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Comparative study of new signal processing to improve S/N ratio of seismic data

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Abstract A prime target of seismic data processing is to improve the signal-to-noise ratio of the seismic data. New signal processing tools such as Wavelet transform, Radon transform, Fan-beam transform, Ridgelet transform and Curvelet transform have proven their results in image processing. A comparative study has been performed with these techniques to test their ability to increase the signal-to-noise ratio of seismic data by removing random noises. We then described the comprehensive mathematical formulation of these algorithms and tested them on both synthetic seismic data, which was created with a known signal-to-noise ratio with desired geologic features, and real seismic data, which contained curved features with random noise. Wavelet transform, which extends the robustness of frequency-dependent filtering by adding time dimension and multi-scale wavelet translation, improves the signal-to noise-ratio through the threshold coefficient filtering of random noise. The Radon transform and Fan-beam transform provide the opportunity of angle-dependent filtering, but produce adverse effects on curved features of seismic data and decrease seismic resolution. Ridgelet and Curvelet transform are more robust than Radon and Fan-beam transform. But Ridgelet transform, which uses Radon transform in its coefficient calculation, also produces adverse effects on curved features and threshold filtering leads to a decrease in the signal-to-noise ratio. The results have shown that the Curvelet transform is robust enough to handle random noise and also preserve the inclined and curved features of seismic data. However, its coefficient calculation requires large computation time and memory space.

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Introduction

Fourier analysis is a well-known tool to decompose a signal into its orthogonal components of Sine and Cosine waves (Eq. 1). It transforms the signal from the time domain to the frequency domain. The amplitude and phase spectrum obtained through the Fourier analysis is the solution of a stationary signal. But, the seismic wavelet is non-stationary and changes its shape and frequency contents as it propagates in subsurface. Short Time Fourier Transform (STFT) is the windowed version of Fourier transform (Eq. 2) (Donoho 1995; Jacobsen and Lyons 2003, 2004). It tries to accommodate the deficiency of the Fourier transform, but the selection of the type and size of the window has its own limitations. A large window size gives more frequency resolution but less time resolution as the product of time-resolution and frequency-resolution is constant. It is not possible to have both high time resolution and frequency resolution at the same instant. The selection of the size of the window is a trade between either high frequency resolution or time resolution. This phenomenon is known as the Gabor–Heisenberg Uncertainty Principle.

$$F(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt \quad (1)$$

$$F(\tau, f : h) = \int_{-\infty}^{\infty} x(t) h^*(t - \tau) e^{-j2\pi f t}\, dt \tag{2}$$

where $x(t)$ is the time domain signal, f the frequency, t the time, $h^*(t)$, the decompostion window, and τ is the translation of window along time axis.

The Wavelet transform provides one of the solutions using the finite duration wavelets with a variable length which defines its central frequency. Both of these techniques (i.e., Short Time Fourier Transform and Wavelet transform) come in the category of spectral decomposition and work on one trace at a time. The rest of the other techniques discussed in the paper are at least two dimensional. Radon and Fan-beam transforms provide the opportunity of the angle-dependent removal of noise from seismic data. The Radon transform computes projections of a two-dimensional image matrix on a straight or curved line. This projection on the line is calculated for $0° - 180°$. Whereas in Fan-beam, this projection can be on a straight or curved line. The projection for each direction is calculated by taking the line integral of the straight lines emerging from a single vertex. This vertex must reside outside of a two-dimensional image matrix. The last two techniques are Ridgelet and Curvelet transform. The Ridgelet transform is obtained by taking the Radon transform of a 2D Fourier transform seismic section. It extends the capability of the Radon transform for the removal of noise from the seismic data through the reconstruction of the filtered coefficients of Ridgelet. Curvelet coefficients are calculated through the application of the Ridgelet transform on each scaled (2D Wavelet transform) version of the seismic data.

Methodology

Signal processing techniques are tested on both synthetic and real seismic data. Testing includes two levels. In the first level, an algorithm has to reconstruct all the seismic features (i.e., linear, inclined and curved features without any distortion) without preconditioning the input data. After the first-level filtering, the qualified algorithm is tested for its improvement of the signal-to-noise ratio using coefficient filtering.

These algorithms have been tested on synthetic and real seismic data. To make the procedure simple, the synthetic seismic data were created using a convolution technique on a thin-bed reflectivity model. This reflectivity model contained both horizontal and inclined layers. Random noise with a signal-to-noise ratio of 3 was added to the reflectivity model before the convolution operation. Figure 1 shows the synthetic seismic data used in the algorithm testing. Whereas, the curved feature was tested on real seismic data as shown in Fig. 2.

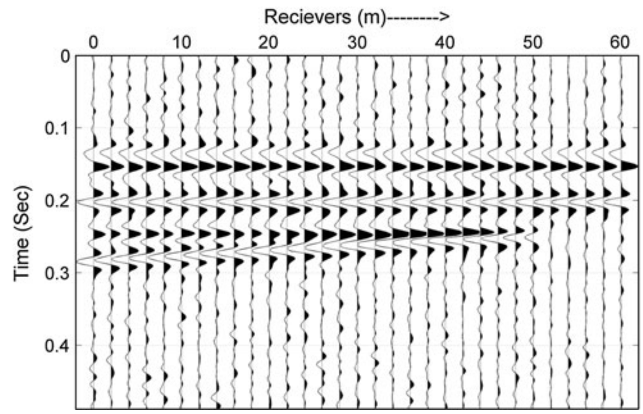


Fig. 1 Synthetic seismic data with a signal-to-noise ratio of 3

Wavelet transform

The Wavelet transform can be calculated using Eq. (3). In this analysis, the Dabachi wavelet 6 was used which satisfied the admissibility condition (Eq. 4). Input 2D synthetic seismic data were decomposed into 4 levels. The first level decomposed the seismic data into detailed (high frequency) and approximation (low frequency) seismic sections. In the second level, the approximation was decomposed into its details and approximation; it was similarly performed for the 3rd and 4th level as shown in Fig. 3a (Mallat 1989; Torrence and Compo 1998).

