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FRP as Strengthening Material for Reinforced Concrete Beams with Openings - A Review

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Abstract

This paper presents the review of studies performed to investigate the behavior of Reinforced Concrete (RC) beams containing different types of openings. For the last 4 decades, research works involved the investigation of the member strength and development of the design approach for simply supported, continuous and T-beams containing large rectangular openings subjected to torsion, bending and shear forces. The proposed design method suggested the installation of diagonal steel reinforcement bars around the opening for providing the required strength. In recent years, Fiber Reinforced Polymers (FRP) have been widely used as an external bonded reinforcement system for upgrading and retrofitting of concrete structures. FRP as externally bonded reinforcement mainly used to repair and retrofit the damaged reinforced concrete member. However, very limited studies showed the application of the FRP laminates as external reinforcement around openings. Therefore, further investigations regarding the application of FRP laminates to strengthen the large openings in reinforced concrete members are very vital.

Keywords: *FRP, opening, reinforced concrete beam, strengthening*

1. Introduction

Utility pipes and ducts are necessary to accommodate essential services in a building such as sewerage, water supply, air-conditioning, power supply, and telecommunication. In the past practices, such pipes and ducts were usually hanged under the concrete slab that used to cover by a suspended ceiling, which formed a dead space. Lately, providing openings through the floor beams for the passage of utility pipes and ducts has become a common practice to avoid the headroom problem caused by hanging of such services pipes and ducts (Mansur and Tan, 1999). Introduction of openings brought up many concerns regarding the structural performance of the members. The provision of openings in reinforced concrete beams changes the simple beam behavior into a more complex one. Due to a sudden change in the cross-sectional dimensions of a beam, the opening corners are subjected to high stress concentration that may lead to wide cracking which is unacceptable from the aesthetic and durability point of view (Mansur and Tan, 1999).

With the introduction of openings in reinforced concrete beams, many issues and concerns have been raised, which induced interest in many researchers for further exploring the area. Many published research focused on the behavior of reinforced concrete beams containing openings of different shapes and sizes in simply supported beams, continuous two spans and three spans beam and T-beams. From the produced data, various models and equations have been developed to estimate the structural capacity of RC beams governed by flexural, shear and/or torsion behavior. In general, inclusion of openings in the web of a reinforced concrete beam reduces its stiffness, which may cause excessive cracking and deflection and severely affects its strength (Tan *et al*., 1996).

Thus, reinforced concrete beams containing openings must be checked for their strength or the region near the opening should be designed adequately, so, the lost strength could be reinstated (Mansur *et al*., 1983). To deal with the opening in RC members, various experimental procedures and numerical techniques have developed to provide adequate reinforcement around the opening (Mansur *et al*., 1985; Tan *et al*., 2001), an updated version of such method was presented in 2006 (Mansur, 2006). All these studies considered the location and size of opening in the pre-design and construction phase. However, many times opening in RC beam is needed after the design and fabrication of the beam, this situation raised many issues regarding the reinstatement of

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the lost strength.

Fiber Reinforced Plastic, FRP materials have been widely accepted by the construction industry worldwide for repair and retrofitting of the structural components (Allam and Ebeido, 2003; Kachlakev and McCurry, 2000; Santhakumar *et al*., 2004). A wide range of published research presented the effects of FRP materials on solid RC beam behavior under flexure (Alagusundaramoorthy *et al*., 2003; Ashour *et al*., 2004; Franca *et al*., 2007) and shear (Li *et al*., 2001; Islam *et al*., 2005; Zhang *et al*., 2004). However, limited number of research related to RC beams with opening reported the application of FRP materials (Mansur *et al*., 1999; Abdalla *et al*., 2003; Allam, 2005; El Maaddawy and Sherif, 2009; Madkour, 2009; Pimanmas, 2010). The investigations were conducted by experiments supported by theoretical analysis (Abdalla *et al*., 2003; Allam, 2005; El Maaddawy and Sherif, 2009) and experiments validated with non-linear Finite Element Analysis (FEA) (Madkour, 2009; Pimanmas, 2010). All such researches highlighted that FRP materials are quite capable to externally strengthen the RC beams with opening and recommended that further studies are necessary.

