees of freedom were recorded by optical tracking system. The test took only few minutes, hence the effects of wave wall reflection were not considered. The progressive mesh beach systems also minimized the interference from reflected waves during tests. The data had been sampled at sampling frequency 100 Hz.



(a) Side view

(a) Plane view

Figure 1: Layout of model test

Initial experiment was conducted by subjecting the model to a unidirectional random wave with significant wave height, $H_s = 0.1 m$ and peak frequency, $f_P = 0.61 Hz$ in head seas orientation. Later on, this random wave is called IRW-1 for convenience. This measured wave height and surge response are selected to be used as model transfer function for dynamic response prediction to fulfill the purpose of this paper. Pre-processing is carried out for both measured time series to remove the noise embedded during the measurement. The empirical mode decomposition (EMD) through Hilbert-Huang transformation is employed for this purpose [5]. It is found out that filtering with EMD method produces similar result with low pass filter (LPF) method with cut-off frequency 0.01 Hz. The results are displayed in Figure 2. It is noted that no detrend or demean process carried out for both time series in order to retain the nonlinearity of the system. From the figure, it can be observed that the wave height seems to be linear random waves because its amplitudes appear to be symmetrical around its mean value. Conversely, since the surge response is not symmetrical and seems to be shifted upward, nonlinearity is present inside.

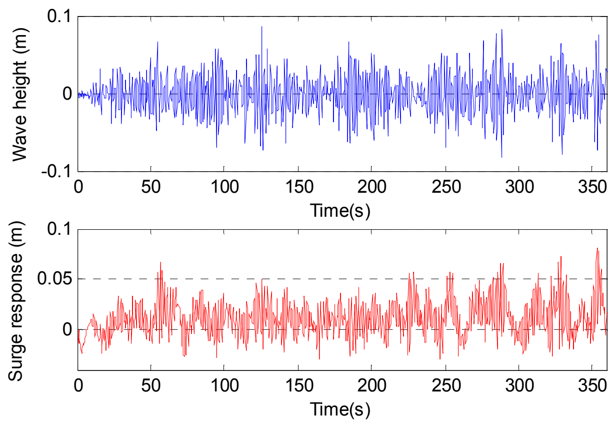


Figure 2: Filtered wave height and surge response of IRW-1

RESULTS

By manipulating equation (1) and (2) with Kalman smoother algorithm [3], TVARX coefficients, $a_i(k)$ and $b_\ell(k)$ for selected model order can be calculated. The coefficients are presented in Figure 3.

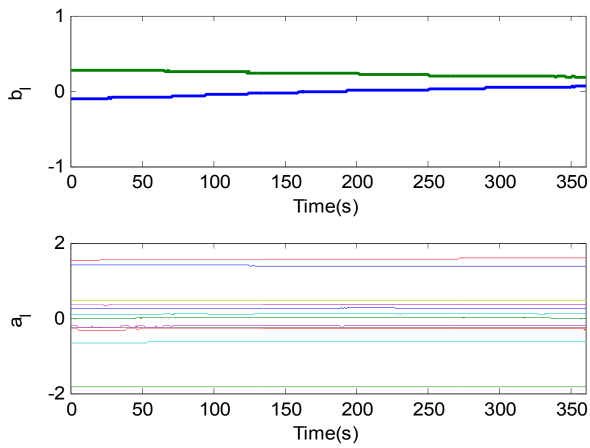


Figure 3: TVARX coefficients for IRW-1

It can be seen that all coefficients evolve over time as an indication that TVARX is an extended form of ARX where the coefficients are time-variant. Those time-varying coefficients are expected to capture the dynamic feature of the model test. Selected model order, P and M can be estimated by adopting some methods such Akaike information criterion (AIC), Bayesian information criterion (BIC) and the others.

The model order estimation is achieved in two phases:

i) By setting up TVARX$(P,2)$, where $(P = 2,\cdots,20)$, model order P is estimated by setting $M = 2$. The proper P order is selected based on minimization of AIC, BIC and other criterion from Figure 4(a).

ii) Following the P order selection, the M order is determined in a similar manner.

