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Nonlinear System Identification of Floating Structures Using Time-Varying ARX–Based Volterra Model

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Abstract. This work presents a new approach for nonlinear system identification of floating structures using time-varying autoregressive with exogenous input (TVARX)–based Volterra model. This method utilizes time series of measured wave heights as system input and surge motion as system output. Solution to the proposed approach is also proposed via Kalman smoother algorithm. The efficacy of the proposed method is then applied in a scaled 1:100 model of a prototype truss spar. It is shown that surge motion obtained from the identification results has good agreement with the experimental results either in time or frequency domain.

Introduction

Floating structures is known as a structural system that exhibit significant nonlinear behavior. Prior researches had been carried out to model its nonlinear behavior through rigorous mathematical models such as [1]-[2] for spar platform. Modeling the nonlinearity through system identification had been carried out by [3]-[4] for tension leg platform. They had implemented the second-order frequency Volterra model successfully and concluded that it was adequate to model the nonlinear relationship between the wave height and surge motion of mini TLP.

However, frequency domain Volterra model needs higher-order spectral moments to estimate the Volterra kernels. If the wave heights are Gaussian, the estimation process is quite straightforward. That is because the only higher-order spectral moments namely cross-bispectrum is needed. The case is getting more complicated when the wave heights are non-Gaussian. One needs to characterize the wave height time series up to fourth-order spectral moments. This process certainly introduces bias and higher variance in the kernels estimation process. Frequency resolution will be trade off in the transfer function (TF) estimation. Sometimes, it leads to untrustworthy TF and affects the reconstructed model output. An alternative solution is offered in this paper by taking benefit on the time domain Volterra models instead of its counterpart. Their structure model in time domain enables the adaptive filter to be implemented. Modification is also proposed in the time domain Volterra model by replacing the linear impulse response function (LIRF) of the Volterra with the TVARX model. It is worth to note that TVARX model is superior in time series analysis and has high-resolution spectral estimation. In this regard, the current work aims to propose the TVARX-based Volterra model. All the identification process is carried out in the framework of wave-to-motion transfer function [5]. The proposed method is then applied to the floating structures, where truss spar platform is taken as case study.

Mathematical Formulations

In this section, mathematical framework as a pavement to achieve the objective of this paper is presented. It covers the concept of proposed approach and its solution method sequentially.

TVARX-based Volterra Model. Second-order Volterra model in a discrete time index n can be expressed in Eq. 1. Notation h_1 is linear impulse response function (LIRF), known as first-order Volterra kernels while term h_2 is quadratic impulse response function (QIRF). This model has memory length K and model error is denoted with $e(n)$, which is Gaussian with zero mean and variance σ_e^2. Further, model structure of TVARX model can be also expressed in Eq. 2,

$$y(n)=h_0+\sum_{i=0}^{K-1}h_1(i)u(n-i)+\sum_{k=0}^{K-1}\sum_{j=k}^{K-1}h_2(j,k)u(n-j)u(n-k)+e(n). \quad (1)$$

$$y(n)=\sum_{i=1}^{P}a_{1(i)}(n)\,y(n-i)+\sum_{\ell=0}^{M}b_\ell(n)u(n-j). \quad (2)$$

In Eq. 2, notations $y(n-i)$ and $u(n-\ell)$ are delayed surge response and wave height variables, called regressors in discrete time index n, respectively. Notation P and M are number of respective delayed regressors which are the order of TVARX model. Values $a_i(n)$ and $b_j(n)$ are the TVARX coefficients which is time-variant. If the coefficients are time-invariant, then it will be equivalent with the well-known linear ARX model. The purpose to make the time-variant coefficients is to accommodate if the nonstationarity exist in the measured system input or output. Modification is put on replacing the RHS of Eq. 1 with TVARX model, Eq. 2 and expressed in Eq. 3,

$$y(n)=\sum_{i=1}^{P}a_{1(i)}(n)\,y(n-i)+\sum_{\ell=0}^{M}b_\ell(n)u(n-j)+\sum_{k=0}^{K-1}\sum_{j=k}^{K-1}h_2(j,k)u(n-j)u(n-k)+e(n). \quad (3)$$

Equation 3 can be expanded into Eq. 4, where the first term of RHS is regressors vector while the second term is Volterra kernels that will be estimated through proposed solution method.

