Experimental and Theoretical Investigation of Wave Forces on Rigid Vertical Cylinders

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***Abstract*— An experimental study has been carried out to investigate the hydrodynamic forces on rigid tubular cantilever cylinders with a free-end condition subjected to regular waves. This paper presents the preliminary results of wave tank model test conducted to determine the hydrodynamic forces on vertical rigid cylinders subjected to regular wave loadings. The paper emphasizes on comparison of experimental results with those determined numerically using Morison equation. The comparison of experimental results with the numerical estimations is necessary for accurate prediction of hydrodynamic coefficients. The experimental data are generated from wave tank model test with a scale factor of 1: 40. The cylinder diameters covered in this paper are 42 mm and 34 mm respectively. The findings presented in this paper are part of on-going experimental investigations intended to determine accurate inertia (C_m) and drag (C_d) coefficients. Generally, the experimental hydrodynamic forces are in good agreement with the theoretical values. The findings of this research can be used to determine accurate hydrodynamic coefficients for Malaysian offshore locations.**

***Keywords*— Drag coefficient; Hydrodynamic forces; Inertia coefficient; Malaysian offshore locations; Morison equation; Tubular cylinders.**

I. Introduction

The hydrodynamic forces on slender tubular cylinders resulting from waves are normally estimated using the semi-empirical Morison equation which treats the inline force on a bluff body as the summation of drag and inertia forces superimposed linearly. The term hydrodynamic forces is commonly used to define all forces acting on the cylinder by the fluid due to the relative motions between the fluid and the structures [1]. According to the existing literature, Morison equation is the only practical method adopted for engineering purposes [2], however, accurate estimation of dynamic response analysis of tubular cylinders subjected to waves remains a cumbersome task as the interaction between moving fluid and structure is very complex phenomena. Reynolds number (*Re*), Keulegan-Carpenter number (*KC)*, cylinder configuration, and characteristics of incoming flows are some of the key parameters that influence the hydrodynamic forces of tubular cylinders [3]. The survey of existing literature revealed that several successful experimental investigations pertaining to estimation of hydrodynamic forces and the corresponding hydrodynamic coefficients of single rigid cylinders have been conducted. The pioneering research on hydrodynamic force acting on slender pipe fixed at the bottom was first published by Morison et al. [4]. They have suggested that the force exerted by unbroken surface wave on a vertical cylinder pile which extends from the bottom through the free surface is composed of drag and inertia forces which can be estimated using the Morison equation. Later, several comprehensive reviews and cconsiderable research have been conducted including articles by [5-10]. In another study, Troesch and Kim [11] have studied theoretically and experimentally the hydrodynamic forces acting on cylinders with different cross sectional geometry such as circle, rounded square, and square oscillating at small amplitude. In addition, several studies have been conducted to determine the effects of hydrodynamic forces on group of circular cylinders such as [12-15], whilst Kudeih et al. [16] have conducted experimental investigations in order to study the hydrodynamic forces exerted by random waves and current on an array of three vertical cylinders in shallow water. Generally, estimation of hydrodynamic forces on tubular cylinders experimentally is not a new topic, but the use of constant hydrodynamic coefficients by designers and consultants during the estimation of hydrodynamic forces equation is questionable, as inertia and drag coefficients are both functions of Reynolds number (*Re*) and Keulegan-Carpenter number (*KC*) which vary with time and with water depth. Hence, in the present paper, comparison of experimental test results with theoretical hydrodynamic forces estimated using Morison equation is presented. The comparison of experimental results with the numerically estimated values is necessary for accurate prediction of hydrodynamic coefficients. However, the scope of the present paper is limited to the comparison of hydrodynamic forces, while the corresponding hydrodynamic coefficients are presented in a separate report.

II. Theoretical Background

In the following paragraphs, the theoretical formulations, method of calculating hydrodynamic forces, and the Airy wave theory are discussed briefly:

A. Calculation of Hydrodynamic Forces

The determination of wave forces exerted on structures is very complex. Morison's equation is generally used for estimating the external viscous hydrodynamic forces on tubular cylinders subjected to wave loadings. Two types of fluid forces result from fluid motions i.e. drag and inertia forces. Wave load per unit length is estimated using Equation 1.

$$F = C_m \frac{\rho \pi D^2}{4}\frac{du}{dt} + C_d \frac{\rho D}{2}|u+U_c|(u+U_c) \quad (1)$$

Where C_m and C_d are the hydrodynamic inertia and drag coefficients respectively, ρ is the water density, D is the pipe diameter, u is the water particle's velocity, U_c is the current velocity and du/dt is the water particle's acceleration.