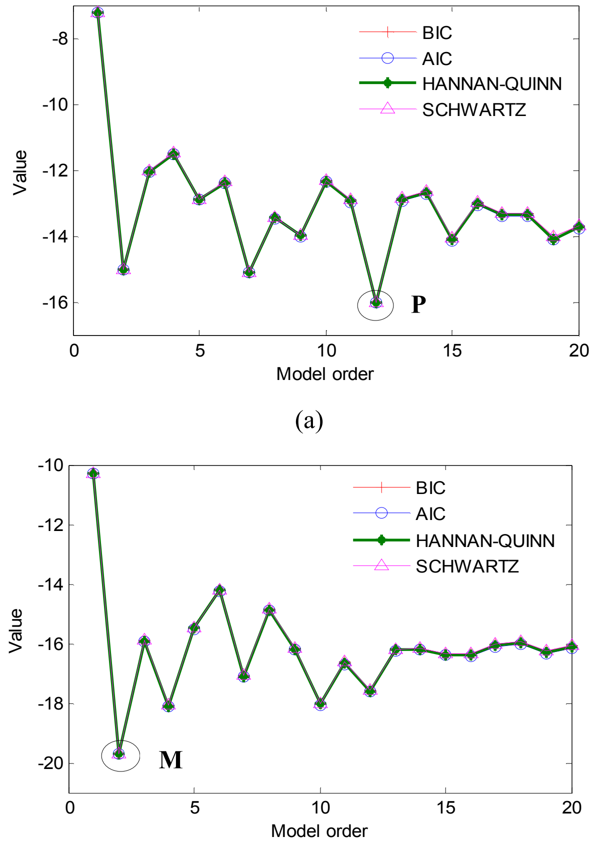


Figure 4: Model order for IRW-1

Overall, it may be observed from Figure 4, that the TVARX model of order (12,2) is optimum for transfer function estimation.

The coefficients are then plugged into the original equation (1) to see the prediction result. Prediction results are then verified with experiment result from the wave tank and presented in the top of the panel of Figure 5. Bottom of the panel is the residuals or prediction error. From those figures, it can be seen that the prediction error is relatively small, meaning that TVARX model can predict the surge response accurately. Further, to obtain a relationship between wave height and surge response in time frequency domain, the coefficients are also plugged into equation (3), the result is displayed in Figure 6 in term of transfer function. From Figure 6, it can be noticed that transfer function of surge response indicates significant response in two principal frequency peaks. The first peak is in the low frequency region at around 0.05 Hz, approximately 0.07 Hz, which corresponds to the surge natural frequency of the semi-submersible model. This frequency may be called as the resonant low frequency response (LF). The second peak is around 0.61 Hz, corresponds to the same frequency exist in the random wave spectrum. This frequency is known as incident wave frequency response (WF). Interpretation of LF and WF may be referred to Figure 7.