$$X_w(a, b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} x(t) \psi^*\left(\frac{t - a}{b}\right) dt, \begin{cases} a\epsilon(-\infty, \infty) \\ b\epsilon(0, \infty) \end{cases} \tag{3}$$

where, $x(t)$, is the time domain signal, a, the location parameter of wavelet translation, b, the wavelet scaling parameter, and $\psi^*(t)$ is the complex conjugate of the mother wavelet $\psi(t)$

Stretching of the mother wavelet gave a long duration with a low central frequency; whereas, compression led to a short analysis wavelet with a high central frequency.

$$C_\psi = \int_0^\infty \frac{|\Psi(f)|^2}{|f|}\, df < \infty \tag{4}$$

$\Psi(f)$ is the Fourier transform of the wavelet $\psi(t)$. Since C_ψ has to be finite, the integral defining C_ψ should be integrable at $f = 0$. This implies that $\Psi(0) = 0$. This means that the average value of the wavelet should be zero i.e. $\int_{\infty}^{-\infty} \psi(t)\, dt = 0$

This decomposition provided the opportunity to filter the noise at a desired level and the seismic data were able to be reconstructed back from its filtered coefficients.

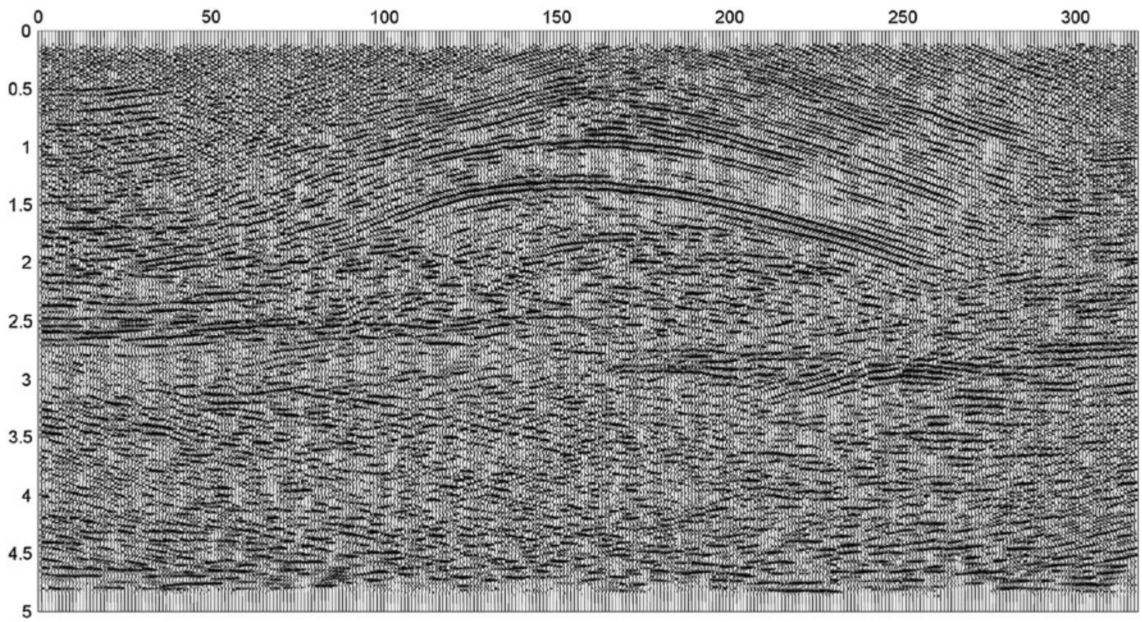
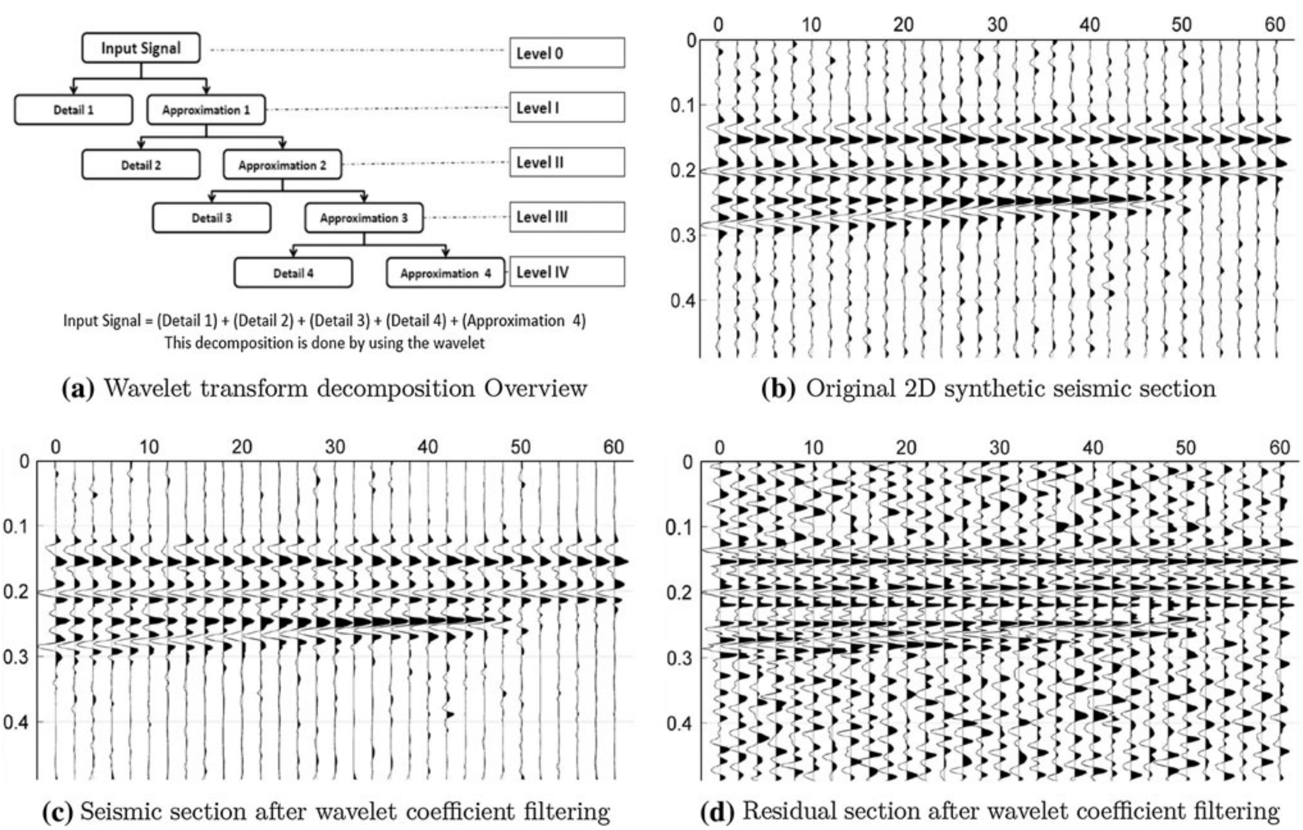


