



# Elementary Lecture on the Pressure Drop Due to Friction in Single and Two-phase Flows

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URAL Federal University – Ekaterinburg – Russia  
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# OUTLINE

- **Definition of pressure drop ( $\Delta p$ ) /friction factor ( $f$ ).**
- **Basic estimation of  $\Delta p$  &  $f$  in single phase flow.**
- **Performance of drilling mud in well drilling.**
- **Cases of Non Newtonian fluids.**
- **Cases of Two phase flow (Newtonian/ non-Newtonian)**
- **Multi layer 2 phase flows.**
- **Conclusions & recommendations**

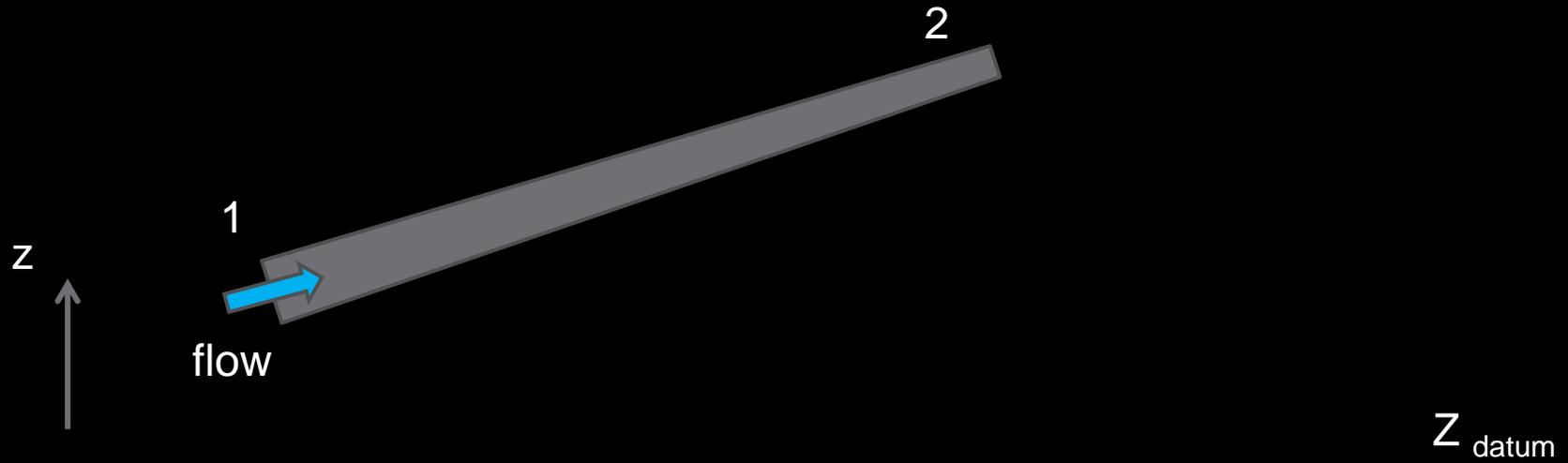
# INTRODUCTION

- Oil and gas pipelines play a critical role in delivering the energy resources needed to power communities around the world.
- According to US (DOT)- more than 2.5 million miles of pipelines enough to circle the earth approximately 100 times.
- Deliver oil and gas to homes and businesses.
- Usually involved multiphase flow such as (oil/water, gas and sand mixtures).



# EXERCISE

Write the flow equation between 1 & 2 in your piece of paper



# FLOW EQUATIONS

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

head

$$p_1 + \frac{v_1^2}{2} \rho + z_1 \rho g = p_2 + \frac{v_2^2}{2} \rho g + z_2 \rho g$$

pressure

Incompressible,  
frictionless

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_l$$

head

$$p_1 + \frac{v_1^2}{2} \rho + z_1 \rho g = p_2 + \frac{v_2^2}{2} \rho g + z_2 \rho g + \Delta p$$

pressure

Incompressible,  
friction due to viscosity

$$\frac{\Delta p}{\rho g} = h_l$$

# FLOW EQUATIONS

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$p_1 + \frac{v_1^2}{2} \rho + z_1 \rho g = p_2 + \frac{v_2^2}{2} \rho g + z_2 \rho g$$

**Bernoulli Eqn.**

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_l$$

$$p_1 + \frac{v_1^2}{2} \rho + z_1 \rho g = p_2 + \frac{v_2^2}{2} \rho g + z_2 \rho g + \Delta p$$