This principal objective of this paper was to bring forward the common issues, concerns and problems in dealing with the pre-decided or post-planned provision of opening in RC beams through literature review. The concluding remarks of this review are referred as the way forward for further research at the Universiti Teknologi PETRONAS. The ongoing research program investigating the shape, size and location of single or multiple openings using non-linear numerical approaches and simulation by experimental testing, effective design of strengthening system using Carbon Fiber Reinforced Plastic (CFRP). The research findings will be reported in the future publications.

2. Review and Analysis

2.1 Behavior of RC Beams containing Openings

Lorensten (1962), tested 4 numbers of simply supported T-beams containing a single large rectangular opening. Nasser *et al.* (1967) extended Lorensten's work by testing of 9 numbers of simple beams of rectangular section incorporating one and two rectangular openings. It is reported that by using the additional bars around rectangular opening, the strength was reinstated; however, the stiffness was still low. Ragan and Warwaruk (1967) reported their results of six simply supported pre-stressed T-beams having several large web openings of rectangular shape.

Somes and Corley (1974) investigated the strength and behavior of circular openings in continuous lightweight aggregate concrete joist floors. The authors reported that circular openings could be accommodated without costly reinforcement. McMullen and Daniel (1975) presented equations for predicting the torsion strength of longitudinal reinforced concrete beams that contained a short and long rectangular shaped opening with and without round corners.

In the 1980s, extensive investigations made on this subject; Mansur *et al.* (1983) performed an experimental research to study reinforced concrete beams with large rectangular openings under pure torsion. The authors found that torsion strength and stiffness of a beam decreased with increasing opening length or depth; however it was marginally influenced by its eccentricity. The beams followed the failure mechanism by forming of four hinges, one at each corner of the opening. Mansur *et al.* (1983) further studied the torsion behavior by developing an analytical model using collapse load analysis for predicting the strength of RC beams containing a rectangular opening. The developed model was found in good agreement with the experimental results. Similarly, other researchers also developed models for predicting the torsion strength (Alwis and Mansur, 1987; Hasnat and Akhtaruzzaman, 1987). Alwis and Mansur (1987) developed a method for predicting the torsion strength of RC beam containing a rectangular opening using the principles of limit analysis. Meanwhile, Hasnat and Akhtaruzzaman (1987) generated a set of equations using the skew bending model that can estimate the torsional strength and failure mode of RC beams with or without a small rectangular transverse opening. Apart from many studies on torsion behavior, various studies considered the effects of bending and shear. Investigations were conducted to develop a design method for the design of RC beams with large rectangular opening subjected to combined bending and torsion (Mansur, 1983) and another study involved bending and shear (Mansur *et al.,* 1985). Mansur and Paramasivam (1984) also designed a method of analysis for calculating the strength of RC beams with a small transverse circular opening under the effects of combined bending and torsion. Another method determined the ultimate strength of beam with a large rectangular opening and subjected to a point load (Mansur *et al*., 1984).

Until 1990s, all researchers investigated the strength of RC beams with openings but none of the studies involved deflection response due to such openings. Mansur *et al.* (1991) tested eight RC continuous beams containing a large transverse opening. The number of span, size of the opening and location along the span were the test parameters. The test results indicated four different stages in the load deflection curve of a continuous beam. A further study was done to verify the deflection response of RC beam containing large rectangular opening and a circular opening by Mansur *et al.* (1991, 1992). The authors determined equivalent stiffness of the opening segment of the beam under service load to find out the deflection. In the late 1990s, such studies also included T-beams with large web openings such as that were done by Tan *et al*. (1996), where openings were made in the positive and negative moment regions by testing 15 numbers of beams.