Fig. 2 Real seismic section contained curved features and random noise



(a) Wavelet transform decomposition Overview **(b)** Original 2D synthetic seismic section **(c)** Seismic section after wavelet coefficient filtering **(d)** Residual section after wavelet coefficient filtering

Fig. 3 Application of the Wavelet transform on a 2D Synthetic seismic

The simple idea of the threshold of the Wavelet coefficients was used in this analysis. Figure 3b–d show the input synthetic seismic data, denoised seismic data after the application of DWT and the residual data, respectively. It can be observed that the seismic data reconstructed from the filtered coefficients of the Wavelet transform had a better signal-to-noise ratio but the residual contained energy of the signal, which can cause

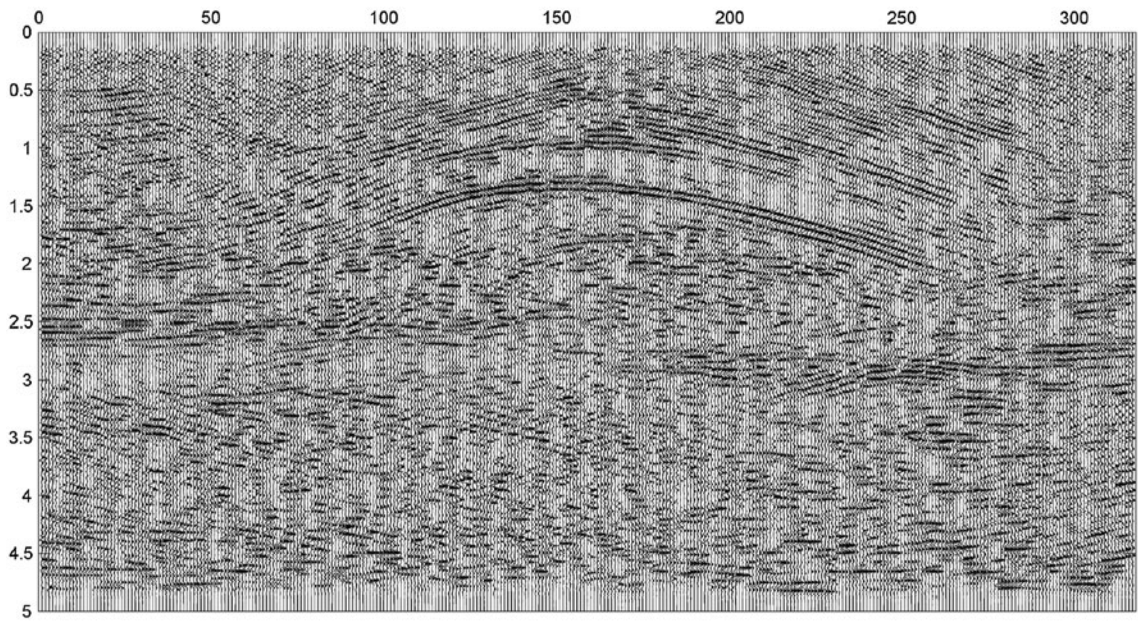


Fig. 4 Real seismic data after the threshold filtering of the Wavelet transform coefficients

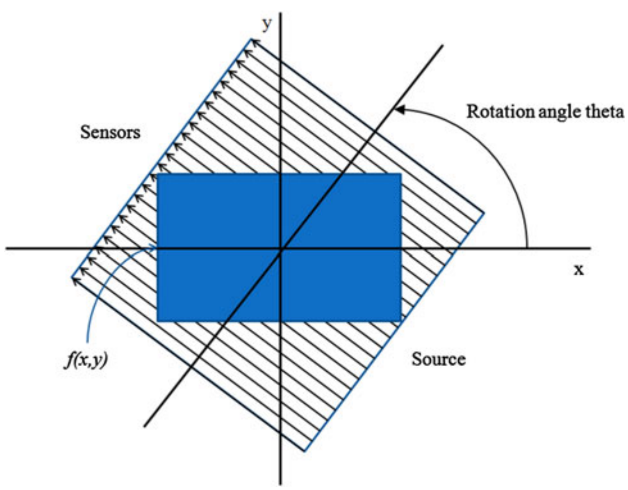


Fig. 5 Radon transform projected the 2D image matrix on a straight line through the line integration of the parallel straight line for each angle of rotation

seismic resolution problems. Figure 4 shows the real seismic data reconstructed after the threshold filtering of the Wavelet transformed coefficients. In comparison with the original seismic section shown in Fig. 1, the signal-to-noise ratio was improved after the Wavelet transform coefficient filtering, and three features, i.e. horizontal, inclined (Fig. 3) and curved (Fig. 4) were recovered.