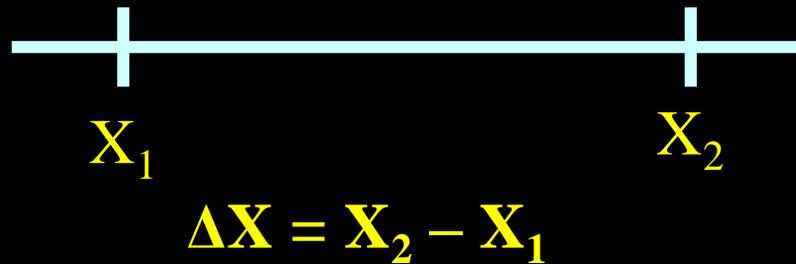
$$\frac{\Delta p}{\rho g} = h_l$$

**Energy Eqn.**

ity

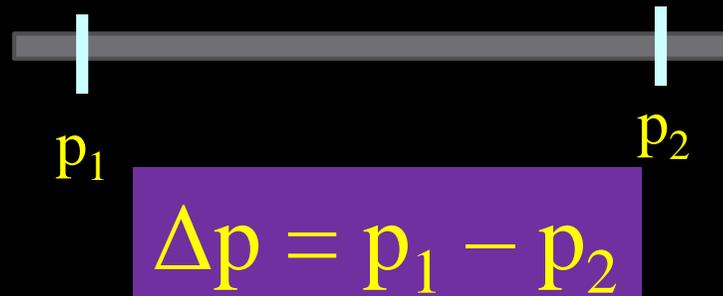
# BASIC DEFINITIONS

$\Delta$  is usually = state 2 – state 1



For the pressure drop in fluid flow

$\Delta p$  is usually = Pressure 1 – Pressure 2



$$\frac{\Delta p}{\rho g} = h_l$$

# FRICTION FACTOR SINGLE PHASE

- Circular , non circular pipes
- Laminar , turbulent flow
- Smooth or rough
- Horizontal, inclined, vertical

$L$  = pipe length

$D_H$  = 4 cross section area/ perimeter

$v$  = mean velocity

$$\frac{\Delta p}{\rho g} = h_l = f \frac{L}{D_H} \times \frac{v^2}{2g}$$

# FRICTION FACTOR SINGLE PHASE

$$\frac{\Delta p}{\rho g} = h_l = f \frac{L}{D_H} \times \frac{v^2}{2g}$$

Laminar or turbulent flow?

**Laminar:**

$$f = \frac{64}{Re}$$

**Turbulent:**

$$f = f(Re, \varepsilon)$$

# FRICTION FACTOR SINGLE PHASE

$$\frac{1}{\sqrt{f}} = -20 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Colebrook Eqn.  
Smooth pipe , or rough pipe

1. Smooth pipe,  $\varepsilon = 0$

$$\frac{1}{\sqrt{f}} = -20 \log\left(\frac{2.51}{Re\sqrt{f}}\right)$$

Prandtl Eqn.