$$\hat{y}(n)=\begin{Bmatrix} y(n-1) \\ \vdots \\ y(n-P) \\ u(n) \\ \vdots \\ u(n-M) \\ u(n-1)^2 \\ \vdots \\ u(n-1)u(n-K-2) \\ \vdots \\ u(n-2)^2 \\ u(n-2)u(n-K-3) \\ \vdots \\ u(n-K)^2 \end{Bmatrix}^T \cdot \begin{Bmatrix} a_{1(1)}(n) \\ \vdots \\ a_{1(P)}(n) \\ b_0(n) \\ \vdots \\ b_M(n) \\ a_{2(1,1)}(n) \\ \vdots \\ a_{2(K-1,1)}(n) \\ \vdots \\ a_{2(K-1,K-1)}(n) \\ a_{2(K-2,K-1)}(n) \\ \vdots \\ a_{2(K,K)}(n) \end{Bmatrix}+e(n). \quad (4)$$

Kalman Smoother. Equation 4 can be formed into more compact form Eq. 5, where vector $\varphi(n)$ is the first term and vector $\theta(n)$ is second term of Eq. 4, respectively.

$$\hat{y}(n)=\varphi(n)^T\cdot\theta(n)+e(n) \quad (5)$$

In order to estimate θ, Eq. 5 can be rewritten in a discrete state-space form and expressed in Eq. 6a,

$$\hat{y}(n)=\varphi(n)\theta(n)+v(n) \quad (6a)$$

$$\theta(n)=A\theta(n-1)+w(n). \quad (6b)$$

If term A is the state transition matrix which will be restricted as an identity matrix, time evolution of $\theta(n)$ is simple random walk, then expressed in Eq. 6b. Terms $v(n)$ and $w(n)$ are the observation and state noise, respectively. At this stage, several adaptive methods can be utilized to estimate the value of $\theta(n)$. Adaptive algorithms such as least mean square (LMS), recursive least square (RLS) may be adopted. Kalman filter (KF) is more superior compared to both. However, as of all other adaptive algorithms, its drawback is the tracking lag. It can be avoided by using the so-called smother algorithm. It is an estimator which utilizes the future measurements in addition to the past ones when computing the estimates at a given time point. Smoother algorithm combined with KF is called Kalman smoother as solution for proposed method in this paper. As in the linear case, difference between model output $\hat{y}(n)$ and the original system output, $y(n)$ must be minimized by defining the cost function in Eq. 7 at each time index,

$$J(n) = \frac{1}{N}\sum_{n=1}^{N}\left(y(n) - \hat{y}(n)\right). \tag{7}$$

Details about LMS, RLS, Kalman smoother algorithms can be found in [6] for reference.

Experimental Setup

The method is then applied to the truss spar model. The model was designed with scale of 1:100 to fit the wave tank by following the Froude's law of similitude. The test model was moored with four mooring lines. The linear springs were connected to load cells to measure the mooring system loads. The model was then tested in the wave tank of the offshore engineering laboratory, Universiti Teknologi PETRONAS. The wave tank has 22 m length, 10 m width and 1.5 m depth. The rigid body motions were measured by optical tracking system, while wave heights were measured with wave probes. Two probes were placed in front of the model and the rest at the back of the model. The sea-keeping characteristic of the model was tested under random waves. The schemes are displayed in Figs. 1-2. The JONSWAP spectrum was used to generate the random wave. The model was then subjected a unidirectional random wave in head seas configuration. The sea state is $H_s = 4.5m$, $f_P = 0.1\,Hz$ and $\gamma = 2$ in full scale. The tests were recorded for 3000 seconds at a sampling frequency of 10 Hz (prototype scale).

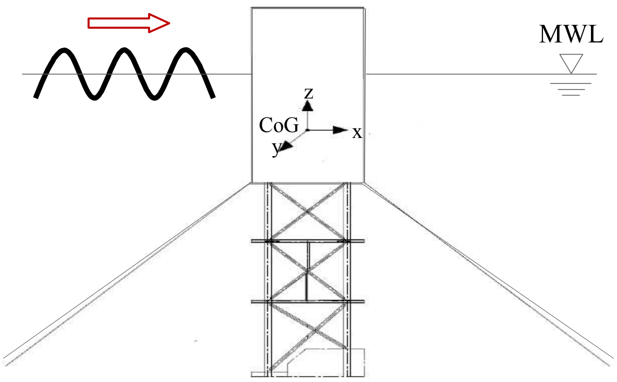

Fig. 1 Truss spar platform model

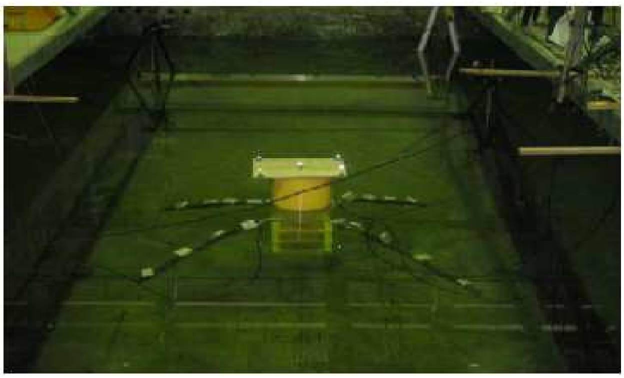

Fig. 2 Truss spar testing model [7]

Results

In order to check the frequency contents of both measured surge motion and wave heights, spectral analysis is then carried out. The results are displayed in Figs. 3-4.

