Information Entropy of Multi-photon Field Interacting with Qubits

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***Abstract*—We study the interaction of the single-Cooper pair box interacting with a cavity field. The solution of the system dynamics is obtained. The entropy as a measure of the entanglement between the qubit and the field is calculated and compared with the field purity. It is shown that the general behavior of the entropy as well as the field purity is affected in the same way under different values of the system parameters.**

I. Introduction

Recently, great interest has suscitated many proposals to build and construct the quantum computer both theoretically and experimentally [1], [2], [3], [4]. Benioff (1982) proposed a first framework of the quantum computer for quantum kinematics and dynamics. The model proposed that at the end of each elementary computational step, no characteristically quantum property of the model (interference, non-separability, or indeterminism) can be detected. The computation can be perfectly simulated by a Turing machine [5]. The theoretical model of quantum computer, universal quantum simulator, consists of a lattice of spin systems with nearest–neighbor interactions and supposing that it can surely simulate any system with a finite-dimensional state space [6]. Also, some algorithms are proposed to simulate the computation problems and the speed of the database search [7], [8], [9], [10].

Experimentally, there are many proposals for the design of the quantum computation such as cavity quantum electrodynamics [11], ion traps (Nobel Prize 2012 Serge Haroche and David J Wineland) [12], NMR quantum computer [13], group IV solid state [14] and silicon circuits [15]. The most promising proposal is Josephson junctions (cooper-pair box) circuits [16], [17], [18], [19], [20]. A basic unit of quantum computer is introduced using a small superconducting electrode (a single–Cooper–pair box) connected to a reservoir via a Josephson junction constitutes an artificial two-level system, in which two charge states that differ by 2e are coupled by tunneling of Cooper pairs. In this experiment they show the basic phenomenons of quantum mechanics in large number of electrons, the two-level system shows coherent superposition and quantum entanglement of the two charge states [17]. Makhlin et al. designed the Josephson junction device to produce charge qubits quantum state which can be manipulated coherently by voltage and current pulses. Thus, they can serve as qubits and the quantum logic gates can be performed. The phase coherence time which is limited, e.g., by the electromagnetic fluctuations in the control circuit, is long enough to allow a series of these manipulations [21]. Electrical superconducting circuit that behaves as a two–level atom with a series of microwave pulses is designed to generate charge qubits with quality factor of quantum coherence Qphi= 25000 which is sufficiently high for a solid-state quantum processor [22].In 2011, physicists at the University of Maryland generated the charged cooper pairs with strong correlations which has a lifetime $= 200\mu S$. This represents a good improvement in the lifetime of the Cooper-pair box and the deferent applications of the quantum information processing. The used circuit consists of a circuit QED experiment in which a separate transmission line is used to address quasilumped element superconducting microwave resonator which in turn coupled to an $Al/AlOx/Al$ Cooper-pair box charge qubit [23]-[24]. The interaction of a single CPB with cavity field and noisy system is studied [25], [26], and the effect of the system parameters in the teleportation process is investigated [27]. The dynamics of the coupled charge qubits with nanomechanical resonator is investigated [28], [29], [24].

The main purpose of this paper is to discuss the different features of the interaction between a single qubit and cavity field. We discuss the quantum field entropy due to changing the number of photon involved. Also, we investigate the field purity of the information that can be transmitted using this model. This paper is organized as follows: in Sec. 2, we present our system as well as its analytical solution. The numerical results are presented in Sec. 3. Finally our results are summarized in Sec. 4.

II. The Model

The S–CPB model consists of a superconducting box connected by a low–capacitance Josephson junction with capacitance C_J and Josephson energy E_J, coupled capacitively to a gate voltage V_g (gate capacitance C_g), placed inside a single-mode microwave cavity. We suppose that the gate capacitance C_g is screened from the quantized radiation field and then the Hamiltonian of the system can be written [16], [18]

$$H = \frac{(Q - C_g V_g - C_J V)^2}{2(C_g + C_J)} - E_J \cos\phi + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right), \quad (1)$$

where $Q = 2Ne$ is the Cooper pair charge on the island, N is the number of Cooper-pairs, ϕ is the phase difference across the junction, ω is the field frequency, and $a^\dagger$ and a are the creation and annihilation operators of the microwave, respectively. The voltage difference V produced by the microwave across the junction may be written as

