

Hydrodynamic Forces on Linear and Multi-Dimensional Arrays of Circular Cylinders

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Abstract. In this paper, major focus is given for determining the accurate wave forces recorded on circular cylinders present in different array configurations. Investigation is made to establish any trend in variation of these forces, when the wave time periods, location of the cylinders in the array and the spacing between cylinders are varied. Wave tank model tests were conducted in regular waves. Tandem cylinder configurations involving a maximum of four cylinders were considered for the present study. In addition to this, two linear arrays of four cylinders each, were installed side by side in the waves and the forces on each of the cylinders were determined. The spacing between the cylinders in this two dimensional array was varied and the trend in the variation of forces are presented. Change of transverse spacing between the cylinders was found to have more effect on the forces on cylinders, than the change in in-line spacing. The wave forces acting on singly existing cylinders were found to be less than that acting on individual cylinders present in the different array configurations considered. The variation of force coefficients with respect to the time periods was found to be of random nature. Morison equation is made use in the determination of these coefficients.

Introduction

The increasing oil and gas demand and the decommissioning of the aged platforms have led to an increase in the offshore platform constructions. On these platforms, many of the members are present in the neighborhood of similar members. To achieve safe and economic design of the offshore platforms, the accurate estimation of the forces on all these members is very essential. The riser tubes are an essential part of offshore platforms and are usually present in groups. Recent investigations on the hydrodynamics of riser interference have been presented in [1,2]. Flow interference between these cylindrical tubes has significant effects on the hydrodynamics of the cylinders [3]. A common practice in the force estimation on the offshore platform members, is to ignore the effect of flow interference on the platform members [4]. Thus the members present in groups are idealized to be isolated members and the isolated member force estimation is done for these kinds of members as well.

The present study is aimed at estimating the wave forces and hydrodynamic force coefficients acting on circular cylinders present in arrays, subjected to regular waves. The influence of various parameters like time period of the waves, position of cylinders in the array, spacing between cylinders, on the wave forces are investigated. Many previous works, including [5,6], presented the relation between the hydrodynamic force coefficients and the Keulegan-Carpenter (KC) number. Since KC number is a function of water depth, it could be inferred that, in the design of cylindrical members to be installed in waves, the designer should consider different coefficient values at different depths of the member considered. Since that is a very tedious and time consuming practice, normally the designers are adopting a conservative value for the force coefficients, that would result in the overdesign of the platform and considering the huge number of members in a

platform, this would also badly affect the economy of the high budget platform design and fabrication. In this project, attempt has been made to estimate the hydrodynamic force coefficients, in relation to the wave time periods, rather than the KC number.

Theoretical Formulations

Estimation of Hydrodynamic Forces: The wave forces acting on circular cylindrical members are estimated using the well known Morison equation [7]. The Morison equation which gives the force on a unit length of a vertical cylinder is given in Eq. 1

$$F = C_M \frac{\rho \pi D^3}{4} \dot{U} + C_D \frac{\rho D}{2} U |U|. \quad (1)$$

Assuming that the cylinder extends from the ocean floor to the still water level, the total force on the cylinder is obtained by integrating Eq. 1 between the water depth limits and making necessary substitutions for the terms. The resulting equation is Eq. 2.

$$F = \rho g V \frac{H}{2d} \tanh kd \left[C_M \sin\phi + C_D \frac{H}{4\pi D} \frac{2kd + \sinh 2kd}{\sinh kd \cdot \sinh kd} \cos\phi |\cos\phi| \right]. \quad (2)$$

where ρ is the water density, g is acceleration due to gravity, V is the volume of cylinder under water, H is wave height, d is water depth, k is wave number, C_M and C_D are inertia and drag coefficients, D is cylinder diameter and ϕ is the phase angle.

Experimentation and Methodology

Model Tests: The experiments were carried out in a 22m long, 10m wide and 1.5m deep wave tank at Universiti Teknologi PETRONAS (UTP), Perak, Malaysia. Unidirectional, regular waves with prototype wave height and maximum prototype time period of 4.125m and 22.25s respectively are studied.