B. Linear Wave Theory

Liner Airy wave theory can be defined as first-order, small amplitude gravity waves with a sinusoidal profile [17]. Fig. 1 depicts the general definitions for two dimensional, linear water wave theory. The surface elevation of an airy wave of amplitude A at any instant of time t and horizontal position x in the direction of travel of wave is denoted by $\eta(x,t)$ and is given by:

$$\eta(x,t) = A\cos(kx - \omega t) \quad (2)$$

where wave number $k = 2\pi / l$ in which l represents the wave length, and angular frequency $\omega = 2\pi / T$ in which T represents the period of the wave. The celerity or speed of the wave is given by L/T or ω / k and the crest to trough wave height is given by $2A$. The linear wave theory is only valid and applicable for non-breaking waves when the amplitude is small compared to the wave length and the water depth.

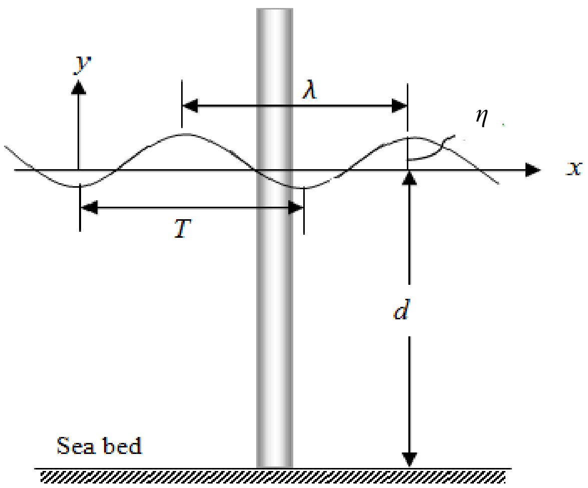

Fig. 1. Airy wave definition diagram

The along wave $u(x,t)$ and vertical $v(x,t)$ water particle velocity in an Airy wave at position y measured from the mean water level in depth of water d are given by:

$$u = \frac{\pi H \cosh ks}{T \sinh kd}\cos\theta \quad (3)$$

$$v = \frac{\pi H \sinh kx}{T \sinh kd}\sin\theta \quad (4)$$

$$\frac{du}{dt} = \frac{2\pi^2 H \cosh ks}{T^2 \sinh kd}\sin\theta \quad (5)$$

$$\frac{dv}{dt} = -\frac{2\pi^2 H \sinh ks}{T^2 \sinh kd}\cos\theta \quad (6)$$

in which, $s = y + d$, $\theta = ks - \omega t$, y is the height of the point of elevation and t is the time instant at which the water particle's kinematics is evaluated, H is the wave height, du/dt and dv/dt are the vertical and horizontal water particle's accelerations respectively.

III. Experimental Setup Details

In the following sections, the wave tank details, experimental model set up and data acquisition system adopted in this experimental investigation are explained:

A. Wave Tank Details

The experimental setup outlined in this paper is a part of ongoing experimental study pertaining to determination of accurate hydrodynamic coefficients. The tests were carried out at University Teknologi PETRONAS wave tank of 22 m long, 10 m wide and 1 m water depth. The wave tank consists of wave maker system with 16 paddles, capable of generating regular, random, bidirectional and multidirectional waves. In addition, the wave tank is also equipped with a dynamic wave absorber which is mainly designed to minimize the reflection of generated waves. In this study, the experimental model was fixed to the soffit of the wave tank bridge, approximately at 10 m from the wave maker. Measurements for wave profile were carried out using wave probes, while load sensors, especially designed by the research group, were used to measure the hydrodynamic forces acting on the model. The load sensors are designed to measure the total hydrodynamic forces on each pipe. An aluminum block of 100 mm (H) x 50 mm (W) x 2 mm thick was fabricated using structural aluminum type 6061, with ultimate tensile strength of 310 MPa. In addition, four 120 Ω general purpose strain gauges, type KFG-3-120- C1-23L1M2R, 3 mm gauge were fixed to the aluminum block to form a wave load sensor [18].