2. Fully rough zone,  $Re = \infty$

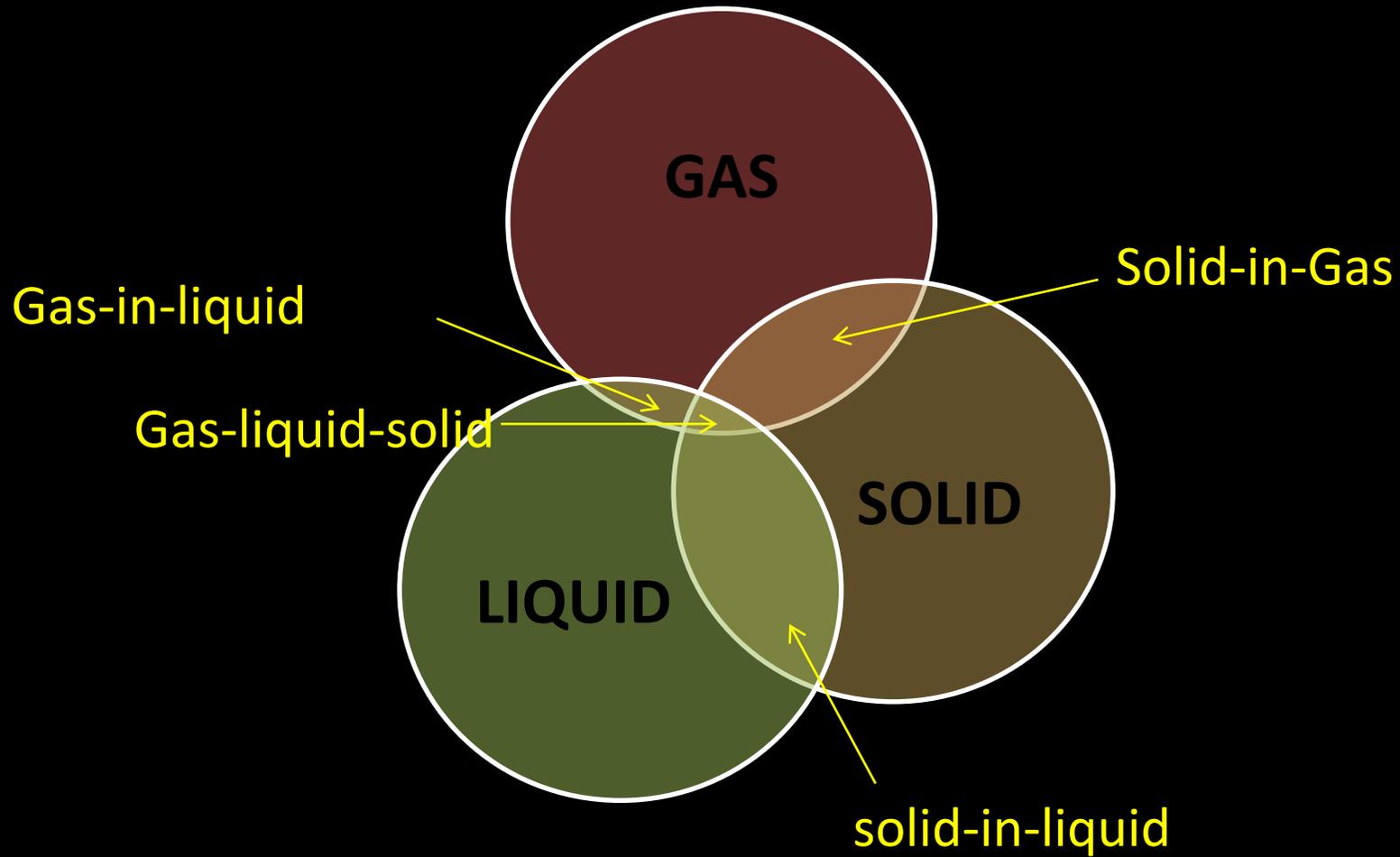
$$\frac{1}{\sqrt{f}} = -20 \log\left(\frac{\varepsilon/D}{3.7}\right)$$

Von Karman Eqn.

**The eqn. is implicit.....**  
and can converge very fast  
Using Haaland Eqn. as first guess

$$\frac{1}{\sqrt{f}} = -1.8 \log\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$$

# BASICS OF TWO PHASE FLOW



# BASICS OF TWO PHASE FLOW

- A phase is simply one of the state of matter and can be either a **gas** , a **liquid** or a **solid**.
- Multiphase flow is a simultaneous flow of several phases . Two-phase flow is the simplest case of multiphase flow .
- Gas - liquid mixtures are referred to as two-phase two - component flow,
- while as liquid - vapor mixture referred to as two-phase single - component mixture.

# BASICS OF TWO PHASE FLOW

- **Steam generators and condensers, steam turbines (Power Plants).**
- **Refrigeration .**
- **Coal & Co-fired furnaces .**
- **Fluidized bed reactors .**
- **Liquid sprays .**
- **Separation of contaminants from a carrier fluid .**
- **Transportation of O & G.**
- **Drilling**