Cylindrical test specimens of model diameter 0.034m were made out of galvanized iron material and were of wet length 0.95m. Each of these cylinders was fitted at its top, with special wave force sensors of dimensions 0.10m x 0.05m x 0.03m, which were capable of measuring wave forces acting on the entire length of the test cylinders, irrespective of the bending moments. Design details of the load cell were published in an earlier paper [8].

These cylinders fitted with the sensors, were installed in the waves as a vertical cantilever fixed at the top. The cylinders were tested in waves with wave height 4.125m and at different time periods as given in Table 1. All the model parameters and forces were scaled up to the corresponding prototype values by making use of Froude's scaling law [9].

Table 1. Time periods and Frequencies of the waves studied

Model T (S)	Model Frequency (Hz)	Prototype T (S)	Prototype Frequency (Hz)
1.0	1.000	7.42	0.135
1.5	0.667	11.12	0.090
2.0	0.500	14.83	0.067
2.5	0.400	18.54	0.054
3.0	0.333	22.25	0.045

Three different linear array configurations of cylinders, involving two, three and four cylinders respectively, were considered in the present study. In all these configurations, the cylinders were arranged in tandem along the direction of the wave propagation; with the in-line cylinder spacing (IS) and transverse spacing (TS) being integral multiples of cylinder diameter (D). Fig.1 shows

these three configurations along with a single cylinder configuration. Two linear array configuration involving four cylinders each, were installed side by side in the waves to obtain a two dimensional array with eight cylinders as shown in Fig.2.

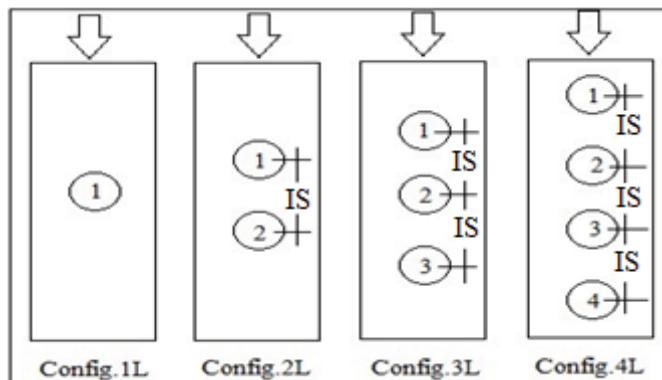


Fig. 1: Linear array configurations

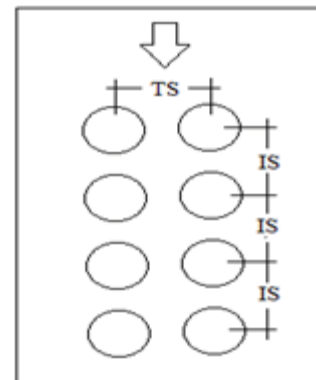


Fig. 2: 2-D array configuration

Methodology: In all the linear array configurations considered (Fig.1), the forces acting on each of the cylinders were plotted against the time period of the waves. Observation is made to find any trend in the variation of wave forces with respect to the position of the cylinder in the array considered. To find the effect of the introduction of a neighboring linear array of four cylinders on the forces acting on an existing similar array, a configuration shown in Fig.2 is made. To study the effect of transverse spacing (TS) between the two linear arrays, on the wave forces acting on each of the cylinders, TS was varied as integral multiples of the cylinder diameter, from 3D through 5D (Config.1TS, 2TS, 3TS), while IS was kept a constant at 3D. In another set of experiments, TS was kept a constant at 3D and IS was varied from 3D through 5D (Config.1IS, 2IS, 3IS). This was to find the effect of the variation of in-line spacing on the wave forces. In addition to all these, forces acting on a single cylinder were also measured for comparison purposes. It is to be noted that Config.1TS and Config.1IS are one and the same with TS and IS both equal to 3D.

The hydrodynamic force coefficients were estimated using numerical computation by adopting the most suitable combination of drag and inertia coefficient and phase angle, such that the deviation of the numerically computed wave forces at the peaks and the zero-crossing points, from the experimental forces, were minimal simultaneously. Eq. 2 was made use for estimating the force coefficients.