$$V = i\sqrt{\frac{\hbar\omega}{2C_F}}\big(a - a^{\dagger}\big)\,, \tag{2}$$

where C_F is the capacitance parameter which depends on the thickness of the junction, the relative dielectric constant of the thin insulating barrier and the dimension of the cavity. Here, we consider the case where the charging energy with scale $E_c = \frac{e^2}{2(C_g+C_J)}$ dominates over the Josephson coupling energy E_J, and concentrates on the value $V_g = \frac{e}{C_g}$ and weak quantized radiation field, so that only the two low–energy charge states $N=0$ and $N=1$ are relevant. In this case the Hamiltonian in a basis of the charge state $|\downarrow\rangle$ and $|\uparrow\rangle$ reduces to a two–state form in a spin–$\frac{1}{2}$ language[30]

$$H = E_c\Big(1+\frac{C_J^2V^2}{e^2}\Big)-2E_c\frac{C_JV}{e}J_z-\frac{1}{2}E_JJ_x+\hbar\omega\big(a^{\dagger}a+\tfrac{1}{2}\big)\,. \tag{3}$$

We denote J_z and J_x as the Pauli matrices in the pseudo–spin basis $\{|\downarrow\rangle, |\uparrow\rangle\}$, i.e.,

$$J_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| \qquad \text{and} \qquad J_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|\,, \tag{4}$$

where the charge states are not the eigenstate of the Hamiltonian (3) even in the absence of the quantized radiation field, i.e., $V=0$. We describe H in the two charge states subspace through new states and denote the corresponding states as [30]

$$|+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle),\ |-\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle + |\downarrow\rangle\big)\,. \tag{5}$$

By making some treatments we find that the hamiltonian reduced to the following form

$$H_I = \frac{1}{2}\Delta\sigma_z + ig\big(a\ \sigma_+ - a^{\dagger}\sigma_-\big)\,, \tag{6}$$

where

$$g = \Big(\frac{e^2\omega}{2C_F\hbar}\Big)^{\frac{1}{2}}\frac{C_J}{(C_g+C_J)}\,, \tag{7}$$

where $\Delta = E_J - \omega$ is the detuning between the Josephson energy and cavity field frequency. We shall work from now on in the basis $\{|+\rangle, |-\rangle\}$, then in the interaction picture, the Hamiltonian (6) is written as [31]

$$U_G = \begin{pmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} \\ \mathcal{U}_{21} & \mathcal{U}_{22} \end{pmatrix}\,, \tag{8}$$

where

$$\begin{aligned}
\mathcal{U}_{11}(t) &= \cos\Omega_{n+k}t - i\frac{\Delta}{2}\frac{\sin\Omega_{n+k}t}{\Omega_{n+k}}\,,\\
\mathcal{U}_{12}(t) &= ga\frac{\sin\Omega_n t}{\Omega_n}\,,\\
\mathcal{U}_{21}(t) &= ga^{\dagger k}\frac{\sin\Omega_{n+k}t}{\Omega_{n+k}}\,,\\
\mathcal{U}_{22}(t) &= \cos\Omega_n t + i\frac{\Delta}{2}\frac{\sin\Omega_n t}{\Omega_n}\,.
\end{aligned} \tag{9}$$

Here, $\Omega_n = \big(\frac{\Delta^2}{4} + g^2 n\big)^{\frac{1}{2}}$, $\Omega_{n+k} = \big(\frac{\Delta^2}{4} + g^2\frac{(n+k)!}{n!}\big)^{\frac{1}{2}}$ and $n = a^{\dagger}a$. The density operator of the system at any time $t > 0$ is given by

$$\rho(t) = \mathcal{U}_t\rho(0)\mathcal{U}_t^{\dagger}, \tag{10}$$

where $\rho(0) = \rho_b(0)\otimes\rho_f(0)$.