# TWO PHASE DRILLING & CUTTING TRANSPORT

**Horizontal or vertical or inclined ?**

**Newtonian or non-Newtonian ?**

**One, two, three layers ?**

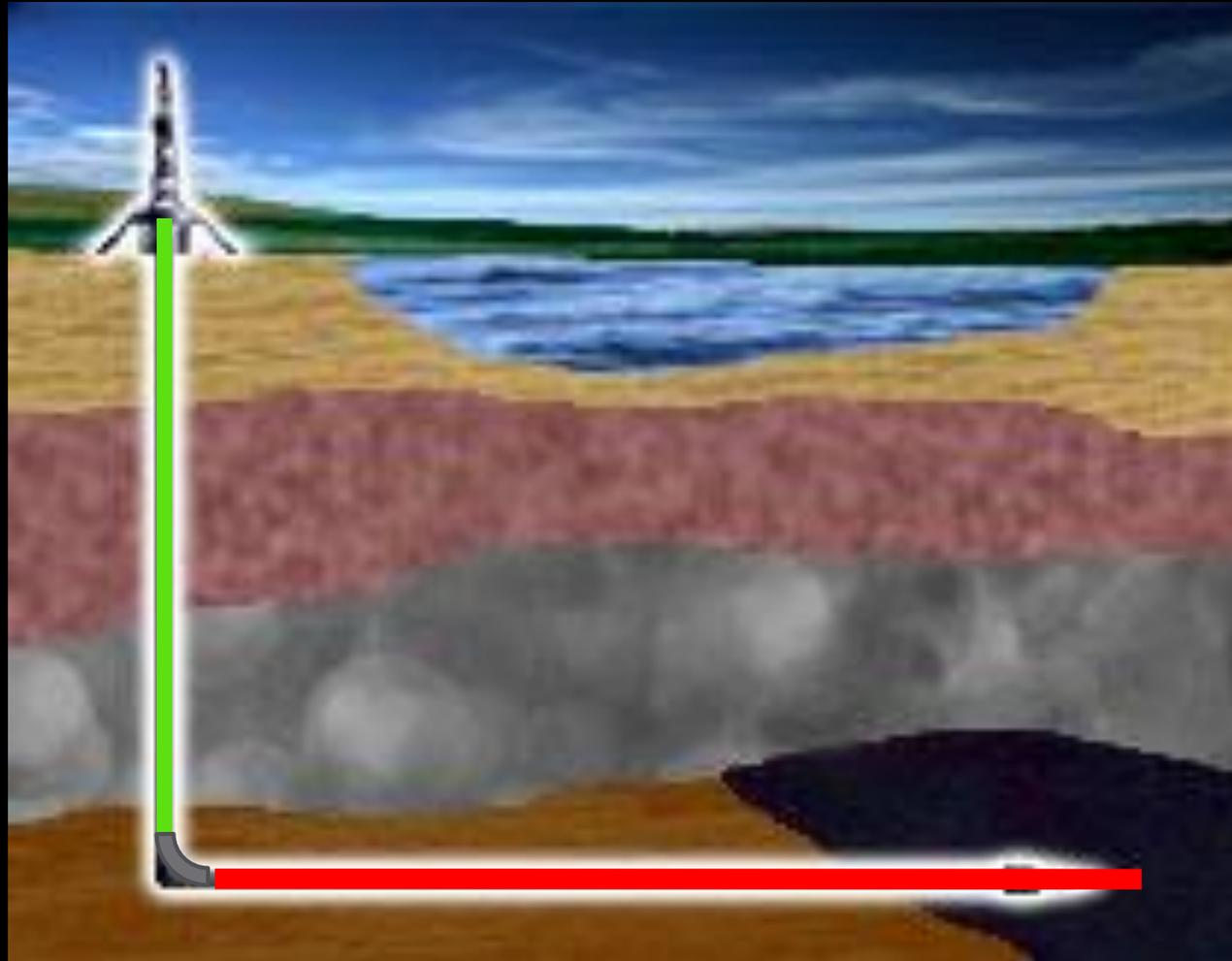
**Pipe or annular flow ?**

# TWO PHASE DRILLING & CUTTING TRANSPORT

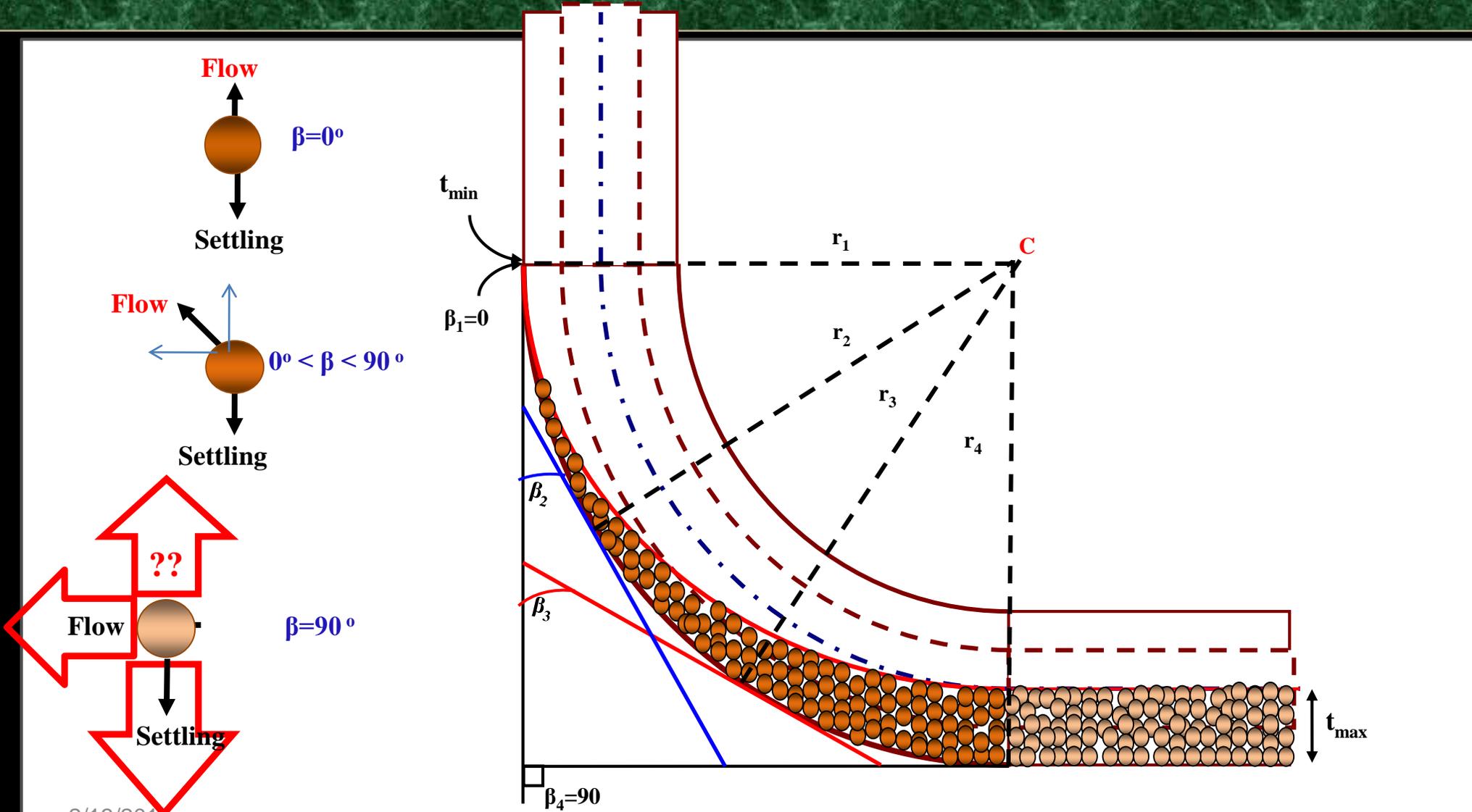
**Vertical**  
 $\beta = 0^\circ$

**Inclined**  
 $0^\circ < \beta < 90^\circ$

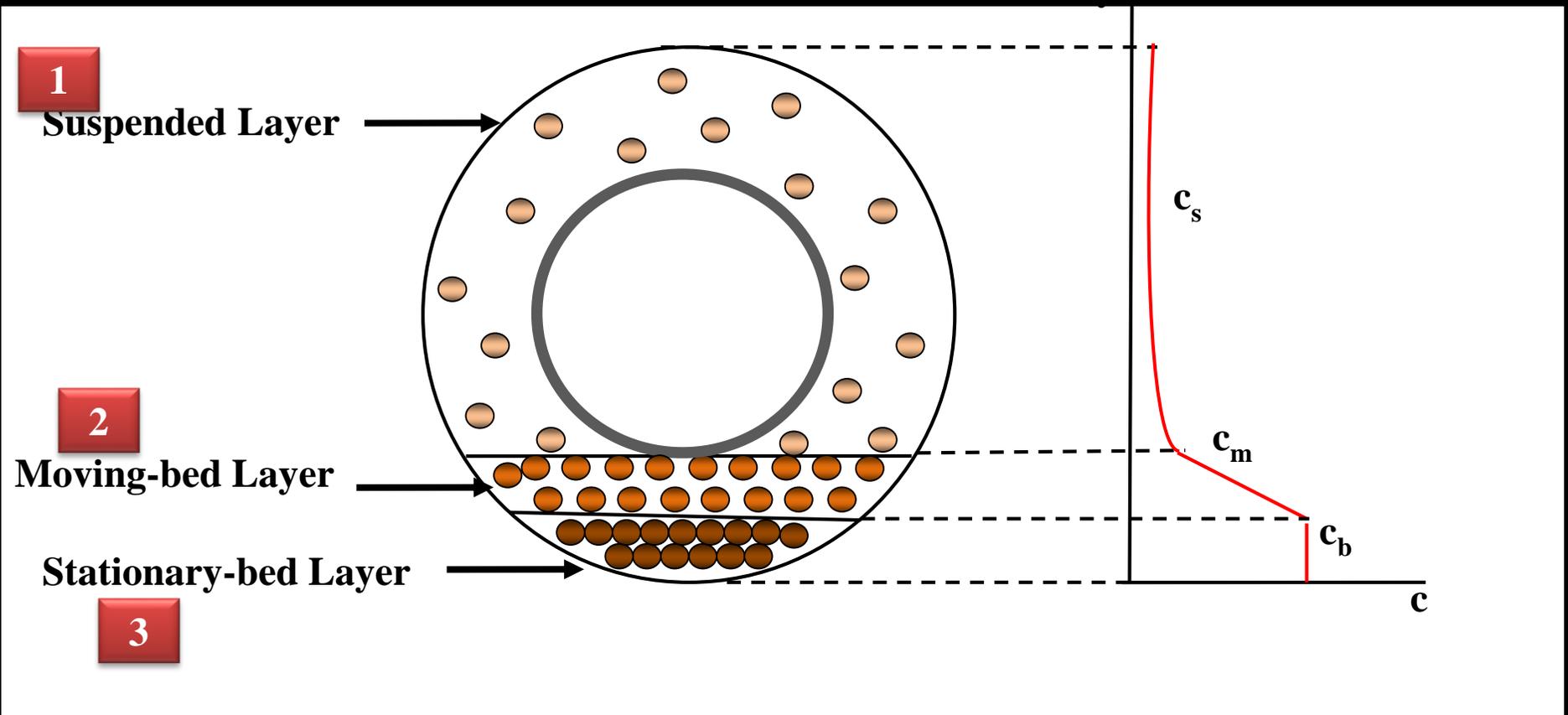
**Horizontal**  
 $\beta = 90^\circ$



# TWO PHASE DRILLING & CUTTING TRANSPORT



# TWO PHASE DRILLING & CUTTING TRANSPORT

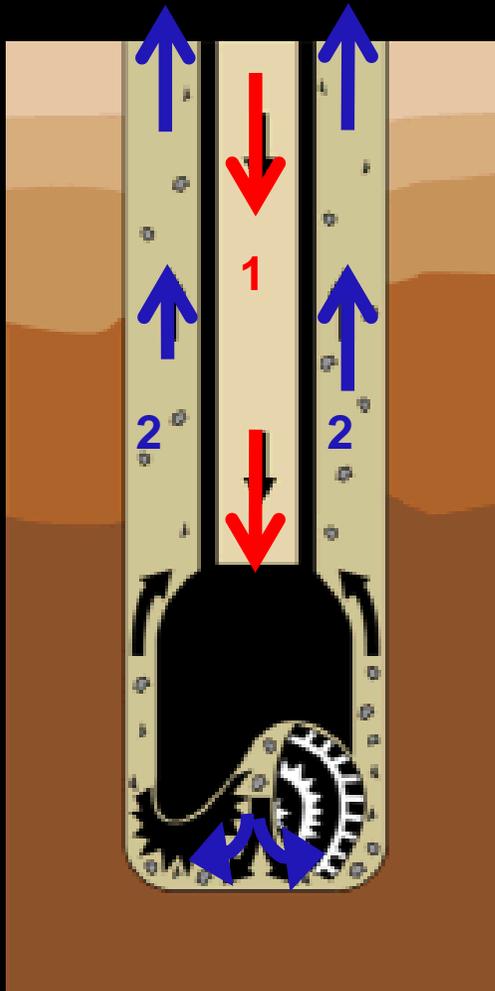


# TWO PHASE DRILLING & CUTTING TRANSPORT

## Two-phase pressure drop relationships- adiabatic

- Empirical correlation based on the homogeneous model
- Empirical correlation based on the two-phase friction multiplier concept
- Direct empirical models
- Flow pattern specific models