In many researches till date [5,6], only a small section of the cylinder will be instrumented and the wave forces acting on this section only will be recorded. The section would normally be near to the water surface where the effect of wave would be felt significantly. The recorded wave forces on the instrumented small portion were used to calculate the hydrodynamic force coefficients. In the present work, wave forces on the entire depth of the cylinders were measured using specially designed wave force sensors and this total wave forces were used for the estimation of force coefficients. Since the KC number is varying with water depth, the presently obtained force coefficients could not be related to a particular KC value. Thus the estimated coefficients were plotted against time periods of the waves.

Results and Discussion

Forces on linear array of cylinders: For all the cylinders in different configurations considered, a decreasing trend, in the wave force, is generally observed when the time periods were increased at the particular wave height considered. Fig.3 shows this decreasing trend recorded for cylinders in different configurations.

The forces on cylinder in Config.1L is found to be less than that acting on cylinders in all the three linear cylinder configurations considered, as can be seen from Fig.3. The dotted line shows the force variation on Config.1L cylinder. This shows that, when the cylinders are present in waves

as a linear array, the forces on them are generally hiked when compared with the forces acting on a singly existing cylinder.

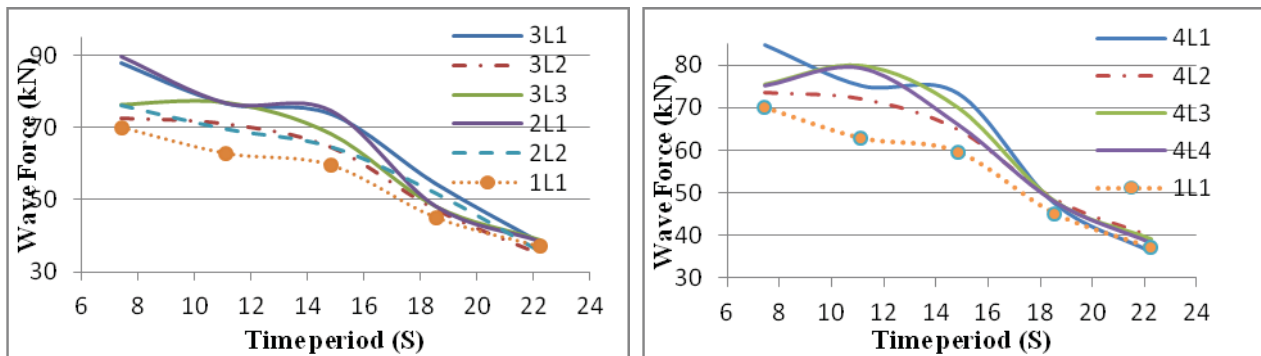


Fig. 3: Variation of wave forces with time periods in linear array configurations

The dashed line in both these figures shows the force variation on the second cylinder in the corresponding configuration indicating that this cylinder is bearing the lowest wave force in the configuration for lower time periods. When time period was increased beyond 14.83s, it was observed that the forces acting on all the cylinders were becoming closer to each other.

Influence of neighbouring cylinders on the wave forces: To study the effect of introduction of leading and trailing cylinders, on the wave forces acting on an existing cylinder, the wave forces acting on first and last cylinders in each of the cylinder configuration is studied. When an individual pipe was tested (Config. 1L in Fig. 2), the maximum force observed was 70 kN at $T = 7.4s$ as can be seen from Fig.3 plot 1L1.

Introduction of leading cylinders: In this section, the cylinder under consideration is referred to as the active cylinder (the last cylinder along the wave direction in every configuration). When a leading cylinder was introduced, as in Config. 2L, the maximum force on the active cylinder was found to be hiked by 8.8%. As can be seen in Config. 3L, when two leading cylinders were introduced, the maximum force recorded on the active cylinder was found to have an increase of 9.3% when compared to the individual cylinder tested in Config. 1L. When three cylinders were introduced in front of the active cylinder as in Config. 4L, the maximum force on active cylinder was raised by 7.5%. All these maximum forces were recorded at a $T = 7.4s$.