Later, studies conducted on reinforced concrete beam with small opening dealing with shear behavior of beams. Mansur (1998) published the results of effects the behavior and strength of RC beams with opening subjected to predominant shear failure. Based on interpretation of results, openings were classified as "large" or "small". The author had identified two types of diagonal tension failure for small openings. Similar other researches include the study of the design of RC beams with small openings under the application of combined loading. Mansur (1999) proposed a strength design method of RC beams containing small opening and subjected to combined torsion, bending and shear forces. The proposed method introduced two different types of failure, known "beam failure" and "frame failure". In 1999, Mansur et al. studied the performance of existing beams by creating circular opening similar to the drilling holes in 9 numbers of T-beams; size and location of openings were the main variables. Results showed that making holes near the support caused the early diagonal cracking and the significant reduction in the strength and stiffness.

It can be summarized that in the last 25 to 30 years, many efforts were made to understand the behavior of RC beams by incorporating openings in different shapes, sizes, configuration (single or multiple) and at different locations. In such studies, single span and multiple spans rectangular and T-sections were tested under bending, shear and torsion dominant forces. Based on test results, various analytical models have been developed. In general, the area of beams with openings has been broadly explored. The next stage of research on strengthening of openings using internal reinforcement or externally applied FRP laminates is still ongoing. The next part of this paper presents the review analysis of different options for strengthening of openings.

2.2 Review of Strengthening of Openings in RC Beams

2.2.1 Strengthening using Internal Reinforcement

As discussed in the first part of the paper that the detrimental effects of openings in beams include the loss of ultimate strength, reduction in stiffness and large deflection response. Therefore, to incorporate openings in RC beams either in the pre-design & construction stage or in the existing beams, an appropriate strengthening option is important. In this section, reviewed results of the internal reinforcement around the openings are discussed.

Mansur *et al*. (1985) proposed a rational design method for RC beams with large rectangular openings and subjected to bending and shear force. Using this method, 12 numbers of beams designed and tested under single point load. In this research, span length, depth of section, load eccentricity, location of openings, and the amount and arrangement of corner reinforcement were main variables. The authors reported that the diagonal bar as corner reinforcement effectively control the crack and deflection than the vertical stirrups. In another study, Tan *et al.* (2001) studied the shear design of reinforced concrete beams with circular openings using the modified ACI Code approach. T-beams with circular web openings designed for moderate to high shear force were tested in an inverted position to simulate the negative moment conditions exists in a continuous beam. The authors remarked that crack control and maintaining of ultimate strength was achieved by providing reinforcement around the opening. Diagonal bars were found to reduce the high stresses in the compression chord, thus premature crushing of the concrete could be avoided.

2.2.2 External Strengthening using FRP Materials

Strengthening and retrofitting using Carbon Fiber Reinforced Polymer (CFRP) has received wider attention from the research community. Various studies have conducted using FRP materials to increase the flexural strength of reinforced concrete beams. Alagusundaramoorthy *et al.* (2003) studied the effectiveness of externally bonded CFRP sheets or carbon fiber fabric in increasing the flexural strength of concrete beams. The behavior of concrete beams strengthened with pre-stressed CFRP by Franca *et al*. (2007) and external bonding of high strength, light weight Fiber Reinforced Plastic (FRP) plates (Ross *et al*., 1999) concerning the flexural strength was studied both experimentally and analytically. The strength of reinforced concrete continuous beams bonded with CFRP laminates (Ashour *et al*., 2004) were studied experimentally. Results show that FRP is very effective for flexural strengthening.

The reinforced concrete structure is often strengthened in flexure. Thus, further investigation was conducted to study the effect of shear strengthening of reinforced concrete structure using FRP materials (Li *et al.*, 2001). The shear behavior of deep beams with externally bonded CFRP shear reinforcement (Zhang *et al.*, 2004) and externally bonded Fiber Reinforced Polymer (FRP) systems (Islam *et al.*, 2005) were studied experimentally. A review was conducted by Khalifa *et al.* (1998) to study the current research on shear strengthening with FRP and design algorithms was proposed to compute the contribution of FRP to the shear capacity of reinforced concrete flexural members.