# TWO PHASE DRILLING & CUTTING TRANSPORT



## Homogenous vertical

Newtonian

**NOTICE:**

- 1.** Injected as pipe flow, single phase
- 2.** Extracted as annular flow, two phase

Drilling mud is non-Newtonian

Cutting Particles are non-spherical

# TWO PHASE DRILLING & CUTTING TRANSPORT

## 1. Non-Newtonian, single phase

Added complication, where the fluid viscosity is not constant.

Viscosity model is required

Sometime, experimental measurement are essential to evaluate the viscosity parameters.

$K$  = consistency index

$n$  = index behavior

$$\mu = K \left( du / dy \right)^{(n-1)}$$

$$Re = \frac{8\rho U^{(2-n)} D_h^n}{K} \left( \frac{6n+2}{n} \right)^{-n}$$

$$\frac{1}{\sqrt{f}} = -20 \log \left( \frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

# TWO PHASE DRILLING & CUTTING TRANSPORT

## 2. Homogeneous model , two phase

Suitable average properties are determined and the mixture is treated as a single fluid

- average values of the properties for both phases .
- e.g., for suspension, foam, mist, dispersed, bubbly flows.
- no detail of the flow is considered

$$\rho_{2p} = \rho_L(1 - \alpha) + \rho_G\alpha$$

$$\mu_{2p} = \alpha\mu_G + (1 - \alpha)\mu_L$$

$$\frac{1}{\sqrt{f}} = -20 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

# TWO PHASE SOLID-IN-LIQUID

## 3- Homogeneous , two phase, Non-Newtonian .

The case of cutting transport from the downhole to the surface

$$\rho_{2p} = \rho_L(1 - \alpha) + \rho_G\alpha$$

~~$$\mu_{2p} = \alpha\mu_G + (1 - \alpha)\mu_L$$~~

$$\mu_{2p} = K_{2p} \left( du / dy \right)^{(n-1)}$$

$K_{2p}$  is the consistency index of the mixture  
 $n$  is the index behavior

$$K_p = K(1 + 2.5c_p)$$

## TWO PHASE SOLID-IN-LIQUID

$$\text{Re}_{2p} = \frac{8\rho_{2p} U_{2p}^{(2-n)} D_h^n}{K_{2p}} \left( \frac{6n+2}{n} \right)^{-n}$$

