VTI Wave Modeling Using Weak Elastic Anisotropy Approximation

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Abstract. *Geological structures commonly exhibit anisotropic behavior which needs to be considered in seismic imaging not only to avoid distortions in imaging, but also to provide valuable information about lithology and fracture networks. Effects of seismic anisotropy in imaging can be studied by employing an anisotropic wave equation. Forward modeling of waves is a fundamental component in both migration and inversion algorithms to study the physics of wave propagation. In this study, we present the role of Thomsen parameters for elastic wave propagation in vertical transverse isotropy (VTI) using weak anisotropy approximation. Wavefield modeling revealed the influence of anisotropy parameter δ in controlling anisotropic features. Moreover, both phase velocity and group velocity are studied which can be employed for ray tracing.*

Keywords: *seismic anisotropy, elastic wave propagation, VTI*

1 Introduction

Hydrocarbon and geothermal reservoirs, and overlying strata are often composed of anisotropic rocks. Considering anisotropy into account is necessary not only to avoid distortions in imaging, but also provides valuable information about lithology and fracture networks. To account for the effects of seismic anisotropy in imaging, an anisotropic wave equation must be employed. Forward modeling of waves is a fundamental component in both migration and inversion algorithms to study the physics of wave propagation and to test hypotheses inferred from observational data [1]. Since there is no general analytic solution to the anisotropic elastic wave equation, various approximate approaches are employed. These are often based on physically-motivated arguments specific to the problem under study [2, 3].

Most of publications on seismic anisotropy present the effect of velocity variation with angle on the amplitudes and traveltimes of seismic waves [2, 4-6]. Backus [4], using averaging, illustrated that fine layering causes elastic anisotropy. Subsequently,

3rd International Conference on INTEGRATED PETROLEUM ENGINEERING AND GEOSCIENCES,
3rd June – 5th June 2014, Kuala Lumpur, Malaysia

transverse isotropy was parameterized using the ‘Thomsen’ parameters for weak elastic anisotropy. The weak anisotropy approximation is an extremely powerful tool in understanding the behavior of seismic wavefields in anisotropic media [7]. Weak anisotropy approximation provides simpler equations compared to strong anisotropy. These equations indicate that anisotropy parameter δ controls most anisotropic phenomena of importance in exploration geophysics, some of which are non-negligible even when the anisotropy is weak [2]. Perturbation theory is another technique which applied to study attributes of elastic waves propagating in weakly anisotropic media. The approximated formula demonstrates that all studied attributes rely on elements of a matrix linearly dependent on parameters of a medium [3].

Since petroleum geophysicists are particularly interested in layered sedimentary rocks, the analysis of events at a planar horizontal interface between two media is given special attention. The principal objective of this research is to provide an approach offering a better understanding of physics of anisotropic media as observed through elastic waves. We present the role of Thomsen parameters for elastic wave propagation in vertical transverse isotropy (VTI) using weak anisotropy approximation. In this study, both phase velocity and group velocity are studied which can be employed for ray tracing [8].

2 Theory

When the wave velocity propagation depends on the angle between the wave vector and the vertical anisotropy symmetry axis, the medium is called VTI. Since most of rocks have anisotropy in the weak to moderate range (anisotropy parameters < 0.2), one can use the approximation of weak anisotropy and applying Taylor series to obtain a set of equations for phase and group velocities [2]. In order to obtain the P and SV wave velocities which depend on the phase angle θ, the phase velocity for weak anisotropy VTI is given by:

$$v_P(\theta) = \alpha_0(1 + \delta sin^2\theta \cos^2\theta + \varepsilon\, sin^4\theta) \tag{1}$$

$$v_{SV}(\theta) = \beta_0\left[1 + \frac{\alpha_0^2}{\beta_0^2}(\varepsilon - \delta)\, sin^2\theta\, cos^2\theta\right] \tag{2}$$

$$v_{SH}(\theta) = \beta_0(1 + \gamma\, sin^2\theta) \tag{3}$$

where α_0 and β_0 are the vertical velocity for P- and S-waves, phase angle θ is the angle between the wavefront normal and the vertical axis, and Thomsen parameters ε, δ and γ are defined by:

$$\varepsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}} \tag{4}$$

$$\delta \equiv \frac{(C_{13}+C_{44})^2-(C_{33}-C_{44})^2}{2C_{33}(C_{33}-C_{44})} \tag{5}$$

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} \tag{6}$$

C_{ij} is elastic modulus tensor which characterizes the elasticity of the medium.