Radon transform

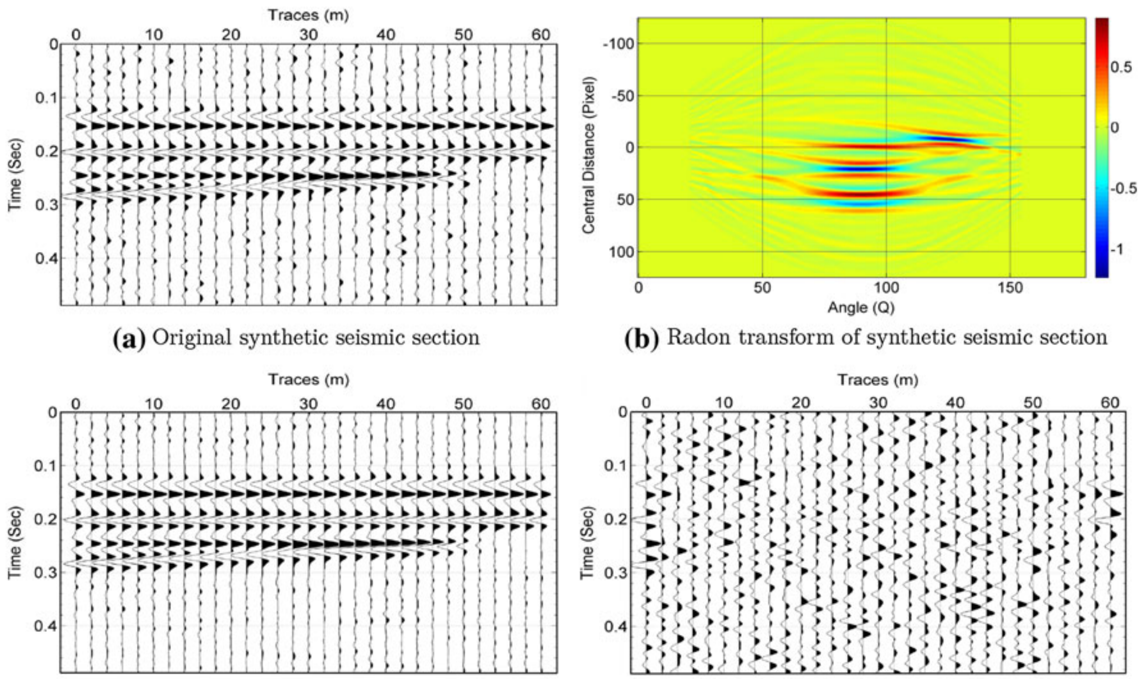
The Radon transform computed the projections of an image matrix along the specified directions. A projection of the two dimensional function $f(x, y)$ was a set of line integrals as shown in Fig. 5. Radon transform computed the line integral of parallel beams for different angle ranges from 0° to 180° with respect to the center of the image (Eq. 5) (Lim and Jae 1990; Bracewell and Ronald 1995). The result of the 2D image was a 2D matrix where each column represented the angle of the beam, the rows represented the distance from the center of the image.

$$Rf(\theta, t) = \int f(x_1, x_2)\delta(x_1 cos\theta + x_2 sin\theta - t)\, \mathrm{d}x_1\, \mathrm{d}x_2 \qquad (5)$$

where, δ, is the Dirac distribution, θ, the angle (0°−180°), and t is the translation.

The Radon transform provided the opportunity of the angle-dependent removal of the noise from the seismic data. Figure 6a shows the seismic section with noise. The Radon transform was calculated with the angle range from 0° to 180° with an interval of 1°. In the Radon domain, the angle ranges from 0° to 20° and 155° to 180° were filtered. Figure 6b shows the filtered section and Fig. 6c shows the seismic section with an improved S/N ratio.

As the Radon transform is a two dimensional operation and uses line integrals at each angle of rotation, the reconstruction of the curved feature becomes challenging and produces noise. Figure 7 shows the real seismic data reconstructed back from the Radon domain without any filtering. In comparison with the original seismic data, the signal-to-noise ratio was decreased and the curved features were adversely affected by the transformation process.