$$\rho_{2p} = \rho_L(1-\alpha) + \rho_G\alpha$$

$$K_{2p} = K(1 + 2.5\alpha)$$

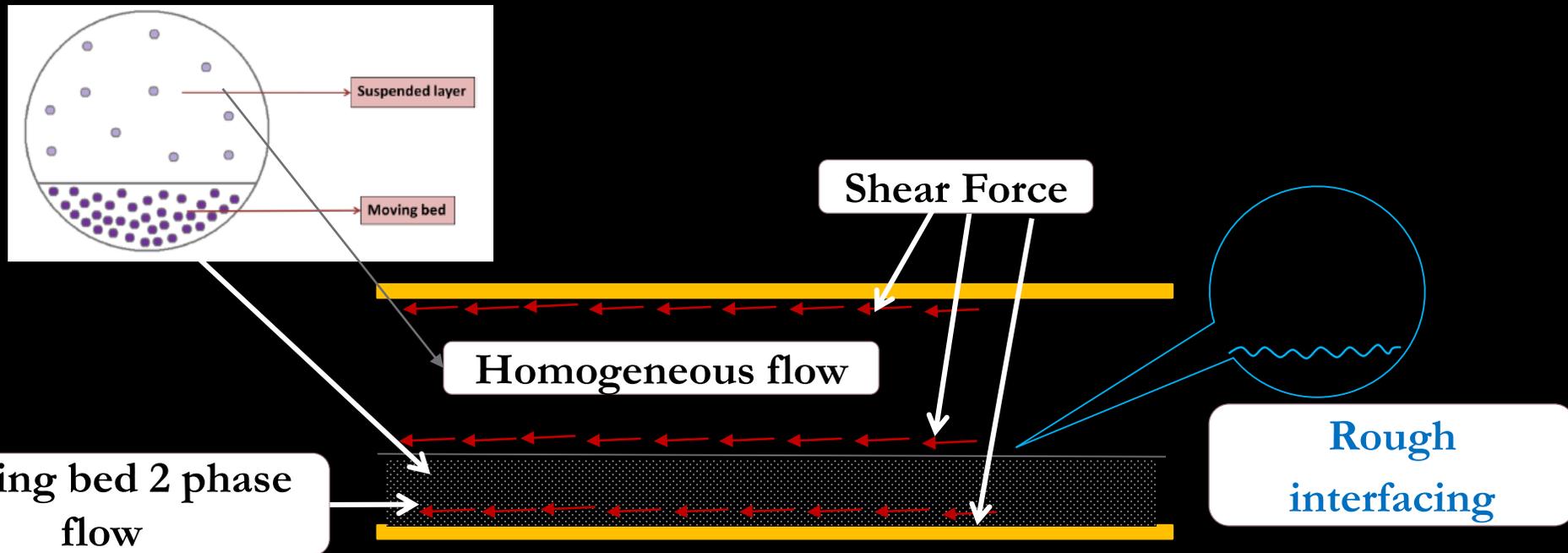
$$\lambda = 1.151^n \left( \frac{0.707}{n} + 2.12 \right) - \frac{4.015}{n} - 1.057$$

Diameter of particle

$$\frac{1}{\sqrt{f}} = -2 \log \left( \frac{10^{-\lambda/2}}{\text{Re}_{2p} f^{(2-n)/2n}} + \frac{d_p}{3.71 D_h} \right)$$

# TWO PHASE, TWO LAYERS SOLID-IN-LIQUID

## 4. 2 layer, 2 phase solid-in-liquid flow



## **TWO PHASE, TWO LAYERS SOLID-IN-LIQUID**

**Sand settling in horizontal pipelines produced stationary sand deposit which creates a pressure drop and affects the rate of production.**

**Formation of sand bed inside the pipeline during the shutdown process creates many engineering challenges particularly during the restart-up process.**