Introduction of trailing cylinders: In this section, the cylinder under consideration is referred to as the active cylinder (cylinder 1 in every configuration). When a trailing cylinder was introduced, as in Config. 2L, the maximum force on the active cylinder was found to be hiked by 27.9%. Another trailing cylinder was introduced and the maximum force recorded on the active cylinder was found to have an increase of 25.4% when compared to the individual cylinder tested in Config. 1L. When three cylinders were introduced behind the active cylinder as in Config. 4L, the maximum force on active cylinder was raised by 20.8%. Here also, these maximum forces were recorded at a $T = 7.4s$. The above results show the significant modifications on the wave forces acting on the cylinders when existing in the vicinity of other cylinders.

Forces on multi-dimensional array of cylinders: To find the effect of the variation of in-line spacing (IS) between the cylinders in configurations involving eight cylinders, the transverse spacing (TS) was kept constant at 3D and the IS was varied as 3D, 4D and 5D and the corresponding three configurations were named as Config. 1IS, 2IS and 3IS. The average wave forces acting on the two 1st and two 3rd cylinders, in all these three cylinder configurations are presented in Fig.4.

It was observed that, the forces acting on all the cylinders were found to be close to each other at all the time periods. This shows that the introduction of a new linear array beside an existing array at a distance of 3D between them, doesn't modify the forces on the existing array to a significant extent. It was also noted that the variation of IS had a very minor effect on the wave forces,

because for all the configurations with different IS, the forces on the array were observed very close to that acting on a linear array having four cylinders (Config. 4L), as can be seen in Fig.4.

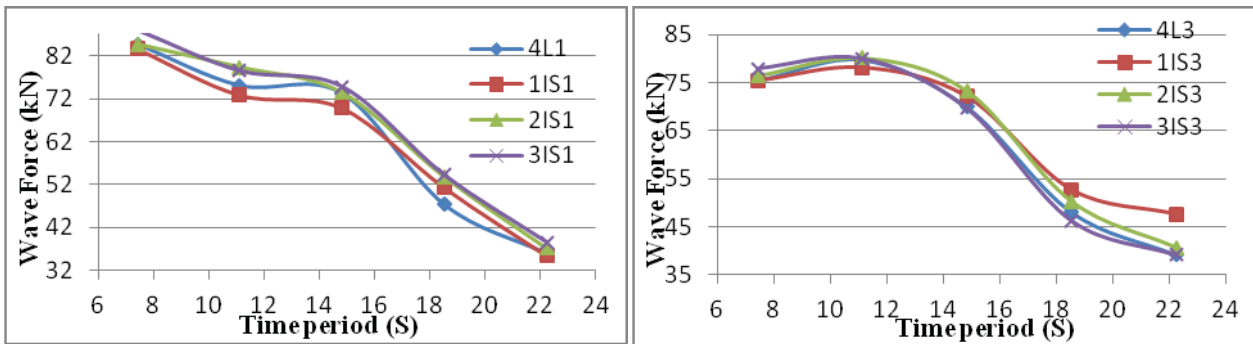


Fig. 4: Variation of wave forces with time periods in 2-D array configurations, showing the effect of variation of in-line spacing

When the TS between the two linear arrays were increased as 3D, 4D and 5D, while keeping IS a constant at 3D, it was observed that for time periods beyond 14.83s, the forces recorded on the cylinders in Config.2TS and Config.3TS are showing a deviation towards the higher side. This is shown in Fig.5.