B. Experimental Model Details

In this experimental study, rigid circular cylinders vertically mounted on the overhead bridge were tested. The tubular cylinders are galvanized steel pipes having an outer diameter D_1 = 34 mm and D_2= 42 mm and a wall thickness of 2.5 mm and 3.2 mm respectively. The immersed length of the model was l = 850 mm, giving an aspect ratio of l/D_1 = 25 and l/D_2 = 20. The models were subjected to regular waves with fixed wave heights of 0.2 m and different frequencies ranging from

0.33 to 1 Hz. However, this paper provides the analysis of hydrodynamic forces for modal frequencies of 0.33, 0.5, and 0.8 Hz, and the recorded forces were scaled up and analyzed. The most common dimensionless scaling law for the fluid structures test is the Froude's Law [19]. Using Froude's law and the scale factor as λ, the suitable multipliers to be used to obtain the prototype values from the model test are calculated. In this study, a scale factor of 40 was chosen, as the maximum water depth in the wave tank is 1m, while area of study has actual prototype water depth varying from 40 to 55 m. The model properties and the scaled up prototype dimensions are presented in Table 1 and the model set up in the wave tank is shown in Fig. 2.

C. *Data Acquisition and Processing*

As the hydrodynamic forces resulting from wave forces are dynamic and change over time, the data were acquired through a standard dynamic data acquisition system type DC-204R. The data logger has a maximum sampling speed of 5μ-sec/channel, upgraded with 10 kHz frequency response and 200 kHz sampling. The sampling rate for the hydrodynamic forces was set to 100 Hz to synchronize the forces sampling rate with that of the wave probes used to record the wave amplitudes. The strain gauges based force transducers used in this study were designed in such a way that the moment developed along the vertical cylinder is neglected. This was proven during the calibration stage of the wave force sensors by placing a well-defined concentrated load at different location along the cantilevered beam and the recoded forces were found to be identical. Further, in order to monitor the wave profiles in the wave tank, four wave gauges with active length of 900 mm wave probe diameter of 6 mm and type EQ-009 R4 were used to measure the rapidly changing water levels during the experimental model test in the wave tank.

D. *Wave Programme*

In this experimental investigation, a series of regular waves were generated in the wave tank as depicted in Table 2. The waves were generated for modal frequency of 0.33 Hz, 0.50 Hz, and 0.80 Hz, while the wave height was fixed to 0.2 m. These wave parameters were selected based on the wave tank capacity, and the actual met-ocean criterion which were scaled down for the wave tank model test.

TABLE I. PROPERTIES OF THE MODEL AND THE PROTOTYPE

Model			**Prototype**		
Pipe Diameter (m)	Pipe Length (m)	Wall Thickness (m)	Pipe Diameter (m)	Pipe Length (m)	Wall Thickness (m)
0.042	1.23	0.0032	1.68	49.2	0.128
0.034	1.23	0.0025	1.36	49.2	0.100