**Transportation of the cutting particles in horizontal well drilling**

# TWO PHASE, TWO LAYERS SOLID-IN-LIQUID

## The Pseudo Hydrostatic Pressure concept

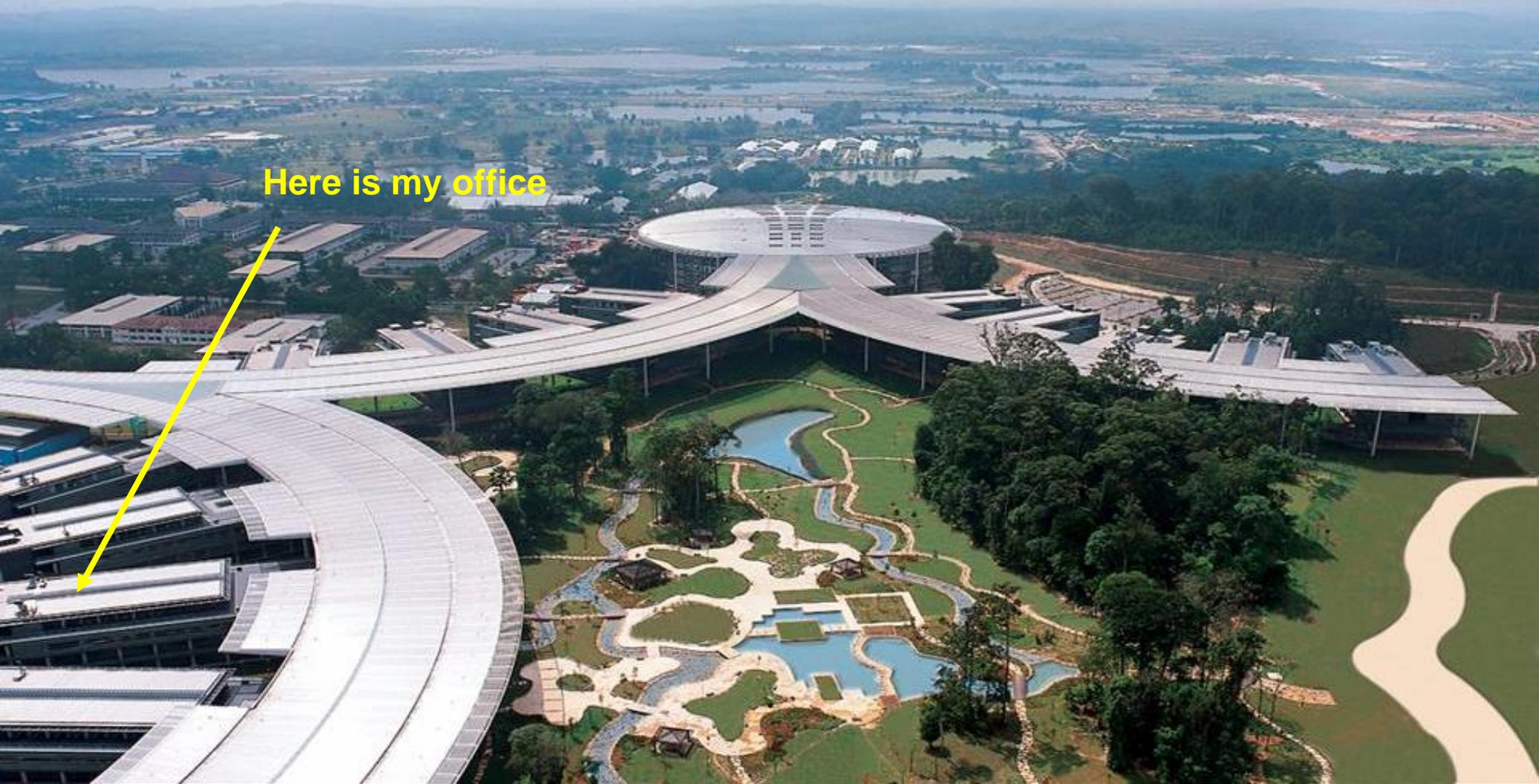
- Based on the pseudo hydrostatic pressure concept, the hydrostatic pressure distribution along the moving bed wall can be defined as:

$$P_{pseudo} = \int_0^{t_m} (\rho_p c_m + \rho_f (1 - c_m)) g dt$$

# TWO PHASE, TWO LAYERS SOLID-IN-LIQUID

**Proposal for  
experimental investigation on the dry friction factor  
between suspended and moving bed**

$$F_d = \mu_d \left[ \rho_p \left( \frac{c_{m,\max} + c_s}{2} \right) + \rho_f \left( 1 - \frac{c_{m,\max} + c_s}{2} \right) \right] g t_m S_{mw}$$



Here is my office



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**THANK YOU**

# TWO PHASE, TWO LAYERS SOLID-IN-LIQUID

**Proposal for  
experimental investigation on the dry friction factor  
between suspended and moving bed**

$$F_d = \mu_d \left[ \rho_p \left( \frac{(c_{m,\max} + c_s)}{2} \right) + \rho_f \left( 1 - \frac{(c_{m,\max} + c_s)}{2} \right) \right] g t_m S_{mv}$$

<http://www.youtube.com/watch?v=a3anE3PEPP8>