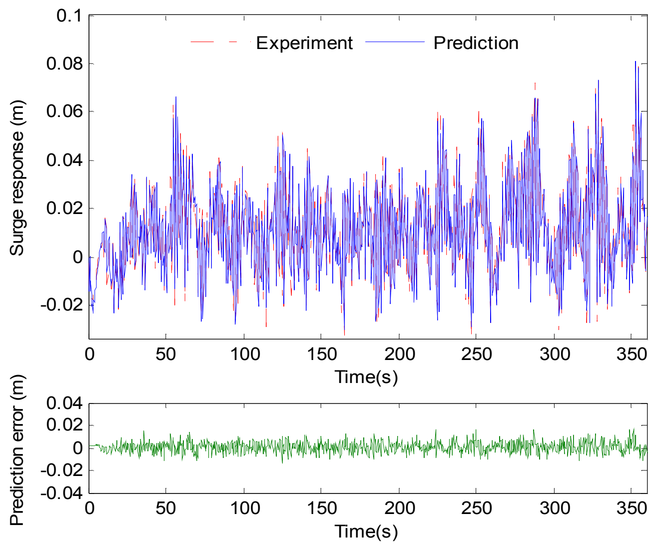


Figure 5: Surge response time series for IRW-1

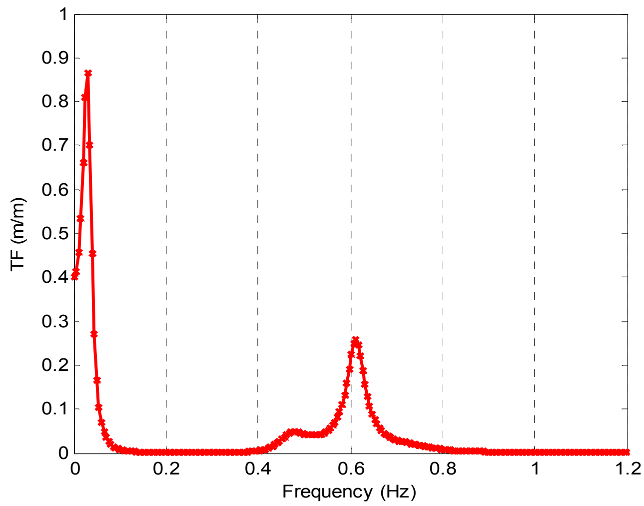


Figure 6: Transfer function for IRW-1

In Figure 7, WF is existed in the wave spectrum; it indicates a linear response. In addition, LF is not existed in the wave spectrum, which suggests a nonlinear response for the model. It implies that there exists a nonlinear effect in term of second-order force (wave drift force), which is proportional to the square of the wave height, exerted on the semi-submersible model. These results are also found by many prior researches, for example [6]-[9]. This nonlinear dynamic can be captured by the transfer function generated from TVARX model. However, to capture the nonlinearity of the semi-submersible model sharply, TVARX model must have higher order model. If the model order is lower than the optimum model order shown in the Figure 3, then the resonant low frequency motion doesn't exist in the transfer function estimate.

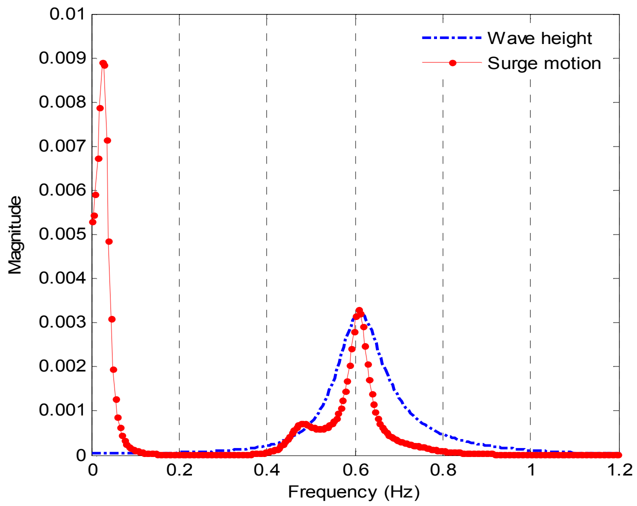


Figure 7: Spectral estimate of IRW-1

Another perspective may be drawn from the coefficients by manipulating the equation (3). By factoring the equation (3), zeros and poles of the system can be obtained. The TF produces one zero and twelve complex poles and plotted in complex plane as shown in Figure 8.