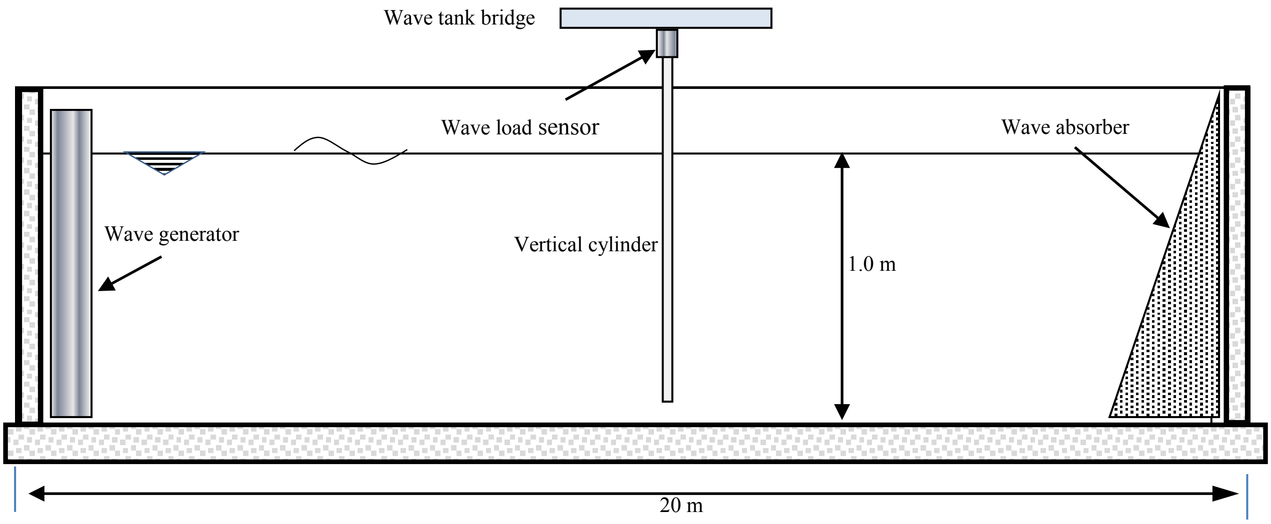

Fig. 2. Cross sectional elevation of wave tank with the model specimen details

TABLE II. REGULAR WAVE PROPERTIES

Model			Prototype		
Wave Height (m)	Wave Period (s)	Wave Frequency (Hz)	Wave Height (m)	Wave Period (s)	Wave Frequency (Hz)
0.20	3.00	0.33	8.00	18.97	0.05
0.20	2.00	0.50	8.00	12.65	0.08
0.20	1.25	0.80	8.00	7.91	0.13

IV. Results and Discussions

In the following paragraphs, the wave tank model test results and the theoretically estimated hydrodynamic forces are discussed.

A. Wave Amplitudes

A typical time domain record for wave amplitude generated in the wave tank for regular waves with H_{max} = 0.20 m and frequency, $f = 0.33$Hz is shown in Fig. 3. This corresponds to scaled up prototype wave with H_{max} = 8 m and $f = 0.0522$ Hz The average recorded wave height was used to estimate the theoretical hydrodynamic forces. The measured data were then post-processed to determine the hydrodynamic forces on the scaled up prototype. Generally, it was observed that the effect of the model on the wave profiles was negligible as the ratio between the pipe diameter (D) and the wave lengths (l) was relatively small i.e. $D/l < 0.2$.

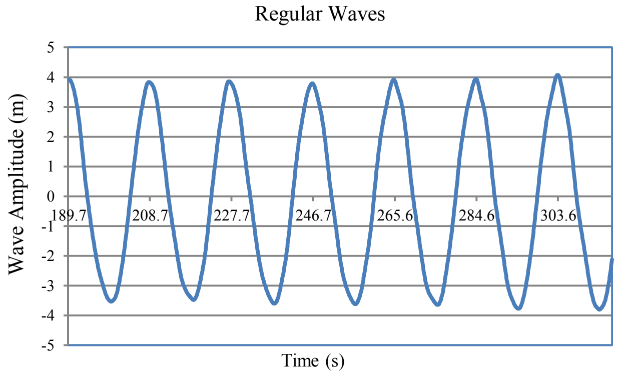

Fig. 3. Typical time series record of regular waves

B. Wave Forces

The experimental results determined from the model test were scaled up to the prototype size using the well-known similitude's law [19] as discussed previously in section III. These values were then compared with the theoretical forces estimated using Morison equation. In this paper, the theoretical hydrodynamic forces were estimated assuming $C_m = 1.60$ and $C_d = 0.65$ [20]. An assumption of $C_m = C_d = 1$ can also be made in order to determine the accurate hydrodynamic coefficients using the least square method, where C_m and C_d values that satisfy both the non-dimensionalized forces i.e. $F_{measured}/F_{thoery}$ and the phase angle θ can be adopted as the optimum inertia and drag coefficients. However, the estimation of hydrodynamic coefficients is beyond the scope of the present paper.