(a) Original synthetic seismic section

(b) Radon transform of synthetic seismic section

(c) Synthetic seismic section after angle dependent filtering

(d) Residual section after angle dependent filtering

Fig. 6 Application of the Radon transform on the synthetic seismic section

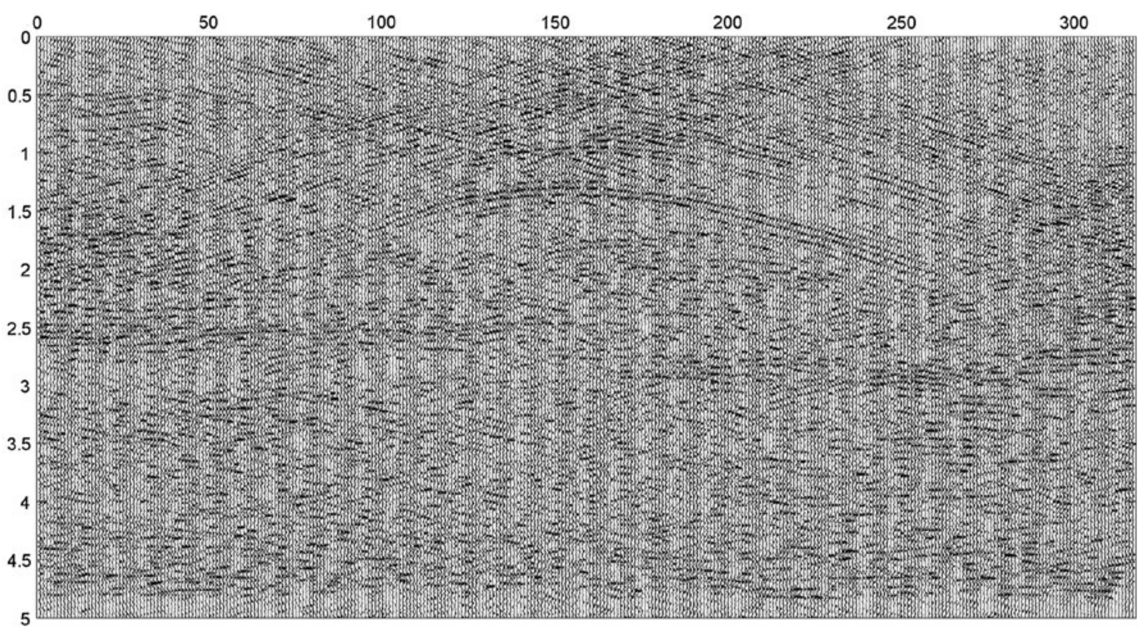
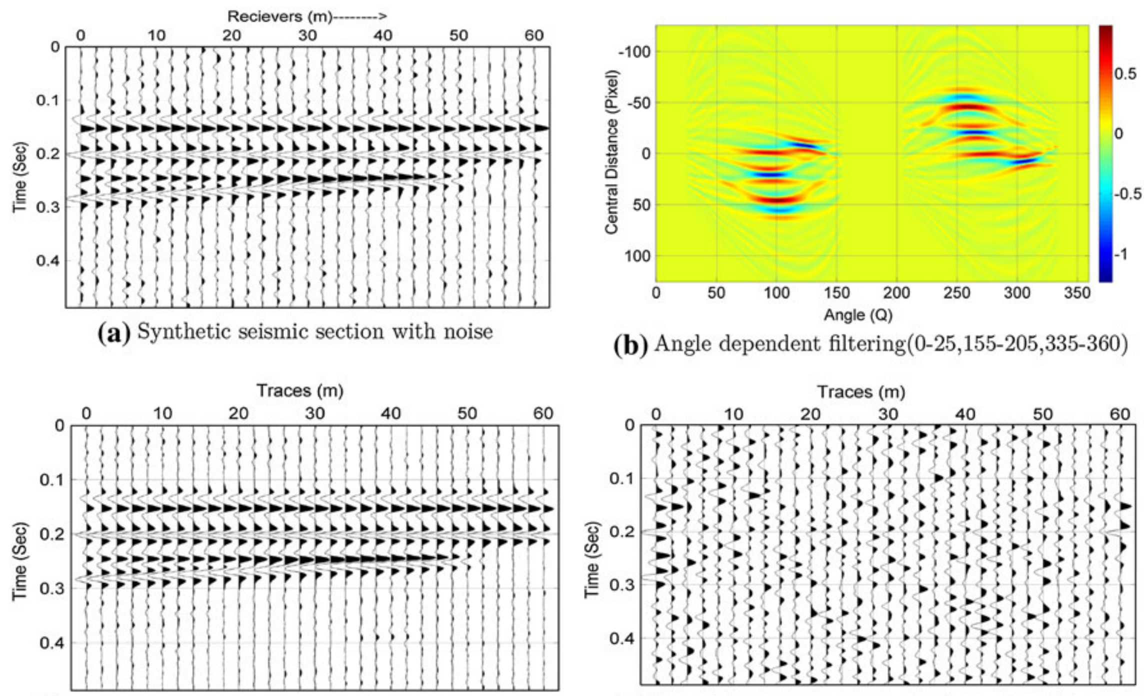


Fig. 7 Curved features of seismic data were not properly reconstructed back from the Radon domain and the signal-to-noise ratio was decreased

Fan-beam transform

Fan-beam used a special X-ray procedure. In its coefficient calculation, the angle of the projection ranged from 0° to 360° (Kak and Slaney 1988), from the center of the image (2D seismic) rather than 0° to 180° for the Radon transform discussed above. This projection can be on a straight or curved line (Matlab 2013). So, Fan-beam provided more flexibility of the angle-dependent filtering than the Radon transform. In this analysis, the Fan-beam coefficients were calculated by projecting on a straight line. The seismic data were reconstructed back after the filtering of the angle



(a) Synthetic seismic section with noise

(b) Angle dependent filtering(0-25,155-205,335-360)