Apart from flexural and shear strengthening of reinforced concrete beams using internal reinforcement, various investigations were carried out to study the behavior of reinforced concrete beams retrofitted using FRP materials. Allam and Ebeido (2003) experimentally studied the behavior of RC beams strengthened with Carbon Fiber Reinforced Polymers (CFRP) sheets. Similarly, Santhakumar *et al.* (2004) conducted a numerical study to simulate the behavior of RC beams retrofitted for enhancement of shear capacity. In both experimental program and analytical study; Norris *et al.* (1997) investigated the behavior of damaged RC beams retrofitted with thin CFRP sheets. Based on the referred researches, it is possible generalized that external lamination of CFRP sheets can significantly enhance the shear capacity of RC beams, whereby the dominating mode of failure of retrofitted beams depends on configuration and design of CFRP schemes. Mahmoud *et al.* (2009) investigated the debonding phenomenon of the CFRP and concrete joints in shear and subjected to cyclic and monotonic loading.

2.2.2.1 Strengthening around Openings using FRP Materials

The literature review concluded that in the past, very limited research dealt with the strengthening around openings using FRP materials. Mansur *et al.* (1999) tested 9 numbers of T-beams containing circular openings simulating the conditions of the

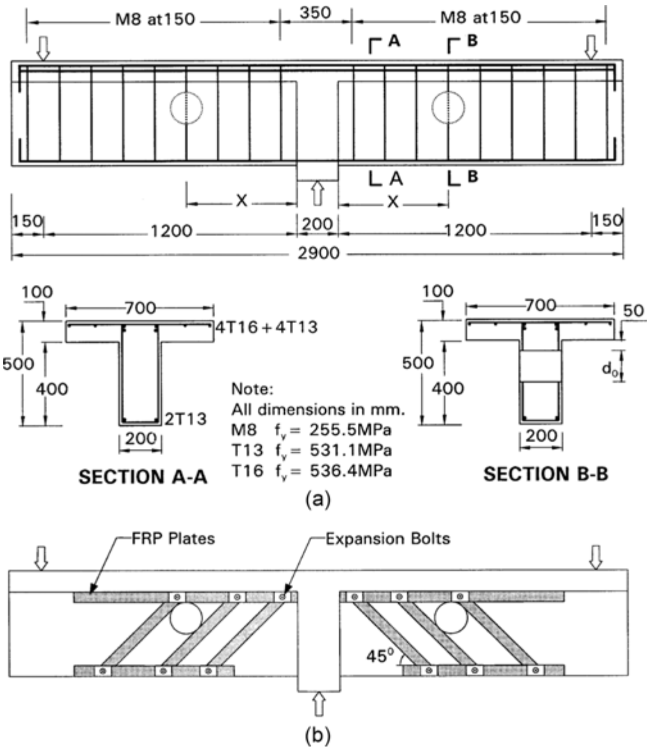

Fig. 1. Details of Beams (Mansur *et al.*, 1999): (a) Reinforcement Details of Beams, (b) Arrangement of FRP for Beam $D_{15}X_4$-F

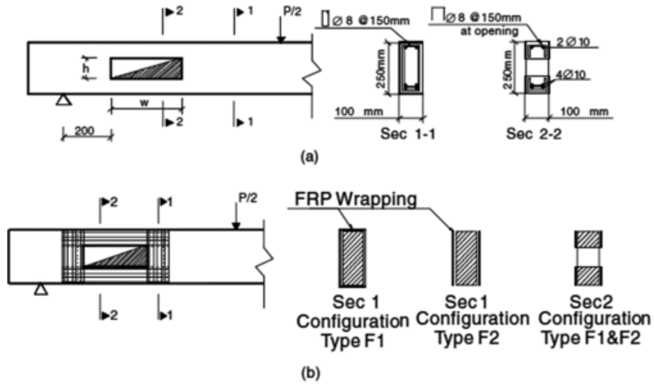

Fig. 2. Details of the Tested Beams: (a) Internal Steel Reinforcement, (b) Types of External CFRP Strengthening (Abdalla *et al.*, 2003)