Figs. 4 to 6 depict the comparison of scaled up experimental and numerical hydrodynamic forces for a 42 mm model pipe (1.68 m prototype) subjected to regular waves with H_{max}= 8 m and wave frequency f = 0.052 , 0.079 , and 0.127 Hz. Initially, a pipe with outer diameter of 1.68 m was subjected to regular waves with H_{max} = 8 m and f = 0.052 Hz, the scaled up measured force was 116.6 kN, and the corresponding theoretical value estimated using Morison equation was 136.5 kN. Then, the wave frequency was increased to 0.079 Hz, the measured force was 345.3 kN and the corresponding theoretical value was 145.9 kN, here, it was observed that the measured force was around 50% higher than the theoretical values. Later, the scaled up prototype frequency was increased to 0.127 Hz. In this case, the total maximum force recorded experimentally was 374.2 kN, and the corresponding theoretical value was 304.9 kN. An increamenent of 18% was obsrved on the measured force.

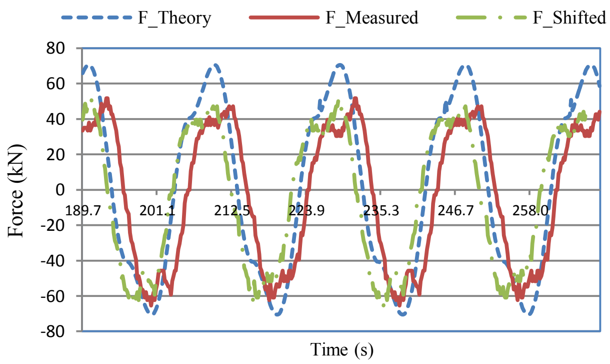

Fig. 4. Comparison of experimental and numerical hydrodynamic forces for 1.68 m diameter cylinder subjected to regular waves with H_{max}= 8 m, and frequency, $f = 0.052$ Hz

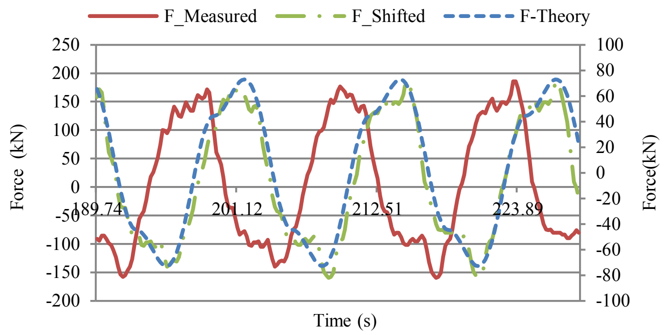

Fig. 5. Comparison of experimental and numerical hydrodynamic forces for 1.68 m diameter cylinder subjected to regular waves with H_{max}= 8 m, and frequency, f = 0.079 Hz

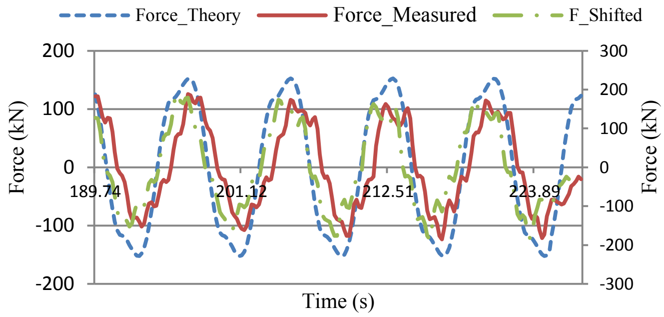

Fig. 6. Comparison of experimental and numerical hydrodynamic forces for 1.68 m diameter cylinder subjected to regular waves with H_{max}= 8 m, and frequency, f = 0.127 Hz

Typically, a model pipe with outer diameter of 34 mm was tested in the wave tank, and the experimental results were compared with the theoretical values estimated numerically using Morison equation. Figs. 7~9 depict the comparison of measured hydrodynamic forces with theoretical values. The scaled up prototype pipe diameter is 1.36 m. The cylinder was initially subjected to regular wave with H_{max} = 8 m and frequency, f = 0.052 Hz, the maximum measured hydrodynamic forces for the prototype was 98.83 kN, while the corresponding theoretical value was 113.2 kN. A difference of 15% was recorded between the two values as depicted in Fig. 7. The wave frequency was then increased to 0.079 Hz. Here, it can be observed that a maximum hydrodynamic force of 290.6 kN was recorded, the corresponding theoretical value at this frequency was estimated as 150.7 kN. The experimental value was 48% higher than the theoretical values as shown in Fig.8. Farther, at frequency of 0.126 Hz the experimental hydrodynamic force was recorded as 255.6 kN, while the corresponding theoretical values was 234.4 kN. The experimental value was 8% higher than the theoretical values as shown in Fig. 9. Generally, the comparison of measured forces with the theoretical values revealed that a phase difference exist between the measured forces and the estimated ones. This is because the wave generator and the data acquisition system were not synchronized.