Assume that the box is initially in its pure state

$$\rho_b(0) = \frac{1}{\sqrt{2}}\big(|e\rangle + |g\rangle\big)\,. \tag{11}$$

We assume the field is initially prepared in Fock state, i.e., the initial density of the field is $\rho_f(0) = |n\rangle\langle n|$. Using Eqs. (6), and (9), one obtains the density operator of the Cooper pair qubit after tracing out the field state as,

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{21}(t) \\ \rho_{12}(t) & \rho_{22}(t) \end{pmatrix}\,, \tag{12}$$

where,

$$\begin{aligned}
\rho_{11}(t) &= (1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}\Big[\cos^2\Omega_{n+k}t\\
&\quad +\Big(\frac{\Delta}{2}\Big)^2\sum_{n=0}^{\infty}\frac{\sin^2\Omega_{n+k}t}{\Omega_{n+k}^2}\Big]\\
&\quad +g^2\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+2k}}{n!}\Big[\frac{\sin^2\Omega_{n+k}t}{\Omega_{n+k}^2}\Big],
\end{aligned} \tag{13}$$

$$\begin{aligned}
\rho_{12}(t) &= -g(1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!}\\
&\quad \times\Big[\frac{\cos\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}}\\
&\quad -i\Big(\frac{\Delta}{2}\Big)\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}\Omega_{n+2k}}\Big]\\
&\quad +g\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!}\Big[-\frac{\sin\Omega_{n+k}t\cos\Omega_n t}{\Omega_{n+k}}\\
&\quad +i\Big(\frac{\Delta}{2}\Big)\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+k}t\sin\Omega_n t}{\Omega_{n+k}\Omega_n}\Big],
\end{aligned} \tag{14}$$

$$\begin{aligned}
\rho_{21}(t) &= g(1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!}\\
&\quad \times\Big[\frac{\cos\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}}\\
&\quad +i\Big(\frac{\Delta}{2}\Big)\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}\Omega_{n+2k}}\Big]\\
&\quad +g\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!}\Big[-\frac{\sin\Omega_{n+k}t\cos\Omega_n t}{\Omega_{n+k}}\\
&\quad -i(\frac{\Delta}{2})\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+k}t\sin\Omega_n t}{\Omega_{n+k}\Omega_n}\Big],
\end{aligned} \tag{15}$$

$$\begin{aligned}
\rho_{22}(t) &= g^2(1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}\Big[\cos\frac{\sin^2\Omega_{n+k}t}{\Omega_{n+k}^2}\Big]\\
&\quad +\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^n}|\alpha|^{2n}}{n!}\Big[\cos^2\Omega_{n+k}t\\
&\quad +\frac{\Delta^2}{4}\frac{\sin^2\Omega_n t}{\Omega_n^2}\Big].
\end{aligned} \tag{16}$$

III. Results and Discussion

In this section we conclude our results and discussions. The Von Neumann entropy is considered as a measure of the entanglement and it can be written as [26]

$$S = -\lambda_i \log \lambda_i \,, \tag{17}$$

where $\lambda_i = \frac{1}{2} \pm \frac{1}{2}\sqrt{[\rho_{22}(t) - \rho_{11}(t)]^2 + 4|\rho_{21}(t)|^2}$. The purity of the transmitted information can be written as

$$P = \mathrm{Tr}|\rho(t)|^2 \,. \tag{18}$$

Fig. 1 shows the dynamics of the quantum field entropy as a function of the scaled time gt. It is clear to see that the quantum entropy starts from 0.1 at $t = 0$. When time increases the entropy decreases to reach $S = 0$ at $gt = 0.5$. The entropy maintains at zero until $gt = 1.3$ and then it starts to increase and reaches its maximum value at $S = 0.6$ at $gt = 2$. The entropy decreases again and reaches 0.3 at $gt = 3$ with small oscillations. When the detuning parameter is increased, $\Delta = 10$, it shows that the entropy starts from the same value, but when time increases the entropy increases to its maximum value, ($S = 0.1$ in this case) at $gt = 0.3$ and beyond that point the entropy maintained at a fixed value. We conclude from this result that by increasing the detuning parameter, Δ, the entropy decreases.