(c) Synthetic seismic section after Fanbeam transform, shows decrease in noise

(d) Residual section after angle dependent filtering

Fig. 8 Application of Fan-beam transform on the real seismic section

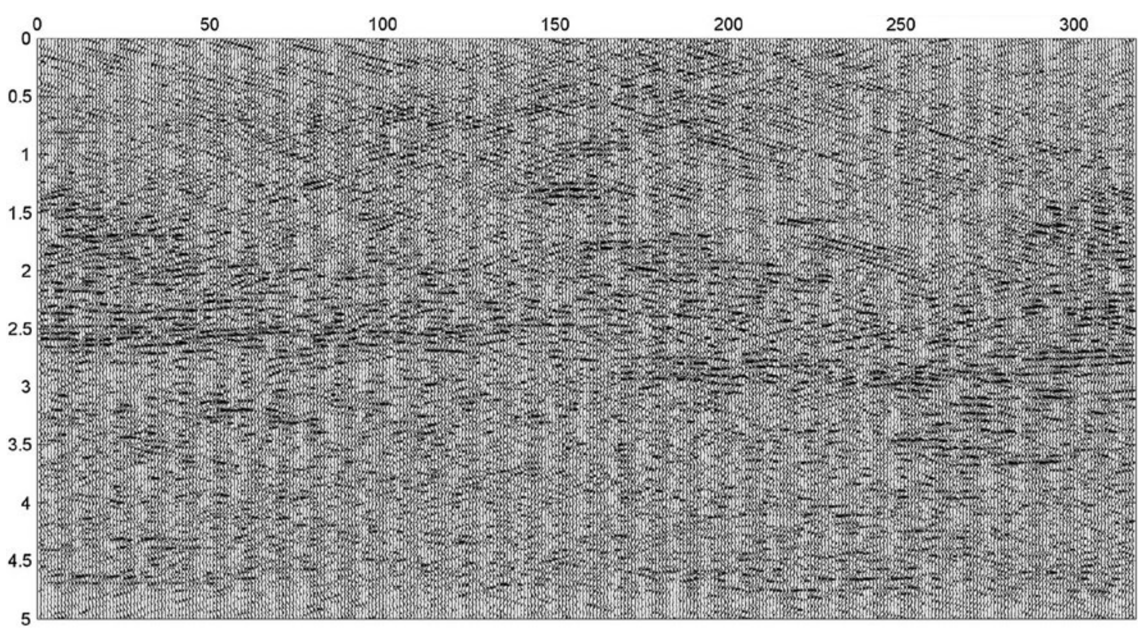
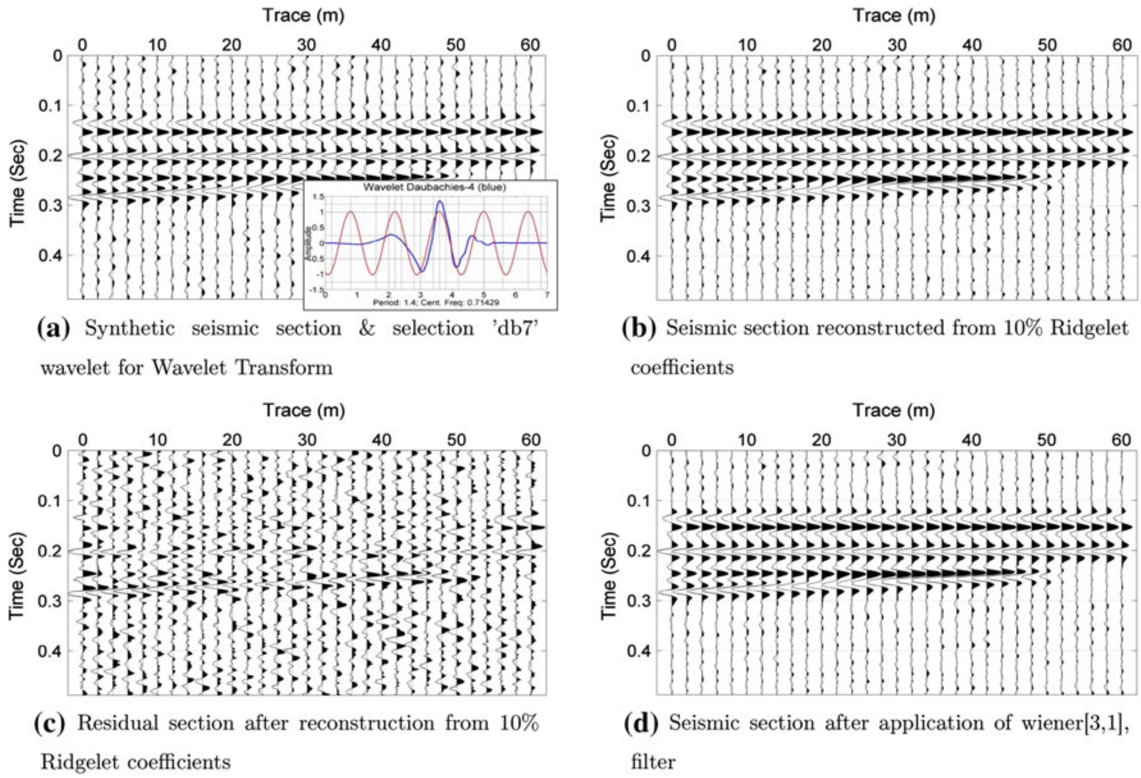


Fig. 9 Curved features of the seismic data were badly effected by the Fan-beam transform

range of 0°−25°, 155°−205°, 335°−360° as shown in Fig. 8. This angle-dependent filtering improved the signal-to-noise ratio of the linear and inclined features of the seismic data; whereas, testing on the real seismic section (Fig. 9) shows that the curved feature of the seismic data was adversely affected by the transformation process. This led to a decrease in the signal-to-noise ratio and false seismic features.



(a) Synthetic seismic section & selection 'db7' wavelet for Wavelet Transform

(b) Seismic section reconstructed from 10% Ridgelet coefficients

(c) Residual section after reconstruction from 10% Ridgelet coefficients

(d) Seismic section after application of wiener[3,1], filter

Fig. 10 Application of the Ridgelet transform on the synthetic seismic data

Ridgelet transform

The Ridgelet transform is good for identification of line discontinuities in seismic data and it can handle edge effects better than the Wavelet transform

$$Rf(\theta,t)=\int f(x_1,x_2)\delta(x_1cos\theta+x_2sin\theta-t)\,\mathrm{d}x_1\mathrm{d}x_2$$
$$R_f(a,b,\theta)=\int Rf(\theta,t).a^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right)\mathrm{d}t \tag{6}$$

where, δ, is the Dirac distribution, t, the translation, θ, the angle (0°−180°), $Rf(\theta, t)$, the Radon transform, $R_f(a, b, \theta)$, the Ridgelet transform, ψ, the wavelet, a, the wavelet scale factor, b, the wavelet translation factor.