Group velocity, which is computed in ray direction (∅), is a key element in driving anisotropy ray-tracing equations. The exact scalar magnitude Vg of the group velocity is given in terms of the phase velocity magnitude v [9] by:

$$Vg = v\sqrt{1+(\frac{1}{V}\frac{dv}{d\theta})^2} \tag{7}$$

Replacing (1) in (7) is given the quasi P-wave group velocity in terms of its phase velocity for the case of weak anisotropy, is as follows:

$$V_P(\emptyset) = v_P(\theta)\left[1+\frac{1}{2v_P^2}\left(\frac{\partial v_P}{\partial \theta}\right)^2\right] \tag{8}$$

The relationship between group angle ∅ and phase angle θ for P, SV and SH is, in the linear approximation,

$$tan\emptyset_P = tan\theta_P[1+2\delta+4(\varepsilon-\delta)\sin^2\theta_P] \tag{9}$$

$$tan\emptyset_{SV} = tan\theta_{SV}\left[1+2\frac{\alpha_0^2}{\beta_0^2}(\varepsilon-\delta)(1-2\sin^2\theta_{SV})\right] \tag{10}$$

$$tan\emptyset_{SH} = tan\theta_{SH}(1+2\gamma) \tag{11}$$

These equations, 1- 3 and 7-11, define the group velocity, at any angle, for each wave type.

3 Numerical Modeling and Examples

The theory, described in the previous section, is implemented in MATLAB. To solve the equations, some MATLAB's build-in functions are applied to calculate and plot the results. Elliptical anisotropy is given by the equality ε = δ. Fig. 1 shows the comparison of wavefields, for phase and group velocities, in a VTI and an isotropic medium. Since ε = δ, the wavefield for anisotropic P-wave is elliptical, however the S-wave wavefield for both media is spherical which can be directly realized from (2). Also, no difference can be seen between phase and group velocity in this condition.

a. b.

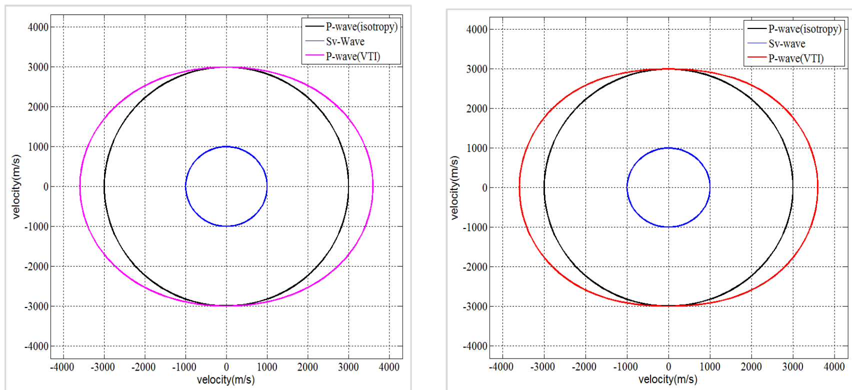

Fig. 1. Comparison of wave propagation in a VTI medium specified by $\varepsilon = 0.15, \delta = 0.15$ and $\gamma = 0.2$, and an isotropy medium. (a) Phase velocity modeling, and (b) group velocity modeling.

In next example, to study the effect of parameter δ, we change the parameter δ, and keep ε and γ fixed. It can clearly be seen that parameter δ along with parameter ε control the propagation of P- and SV-wave in VTI medium (Fig. 2). However, the equation (1) indicates that, for near-vertical P-wave propagation, the δ contribution entirely dominates the ε contribution [2].

a. b.

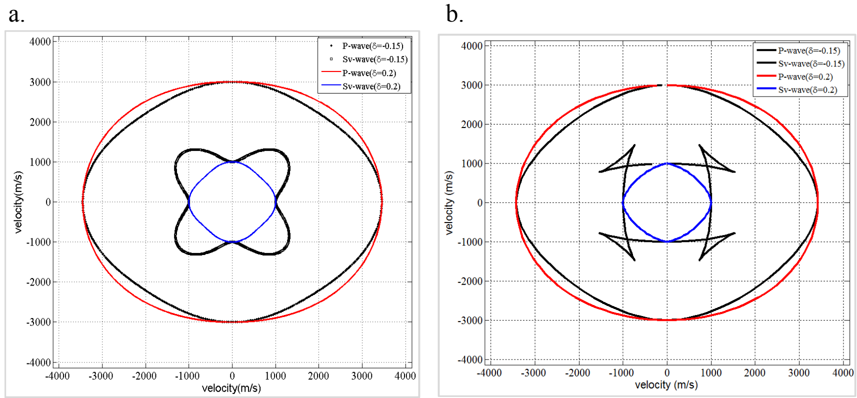

Fig. 2. Wave propagation in a VTI medium for different $\delta = -0.15$ and $\delta = 0.2$ ($\varepsilon = 0.15$ and $\gamma = 0.2$). (a) Phase velocity modeling, and (b) group velocity modeling.