Spectral Analysis. The spectrum of wave elevation is shown in Fig. 3b. It can be seen that the target based on JONSWAP spectra and measured spectra agree well. Further, in the Fig. 4b, surge motion has two principal frequency peaks. The first peak is around 0.016 Hz. This frequency is known as low frequency region (LFR) created through the difference frequency interaction. This frequency is still above the surge resonant frequency of the prototype at 0.008 Hz obtained from surge free decay test in still water. The second peak is around 0.1 Hz, corresponds to the frequency exist in the random wave spectrum. This frequency is known as wave frequency region (WFR). Most of wave energy is located in this frequency. Direct assessment may be drawn directly from the results above, that the low frequency is not present in the wave spectrum, but the reverse is true. In the perspective of dynamic system, truss spar platform is a nonlinear system.

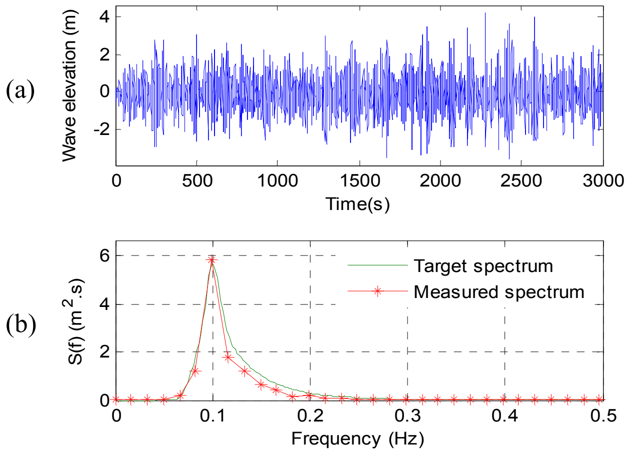

Fig. 3 Random wave (a) measured time history (b) wave spectrum

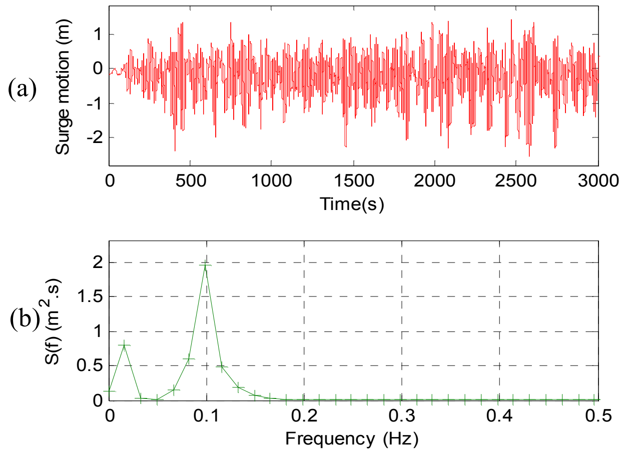

Fig. 4 Surge motion (a) measured time history (b) surge motion spectrum

Identification Results. After Volterra kernels are estimated via Eqs. 6a-6b, LIRF and QIRF may be generated. Respective tranfer functions and their coherence functions can be estimated from those results. However, only identified surge motion is reconstructed and presented in this paper to show the efficacy of the proposed method. To acces this, comparisons between experiment and reconstructed data in time domain are depicted in Figs. 5-6.