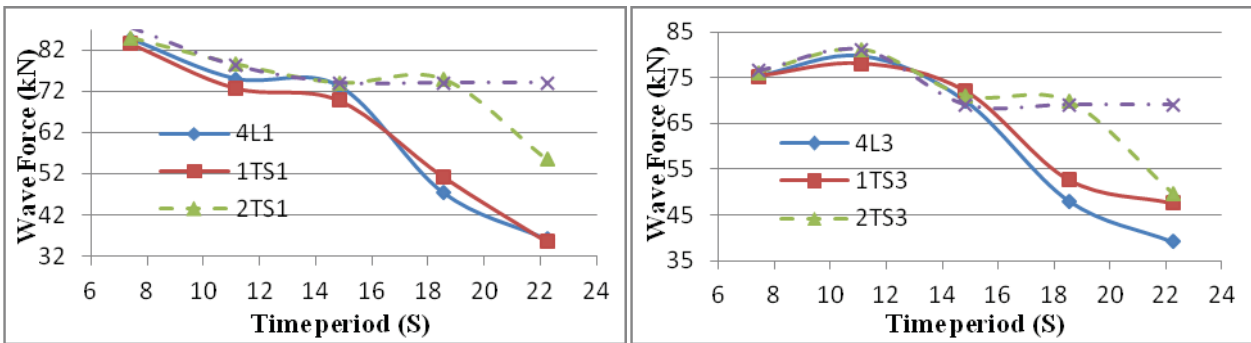


Fig. 5: Variation of wave forces with time periods in 2-D array configurations, showing the effect of variation of transverse spacing

All the above shown force variation recorded on the cylinders when existing in groups calls for some refinements to be made in the force coefficients to be used in the Morison equation for the estimation of wave forces for the purpose of design of jacket platforms. One of the major aims of the ongoing project is to develop a semi-empirical model to predict the hydrodynamic coefficients for group of tubular cylinders. As an initial step towards this the variation of drag and inertia coefficients for the linear arrays studied, are presented here.

Fig.6 shows the scatter of the drag and inertia coefficients acting on cylinders in all the linear array configurations, for the wave height and time periods considered.

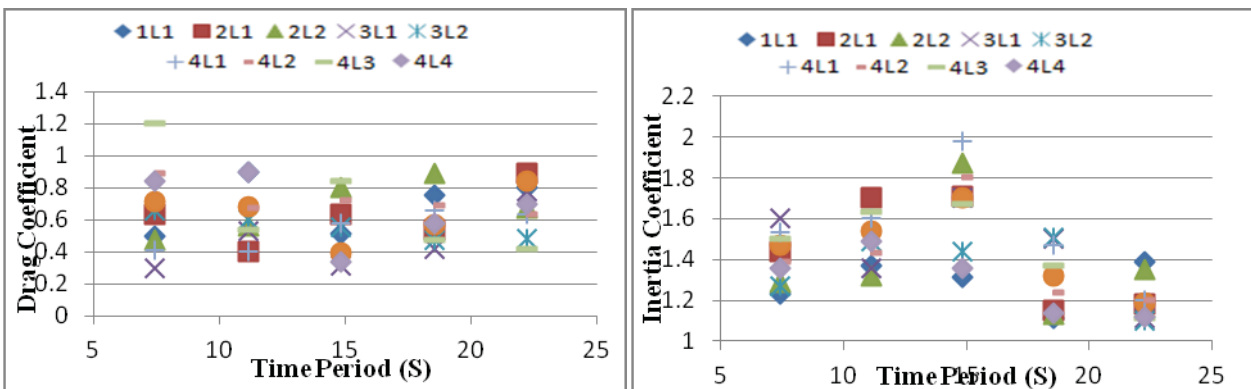


Fig. 6: Variation of drag and inertia coefficients with time periods for cylinders in linear array configurations

The wave forces acting on all these cylinders, as shown in Fig.3, are close to each other and the force variation on each cylinder is similar to one another. Unlike this trend, the drag and inertia coefficients are showing a very random variation with respect to the time periods considered.

Conclusions

The major conclusions emerging out of the present study are listed here:

1. When the cylinders are present in waves as a linear or multi-dimensional array, the forces on them are generally hiked when compared with the forces acting on a singly existing cylinder.
2. In a two dimensional array,
 - a. when the in-line spacing between the cylinders are increased keeping the transverse spacing as a constant, the force acting on the cylinders remain close to that acting on the corresponding cylinders in a single linear array.
 - b. when the transverse spacing between the cylinders are increased keeping the in-line spacing as a constant, the forces acting on the cylinders remain close to that acting on the corresponding cylinders in a single linear array, except for high time periods.
3. Variation of force coefficients with time periods, for cylinders in all the linear array configurations is random in nature.

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