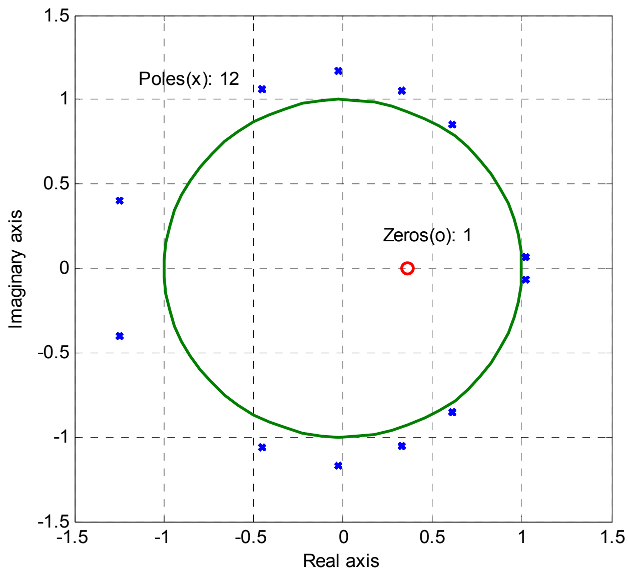


Figure 8: Plot of poles and zeros for IRW-1

The figure describes that the system is poles dominant, where all the poles spread in the complex plane, either in the left or right plane. They all locate outside of the unit circle periphery. The poles are also complex-conjugate pair, represents oscillating behaviour mode. Because the stability is restricted on the poles, not the zeros, then the figure also indicates that the semi-submersible model is unstable system, which is a significant dynamic characteristic of uncontrolled moored floating offshore structures. This oscillatory motion is imparted to the semi-submersible model due to fluid-structure interaction.

Based on the results above, the application of the proposed method in dynamic response prediction is tested in three sets of random wave as shown in Table 1. The wave frequency and the wave heights are taken based upon the limitation of the wave tank and wave maker. By taking the transfer function from Figure 6 as a model transfer function, surge response in time and frequency domain can be predicted.

Table 1: Random wave parameters

Data	Significant Height (m)	Peak Frequency (Hz)
IRW-2	0.06	0.83
IRW-3	0.08	0.77
IRW-4	0.09	0.71

By utilizing equation (4), response prediction is carried out for every data in Table 1. It can be simply calculated by replacing $H\left(k, e^{j\omega}\right)$ with the model TF as per Figure 6 and $S_i\left(k, e^{j\omega}\right)$ with IRW-2, IRW-3 and IRW-4, respectively. Surge response in frequency domain can be obtained. As a first prediction, surge response spectrum under IRW-2 data is presented in Figure 9. It is clearly observed that predicted surge response spectrum is in good agreement with the experimental result. Peak either in LF or WF is overlapped each other. There is no significant difference in magnitude for both spectrums.