negative moment of a continuous beam. By creating an opening using coring technique in the existing beam, it did not affect the integrity of the surrounding concrete. FRP plates in the form of a truss around the opening were used to achieve the original capacity of the beam. To prevent premature debonding, two horizontal plates with an expansion bolt anchored three diagonal plates on each face, which is illustrated in Fig. 1. Test results showed that appearance of excessive cracks, large deflection, and the loss of ultimate strength by creating an opening in existing beams could be eliminated by strengthening using FRP plates. Based on the test results of the strengthened beams, the contribution of the FRP plates V_F to ultimate shear resistance was obtained as:

$$V_F = A_F f_{FS} sin\alpha\left(\frac{E_F}{E_S}\right) \quad (1)$$

in which, A_F, f_{FS}, and α are the area, tensile stress, and angle of inclination of the FRP plates, respectively, and E_F and E_s are the modulus of elasticity of the FRP plates and stirrups, respectively. The calculation of f_{FS} was based on the average strains measured for the FRP plates at ultimate condition.

Abdalla *et al.* (2003) investigated the application of FRP laminates to reinstate the lost capacity of beam containing openings. In this study, openings made in the shear zone were 200 mm away from the support. The main variables of the study were; the size of opening (width and depth), and the amount and configuration of the FRP sheets. Ten beams of the size of 100 mm × 250 mm in section and 2000 mm clear span were tested as the simply supported beams using 2-point loading, the opening height was 60% of the beam depth. With the help of deflection, strain, cracking and ultimate load results; an analytical procedure estimating the strength of the beam was developed. The research concluded that creating an opening in the shear zone caused a huge reduction in the ultimate capacity of the beams. Fig. 2 shows the arrangement of carbon fiber reinforced plastic, CFRP laminated around the opening. The strengthening technique resulted in reduction of maximum deflection and cracking around the opening, and enhanced the ultimate capacity of the beam. It was observed that the shear failure at the chords of strengthened openings occurred due to a combination of shear cracking of concrete and the bond failure of the glued FRP sheets. A conservative design method effectively estimated the shear capacity of the beams with strengthened openings.

Allam (2005) reported the study of 9 simple span RC beams containing externally strengthened large openings in the shear zone. The beams were tested to failure under two concentrated loads. This study concluded that creating an opening in shear zone substantially reduces its strength. Therefore, strengthening using internal reinforcement is not an adequate method; hence, the lost capacity was reinstated using externally applied reinforcement. Fig. 3 shows the details of externally strengthening around the opening using CFRP and steel plates. Externally applied steel plates showed high efficiency as compared to performance of externally applied CFRP sheets, the main reason is that the steel is the isotropic material, where in CFRP sheets, carbon fibers are unidirectional oriented. In the final part of this paper, theoretical analysis was performed; according to the Egyptian code, shear capacity of beams with opening in shear zone is calculated as:

$$V_c = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}\delta_c(bd) \quad (2)$$

$$\delta_c = 1 + 0.7\left(\frac{N_c}{A_c}\right) \quad (3)$$

where, δ_c represents the increase in the shear force capacity due to the effect of the axial compressive force N_c, and A_c is the area