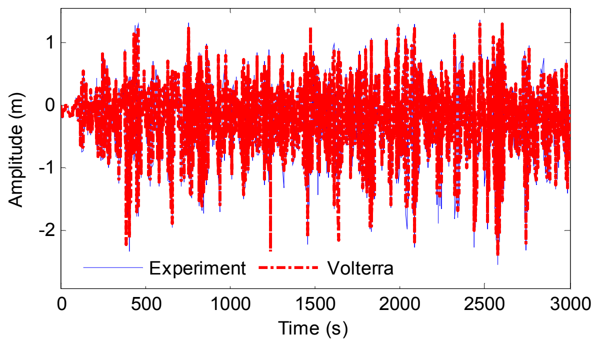

Fig. 5 Surge motion obtained from Volterra model

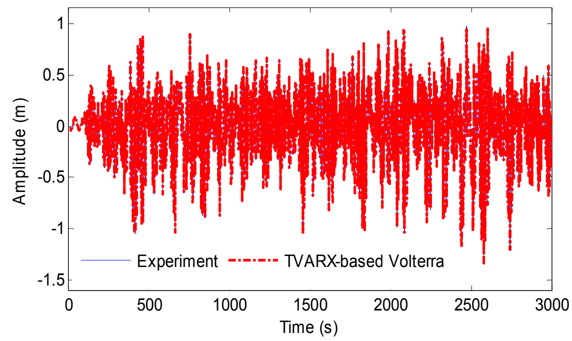

Fig. 6 Surge motion obtained from Volterra-based TVARX model

Reconstruction for time domain comes from Eq. 3. Once the surge motion is reconstructed, by using existing spectral analysis, frequency domain of identified results can be carried out via multitaper and displayed in Figs. 7-8. To compare the prediction results with its respective measured time series qualitatively, the normalized mean square error (NMSE) is calculated as a statistical comparison. The NMSE value is shown in Table 1. NMSE values of the proposed method have smaller NMSE value compared to the stand-alone time domainVolterra model. It shows that the proposed method has better accuracy in modeling the nonlinearity of the system than the stand-alone time domain Volterra model. These approaches are different with frequency domain Volterra model via higher-order spectral analysis. Reconstructed surge motion is obtained from identified linear TF, quadratic TF and input wave height spectrum and inverse FFT is then employed.

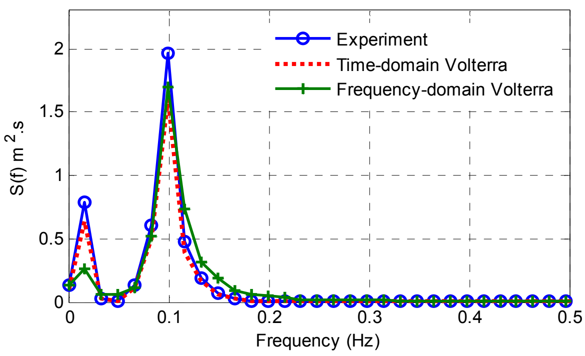

Fig. 7 Surge spectrum obtained from Volterra model

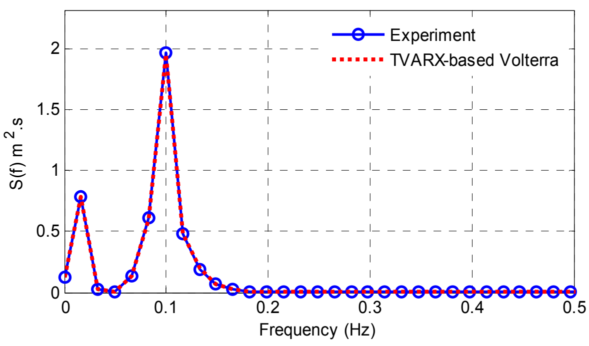

Fig. 8 Surge spectrum obtained from Volterra-based TVARX model

Table 1: NMSE Value

Method	NMSE	
	Time domain	Frequency domain
Time-domain Volterra model	0.213	0.181
Volterra-based TVARX model	0.103	0.0067

Summary

In this paper, the application of TVARX-based Volterra model for nonlinear system identification is proposed and applied in floating structures. The high correspondence between the predicted and actual surge response is achieved either in time or frequency domain through proposed method. By having more accurate identification results of the system in term of empirical model, dynamic response prediction of a moored floating structure such as truss spar can be carried out across many wave frequencies in a most efficient manner.

References

[1] P.D. Spanos, R. Ghosh, L.D. Finn and J.E. Halkyard, Coupled analysis of a spar structure: Monte Carlo and statistical linearization solutions, Journal of Offshore Mechanics and Arctic Engineering, ASME. 127 (2005) 11–16.

[2] Y.M. Low and R.S. Langley, Understanding the dynamic coupling effects in deep water floating structures using a simplified model, Journal of Offshore Mechanics and Arctic Engineering, ASME. 130(2008) 1-10.

[3] Zhang, N. Kim, Extraction of the second-order nonlinear response from model test data in random seas and comparison of the Gaussian and non-Gaussian models, PhD dissertation, Texas A&M University, Texas, America, 2004.

[4] Y. Birkelund, E.J. Power and A. Hansen, On the estimation of nonlinear Volterra models in offshore engineering, *Proc. ISOPE*, pp. 180-187, 2002.

[5] R. Taghipour, T. Perez and T. Moan, Hybrid frequency-time domain models for dynamic response analysis of marine structure, Ocean Engineering. 35 (2008) 685-705.

[6] E. Yazid, M.S. Liew and S. Parman, Field measurements based RAO estimation via the TVAR model, *Proc. OMAE*, pp. 180-187, 2012.

[7] M.A. Yassir, Numerical and Experimental Studies on the Slow Drift Motions and The Mooring Line Responses of Truss Spar Platforms, PhD dissertation, University Teknologi Petronas, Malaysia, 2012.