Parameter γ is another anisotropy parameter, influencing elastic wave propagation, which is studied in this step. To achieve our goal, we keep ε and δ unchanged in both plotting, and only parameter γ is altered. As it is illustrated in Fig. 3., there is no difference in wavefields of both figures and wavefields are overlapped each other. Hence, γ does not affect the propagation of P- and SV-waves, and it only corresponds to the conventional meaning of S_H anisotropy (equation 3) [2]. For small value of γ, phase velocity and group velocity of S_H almost propagate similarly, although increasing γ causes the distinct wavefields of S_H.

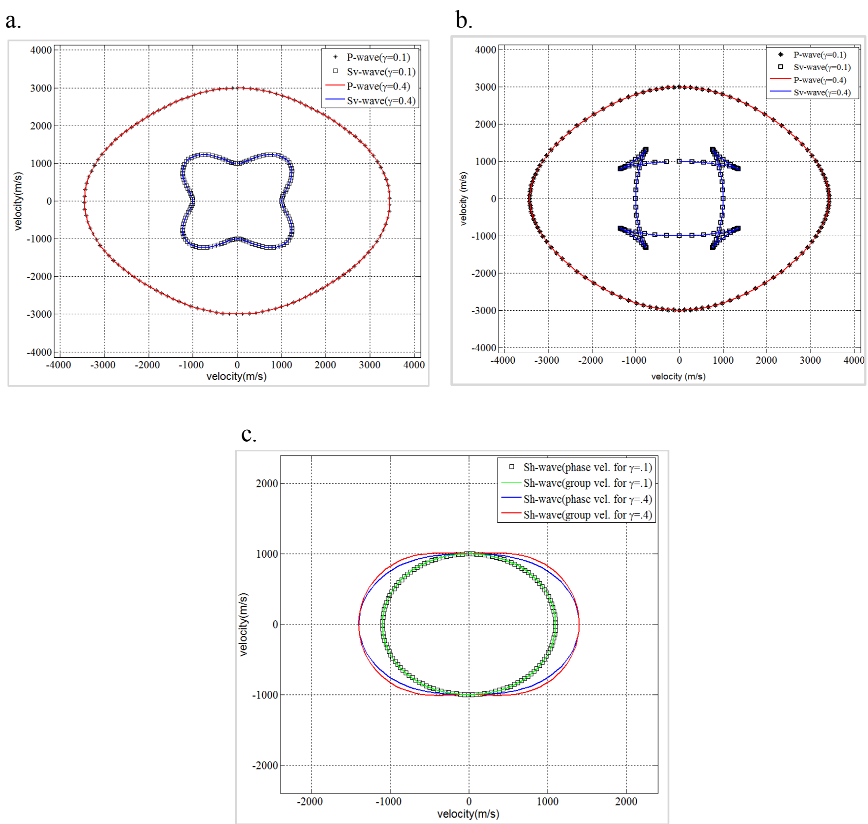

Fig. 3. Wavefield snapshots for different parameters $\gamma = 0.1$ and $\gamma = 0.4$ ($\varepsilon = 0.15, \delta = -0.1$). (a) Phase velocity modelling of P-wave and Sv-wave, (b) group velocity modelling of P-wave and Sv-wave, and (c) group and phase velocity wavefields of S_H-wave.

4 Conclusions

I presented an elastic wave equation for VTI media with the weak anisotropy approximation. Solving exact elastic anisotropy equations are certainly more expensive in terms of computing time and memory requirement. A simplification of the phase-velocity and group-velocity formula under the assumption of weak anisotropy allows one to express clearly the propagation of wave in anisotropy media. This yields a straightforward method for calculating phase and group velocity. Equation (1) shows that, for weakly anisotropic media, the anisotropy parameter δ, for near-vertical P-wave propagation, completely dominates the ε contribution. Because of this, δ (rather than ε) controls the anisotropic features of most situations in exploration geophysics. Nevertheless, the parameter γ does not influence the P- and SV-wave wavefronts since it does not appear in their wave equations. In addition, S_H wave propagation is only affected by parameter γ. Another achievement of this study is the ability to compute P and S wave velocities as functions of the ray direction which is demanding for the implementation of ray-tracing in anisotropic media.

Acknowledgment

We gratefully acknowledge members of Centre of Seismic Imaging (CSI) at UTP for their helpful discussions. The support for this work is provided by PETRONAS on the project Seismic Anisotropy Imaging for Deep Reservoir and Fractured Basement.

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