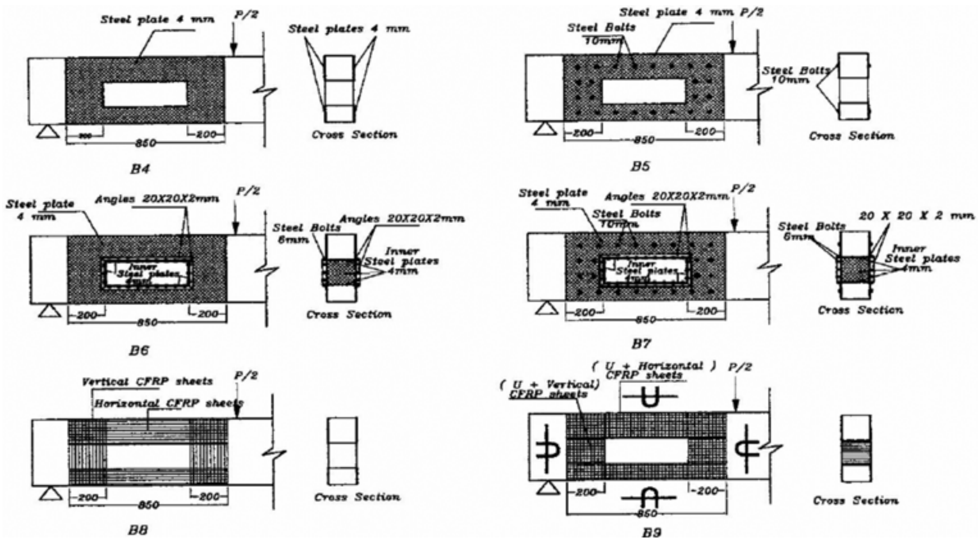

Fig. 3. Details of Strengthening around Opening using CFRP and Steel Plates (Allam, 2005)

of concrete section as shown in Eq. (3). The contribution of the strengthening using steel plates, V_p can be estimated using Eq. (4), whereas Eq. (5) used to determine the contribution of the FRP laminates, V_F.

$$V_p = 2\tau\left(\frac{dh_p}{2}\right) \tag{4}$$

$$V_F = 2t_f f_{Fe} d_f \tag{5}$$

El Maaddawy and Sherif (2009) also used externally bonded CFRP sheets to upgrade 13 RC deep beams with openings, which tested under 4-point bending. The beam dimensions 80 mm × 500 mm in section and 1200 mm long. The clear span was set as 1000 mm whereas the shear span was 400 mm. Two square openings, one in each shear span, placed symmetrically about the midpoint of the beam. The test parameters included the opening size, location and the presence of the CFRP sheets. Strengthening using CFRP around the opening helped to enhance the shear capacity in the range of 35-73%. The experimental results compared with analytical model based on structural idealization of RC deep beams with openings as shown in Fig. 4. The shear strength of strengthened beam with opening, V_{anl}, can be estimated by:

$$V_{anl} = V_c + V_{sd} + V_f \tag{6}$$

where, V_c is the contribution of concrete to shear strength, V_{sd} is the contribution of tension steel to shear strength (dowel action), and V_f is the contribution of CFRP sheets to shear strength. A set of equations used to determine the components of Eq. (6) is given in the referred publication.

Madkour (2009) discussed the nonlinear behavior of strengthened RC beams containing rectangular openings using a numerical model of damage-non-linear elastic theory. The model analyzed the efficiency of externally applied CFRP lamination around the openings. The model considered the effects of different opening heights and the CFRP configurations around the opening. In numerical solutions, the author performed 2-D and 3-D analysis and obtained very close agreement with the experimental test results.

In a recent study, Pimanmas (2010) tested the RC beams with opening those strengthened FRP rods, for which he used 13 RC beams. The opening was provided in the shear zone significantly reduced the capacity of the beam. The FRP rods were applied in two different patterns; in the first pattern, rods surrounded the

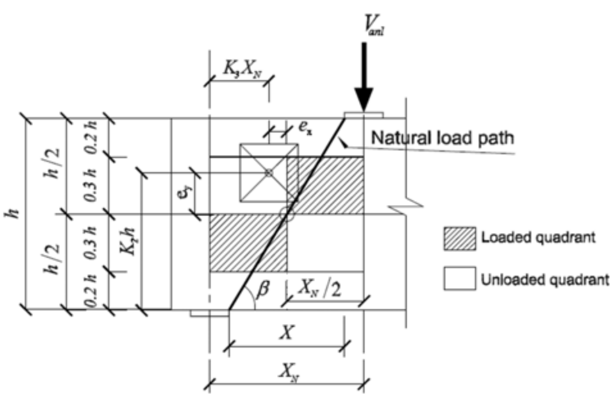

Fig. 4. Structural Idealization with Opening (El Maaddawy and Sherif, 2009)