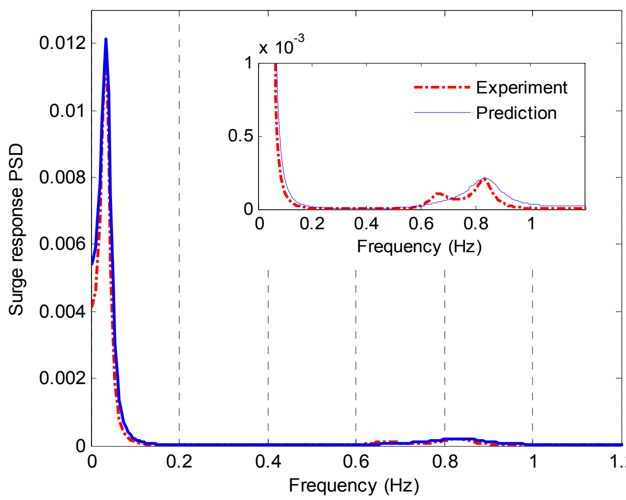


Figure 9: Surge response spectrum for IRW-2

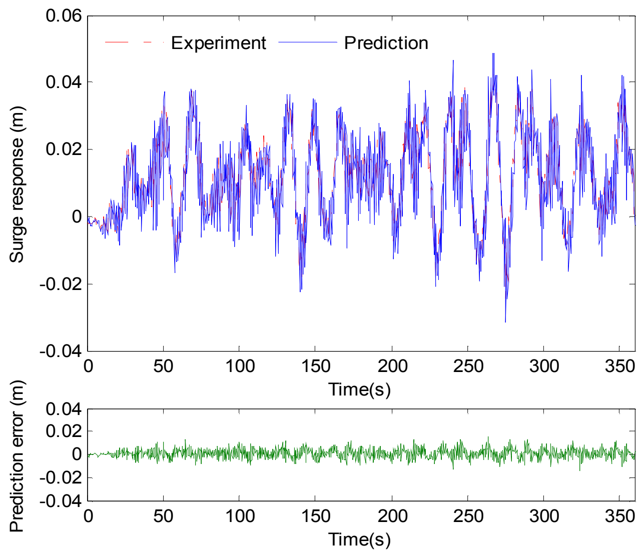


Figure 10: Surge response time series for IRW-2

By converting the result from Figure 9 into time domain, time series of surge response can be obtained. Prediction results are then verified with experiment result from the wave tank and presented in Figure 10. Prediction result is also in good agreement as frequency domain. Next prediction is carried out for IRW-3 data. Similar procedure with IRW-2 data, surge response spectrum is predicted and displayed in Figure 11.

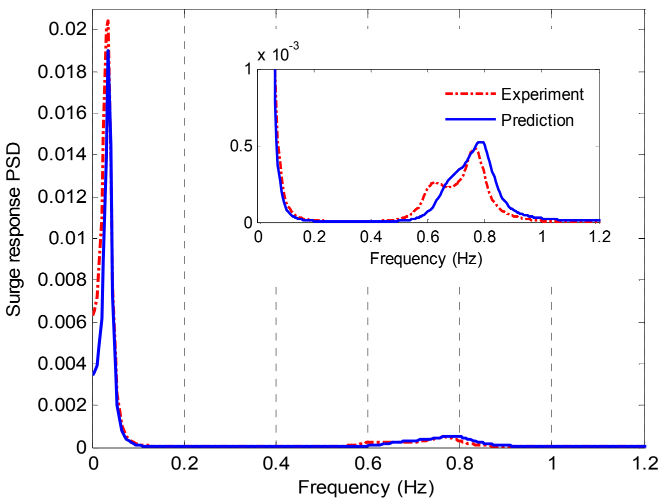


Figure 11: Surge response spectrum for IRW-3

In Figure 11, peak frequency in LF region around 0.05 Hz and WF region around 0.78 Hz is clearly shown by predicted result. Discrepancy is noticed in the magnitude in LF region, where prediction result is lower around 9.8% than experimental results. Further, prediction result produces fewer ripples in LF region than experimental result.

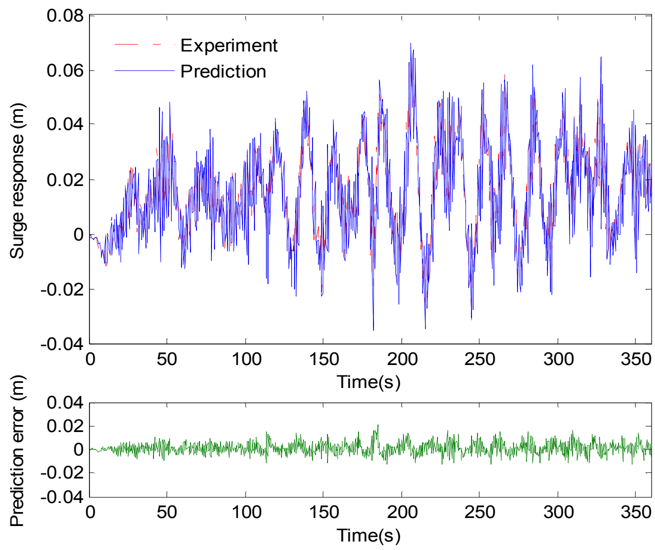


Figure 12: Surge response time series for IRW-3

Last, prediction is carried out for IRW-4, where the wave frequency is 0.71 Hz. Discrepancy is observed the magnitude either in LF or WF region, where the magnitude for prediction results is bigger around 12.5% in LF region and 6,3% in WF region than the experiment, respectively. It also can be noticed that ripple is more apparent in the prediction result than experimental result.