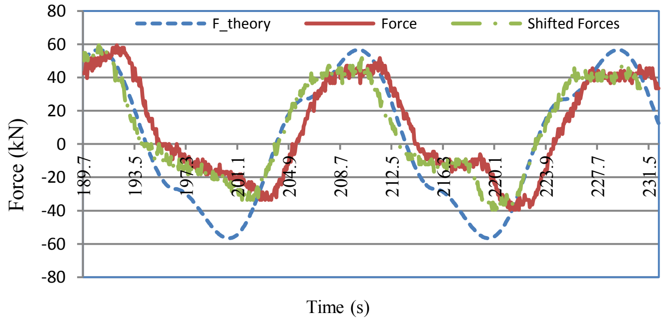

Fig. 7. Comparison of experimental and numerical hydrodynamic forces for 1.36 m diameter cylinder subjected to regular waves with H_{max}= 8 m, and frequency, f = 0.052 Hz

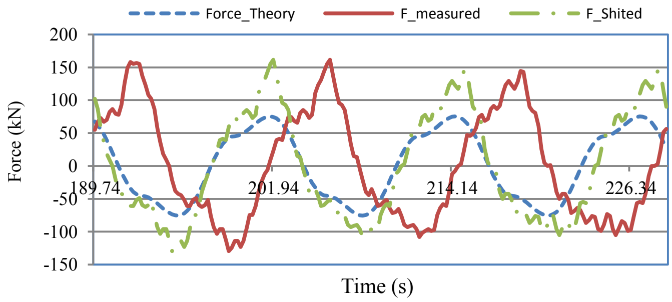

Fig. 8. Comparison of experimental and numerical hydrodynamic forces for 1.36 m diameter cylinder subjected to regular waves with H_{max}= 8 m and frequency, f = 0.079 Hz

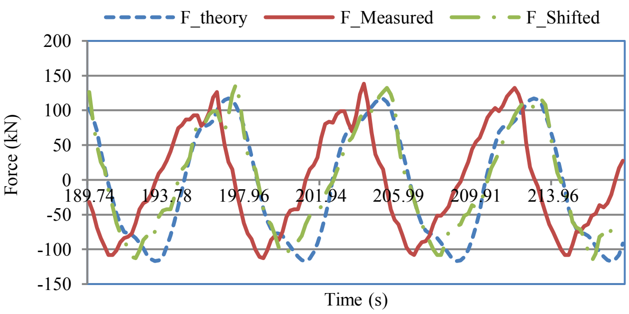

Fig. 9. Comparison of experimental and numerical hydrodynamic forces for 1.36 m diameter cylinder subjected to regular waves with H_{max}= 8 m and frequency, f = 0.127 Hz

The phase angles can be estimated based on the time lag between the graphs or using Fast Fourier Transform (FFT) method. The phase angle has no impact on the hydrodynamic forces, but it should be taken into the consideration when estimating the hydrodynamic coefficients. The shifted hydrodynamic forces are plotted against the theoretical force for better comparison as shown in the above graphs.

V. CONCLUSIONS

Experimental investigation was conducted in the wave tank to investigate the effects of waves on hydrodynamic forces of rigid tubular cylinders, and the findings were compared with the theoretical values estimated using Morison equation. From this study the flowing conclusions can be drawn:

1. The hydrodynamic forces resulting from regular waves on tubular cylinder with outer diameter of 42 mm (1.68 m) and 34 mm (1.36 m) were determined both experimentally and theoretically for H_{max} = 8 m, with different wave frequencies.

2. There is fair agreement between the theoretical forces determined using Morison equation and the experimental results. Generally, the experimental hydrodynamic results have shown comparatively higher values than the theoretical forces. The finding can be used to determine accurate hydrodynamic coefficients.

3. In general, a phase difference was observed between the measured forces and the theoretical results. This is because the wave generator and the data acquisition system were not synchronized. The phase angle is not affecting the hydrodynamic forces, but it should be taken into account when estimating drag and inertia coefficients.

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