Ridgelet coefficients can be obtained by taking the Wavelet transform of the Radon transformed seismic data (Eq. 6) (Do and Vetterli 2003; Jean-Luc et al. 2002). Figure 10a shows the input seismic section along with the wavelet used for its decomposition. Figure 10b–d show the reconstructed seismic section from the filtered Ridgelet coefficients, residual section and reconstructed seismic section after the application of the wiener filter, respectively. The analysis shows that the Ridgelet transform had an adverse effect on the inclined features as observed in Fig. 10c (where the residual section contained the energy of inclined features). The wiener filter (which is a low pass-filter) was applied to filtered seismic section to remove the effect of jaggedness. It used the pixel-wise adaptive wiener method based on the statistics (mean & standard deviation) estimated from a location's neighborhood. In this study, the neighborhood was defined by the window. The Ridgelet threshold coefficient filtering produced noise in the seismic section which led to a decrease in the signal-to-noise ratio as shown in Fig. 11.

Curvelet transform

The Curvelet transform is a powerful tool to improve the signal-to-noise ratio of seismic data. The Curvelet transform used the 2D Wavelet transform before the implementation of the Ridgelet transform on each scaled version of the seismic section (Do and Vetterli 2003). Figure 12 shows the application of the Wrapping Curvelet transform on the noisy data. Figure 12a shows the synthetic seismic data with no noise; whereas 12b shows the same synthetic seismic data with the addition of normally distributed noise with an S/N ratio of 3. The Curvelet transform of the synthetic seismic data was calculated and the synthetic data were reconstructed with the 0.008 % Curvelet coefficients shown in Fig. 12c.

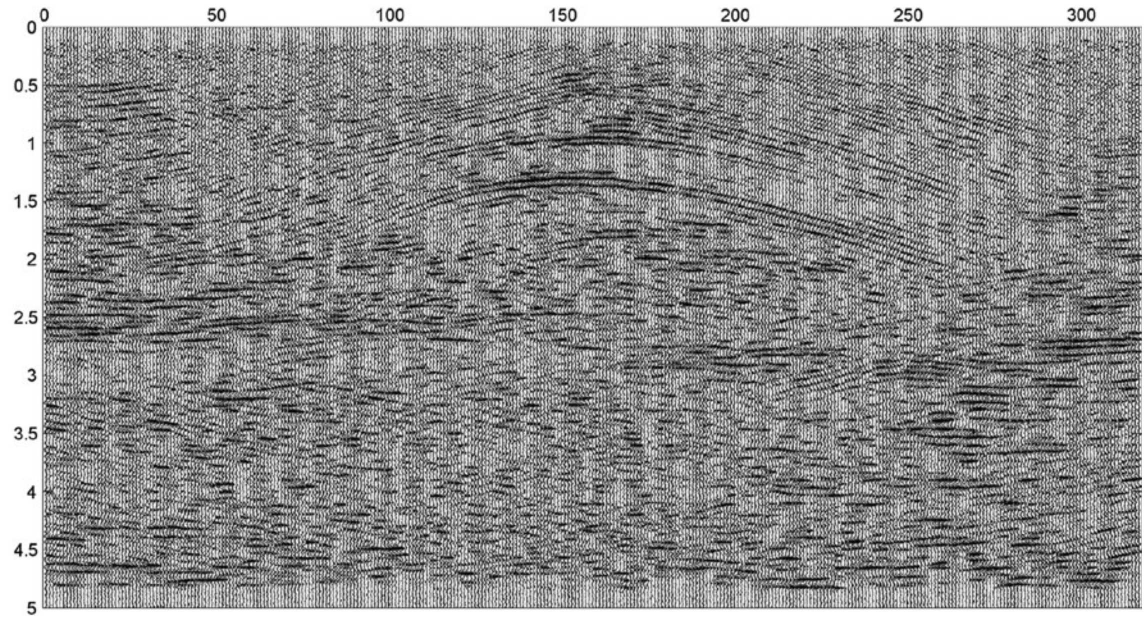


Fig. 11 Ridgelet transform reconstructed the curved feature of the seismic section but the threshold coefficient filtering decreased the signal-to-noise ratio