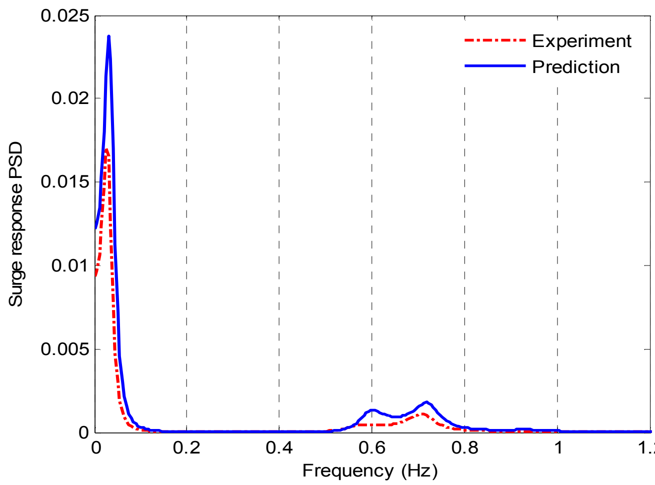


Figure 13: Surge response spectrum for IRW-4

Also, from all time series of surge response obtained from prediction, it can be seen that the prediction error in the bottom panel of each figure are relatively small, meaning that TVARX model can predict the surge response well. To compare the prediction results with its respective measured time series qualitatively, the normalized mean square error (NMSE) is calculated as a statistical comparison. The NMSE value is shown in Table 2.

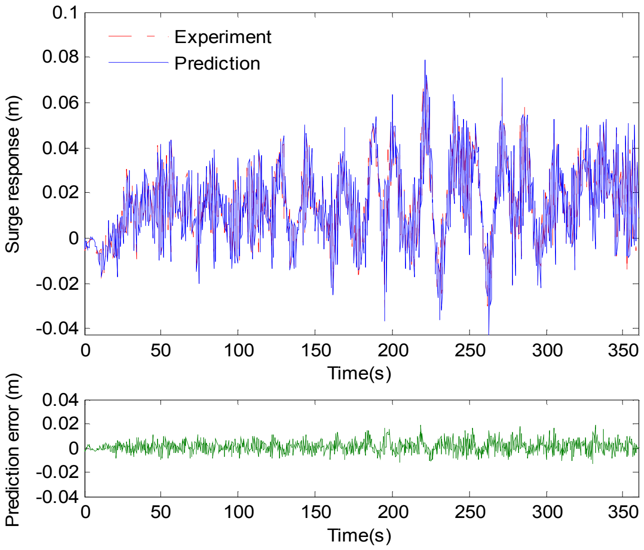


Figure 14: Surge response time series for IRW-4

Table 2: NMSE value

Data	NMSE
IRW-1	0.2692
IRW-2	0.3101
IRW-3	0.3044
IRW-4	0.2948

NMSE values in Table 2 shows a trend that slight degradation occurs when dynamic response prediction is carried out into higher wave frequency. However, it is still acceptable since LF and WF region can be identified clearly from all tested wave frequency range.

CONCLUSIONS

In this paper, the application of TVARX model for dynamic response prediction is carried out. The high correspondence between the predicted and actual surge response is achieved either in time or frequency domain. It has been shown that the time-varying transfer function obtained from TVARX model can be used for dynamic response prediction of the model test accurately. However, to get such a TF, higher model order is required. In this model test, optimum model order is found (12,2) to capture the nonlinearity of semi-submersible model. By having an accurate empirical model of the system in term of TF, prediction of a moored floating structure's dynamic can be carried out across many wave frequencies in a most efficient manner. It leads to a significant saving in test time and test cost. The method is also potential for analyzing the typical structure's current state of health for vibration offshore monitoring in full-scale application.

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