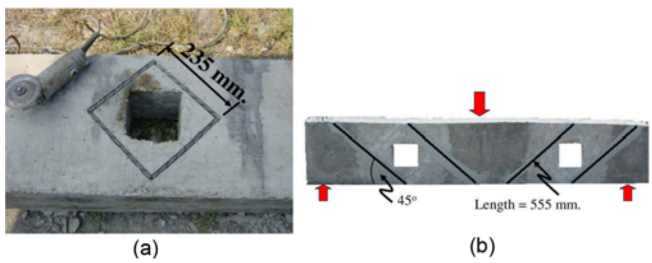

Fig. 5. Patterns of FRP Rods (Pimanmas, 2010): (a) Beam R-Rod1-s1, (b) Beam R-Rod2-s1

Table 1. Summary of FRP as Strengthening Material in Reinforced Concrete Beams with Opening

Country	Authors	Year	Opening Shape	Strengthening material	Type of beam	Bending/ Shear/Torsion	Investigations
Malaysia/ Singapore	Mansur *et al.*	1999	Circular	FRP plates, expansion bolts	T-beam	Shear	Experiment
Egypt	Abdalla *et al.*	2003	Rectangular	CFRP sheets	Simply supported	Shear	Experiment and theoretical analysis
Egypt	Allam S.M.	2005	Rectangular	Steel plates, CFRP sheets	Simply supported	Shear	Experiment and Theoretical analysis
UAE	El Maaddawy and Sherif	2009	Square	CFRP sheets	Deep beam	Shear	Experiment and Theoretical analysis
Egypt	Madkour H.	2009	Rectangular	CFRP laminates	Simply supported	Shear	Numerical Modelling
Thailand	Pimanmas A.	2010	Circular and square	FRP rods	Simply supported	Shear	Experiment and Numerical Modelling

opening whereas in the second group FRP rods were diagonally placed along the entire depth of the beam. Test results of un-strengthened beams showed a huge reduction in the capacity of the beam and experienced the brittle shear failure. Strengthening using first pattern of FRP rods found inadequate due to propagation of the diagonal cracks throughout the beam. Due to diagonally placed FRP rods, substantial improvement in strength and ductility was achieved. Fig. 5 shows the strengthening patterns around opening by using FRP rods. Table 1 summarizes the research activities involving externally bonded FRP laminates to strengthen the RC beams containing different types of openings.

3. The Way Forward

The detailed review of the past researches has identified few gaps, which need further investigations. Currently, the authors are working on a comprehensive project dealing with the identified gaps extracted from the reviewed literature. The author's scope of work comprises of the numerical analysis for effective strengthening using CFRP laminates around openings provided in the shear and bending zone and experimental testing. The numerical analysis involved the 2-D non-linear finite element analysis using ATENA software.

4. Conclusions

In the last five decades, extensive research activities involved to determine the effects of openings in terms of shape (circular, square and rectangular), size and location (bending, shear, and torsion) on the strength and stiffness of RC beams. Effects of large square and circular openings placed at critical locations in bending and shear are identified as the main gaps.

1. In the past, most of the studies focused on experimental investigation, there were very little studies presented theoretical analysis and numerical modeling. For the future research, there is a great need to investigate the effects of openings using numerical analysis such as 2-D and 3-D finite element analysis and developed empirical equations that can be used as the design aids.
2. As discussed, there is very limited research found on strengthening of openings using externally applied FRP sheets. It was obtained that providing of opening in shear zone caused huge reduction in the strength and stiffness that could not be recovered by internal reinforcement. Therefore, further exploration of the option of externally applied FRP sheets has great potential particularly effective strengthening.
3. From literature review, it is obtained that almost in all of the available researches; beams were tested under monotonic loading. Beams subjected to fatigue loading are identified as the needed area for future research.

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