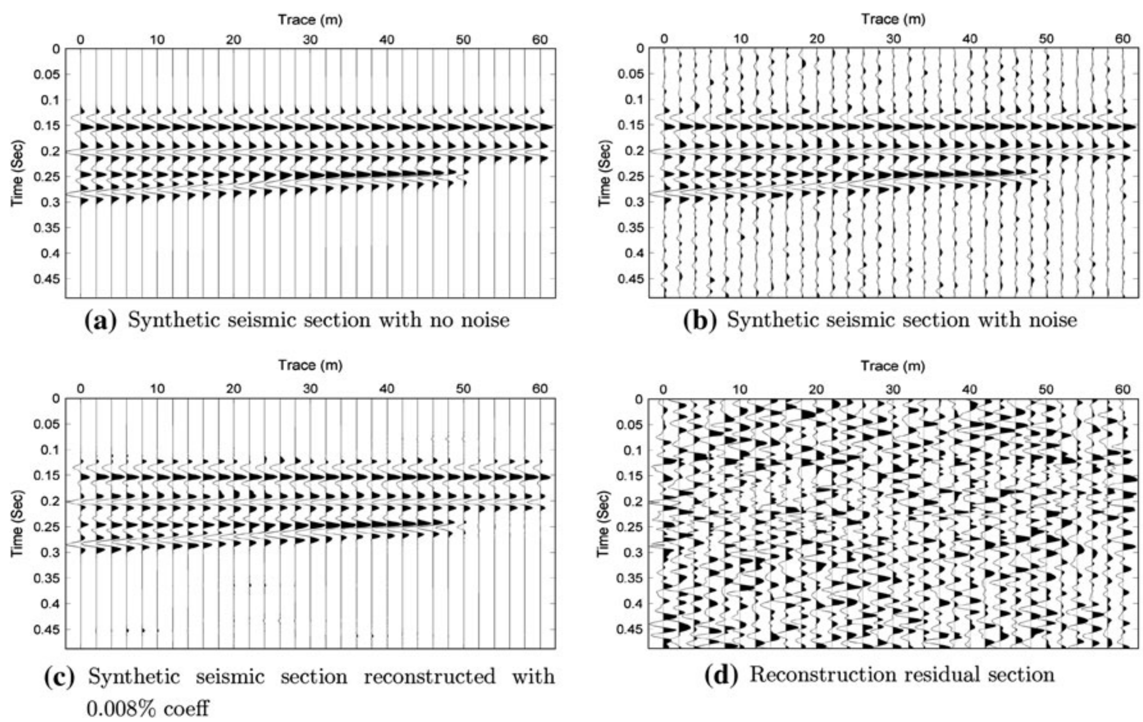


Fig. 12 Application of the wrapping Curvelet transform on the synthetic seismic data

It can be observed that random noise was considerably removed from the synthetic seismic section. The Curvelet threshold coefficient filtering effectively removed the random noise from the seismic section without much effect to the curved features of the seismic section, as shown in Fig. 13.

Results and discussion

A comparative study between new signal processing techniques has been performed to improve the signal-to-noise ratio of seismic data. The Fourier transform is a powerful tool to transform the signal from the time

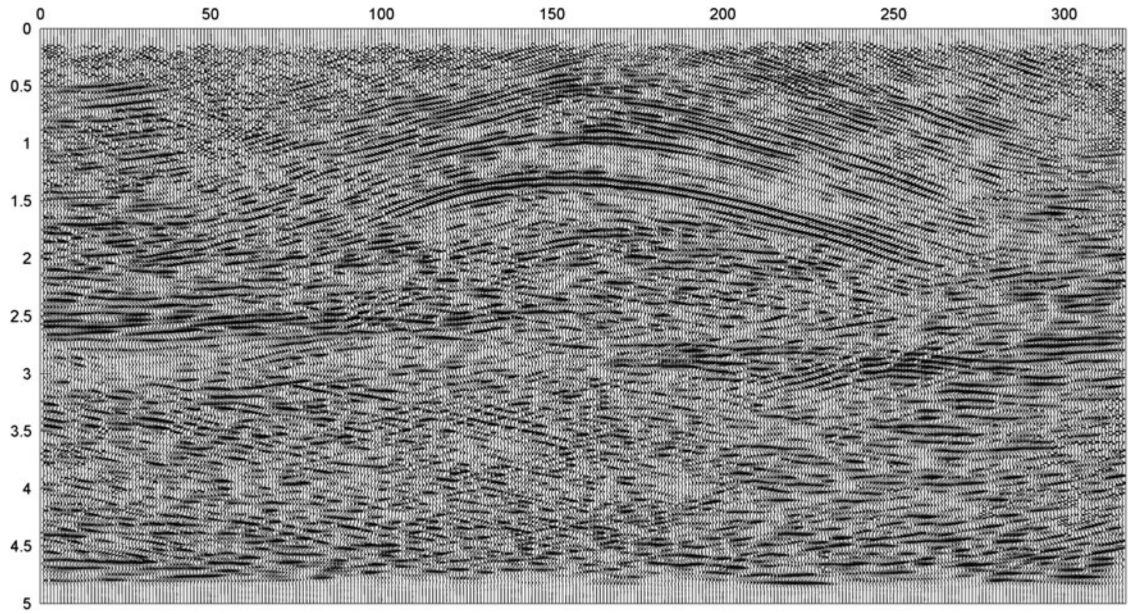


Fig. 13 Reconstructed seismic section from the filtered Curvelet coefficients effectively reduced the random noise

domain to the frequency domain, but it has a lack of time localization of the seismic event in the Fourier domain. So, in non-stationary seismic signals, the Fourier transform is not the solution. The Short-Time-Fourier-Transform, STFT, has extended the capability of the Fourier transform by adding the time dimension. This has helped to filter the desire frequency at a known time; but, STFT decomposition depends upon the size and shape of the spectral decomposing window. One of the solutions to this uncertainty principle is a Wavelet transform. But the Wavelet transform cannot handle edge effects when the seismic data are reconstructed back from its filtered coefficients because the edges in the seismic data are repeated scale after scale. So, filtering though the Wavelet transform can cause resolution problems in seismic data. The Radon transform and Fan-beam transform provide the opportunity of angle-dependent filtering using a technique of projection of a beam on line. These techniques have been found to be good at improving the signal-to-noise ratio, but have adverse effects on curved features of seismic data. Secondly, both of the techniques use interpolation for the reconstruction from their coefficients which causes resolution problems in seismic data. The Ridgelet transform, which uses the Radon transform in calculation of Ridgelet coefficients, solves the problem of edges in the seismic data but produces adverse effects on the inclined features and curved features when seismic data are reconstructed back from their filter coefficients. The Curvelet transform (uses the 2D Wavelet transform before the application of the Ridgelet transform on each scaled version of the seismic data) is more robust to handle inclined and curved features. It handles the horizontal, inclined and curved features of seismic data more effectively when a seismic section is reconstructed back from its filter coefficients. But this technique requires a long processing time and memory. So, a faster and more memory efficient algorithm is needed to be developed so that it can be implemented on a large seismic volume.

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