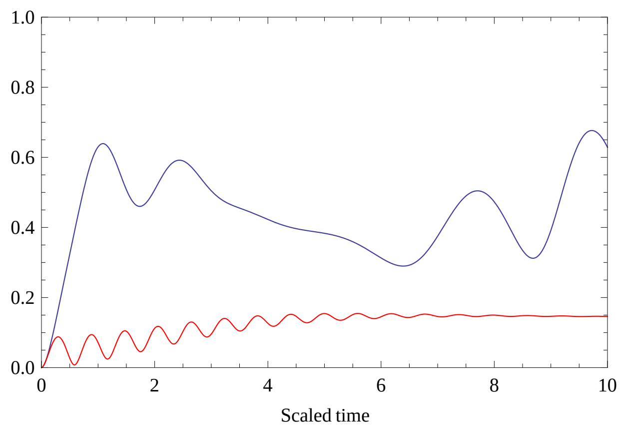

Fig. 1. The entropy as a function of the scaled time for different values of the detuning parameters, where $\Delta = 0.5$ and $\Delta = 10$ with a single photon process.

Fig. 2 depicted the purity of the transmitted information as a function of the scaled time gt. We find that the purity starts from its maximum value when $gt = 0$. As time goes on, the purity decreases and reaches its minimum value when the box is totally in a mixed state and the purity repeated periodically. The effect of the detuning affect the purity in an opposite way from the entropy, where the purity is increased when the detuning increases.

Figs. 3 and 4 represent the effect of the number of photon on the entropy and purity. It shows that the periodic behavior of the entropy is manifested for the two–photon process while this periodicity is no longer exist when the number of photon is increased. It is interesting to see that the same behavior is observed for both entropy and purity corresponding to the photon processes.

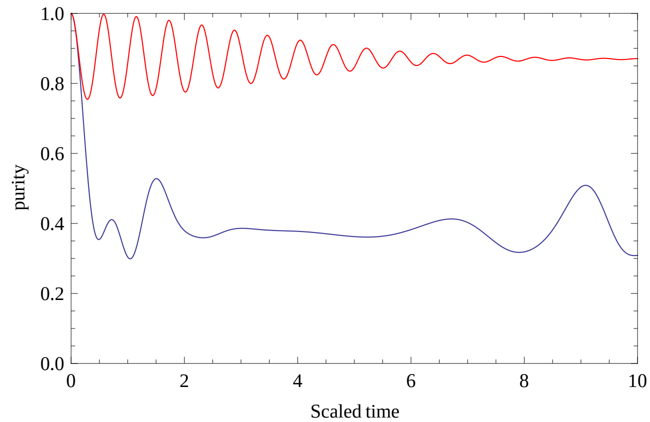

Fig. 2. The purity of the transmitted information as a function of the scaled time. Same parameters are applied as in Fig. 1.

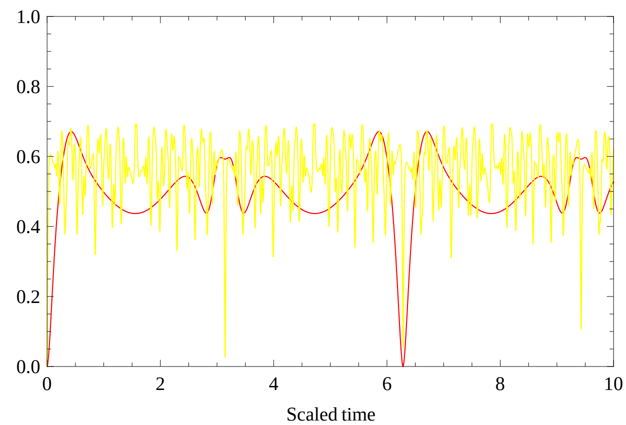

Fig. 3. The entropy as a function of the scaled time for different values of the number of photons, where $k = 2$ (two-photon process) and $k = 4$ (four-photon process). The other paternosters are the same as in the case in Fig. 1 but $\Delta = 1$.

IV. Conclusion

We considered the interaction between a single Cooper–pair box and the cavity field. The effect of the system parameters on the entropy and the purity of the transmitted information are studied. We discussed the effect of the multi–photon processes on the entropy and purity and find that the photon process plays the same role on both entropy and purity. We also find that the number of photons have a big effect in the information entropy and the purity of the transmitted information as well as the detuning parameter. In general, the information entropy and the purity of the transmitted information for the system consists of the SCPB with multi–photon field can be controlled by adjusting the system parameters.

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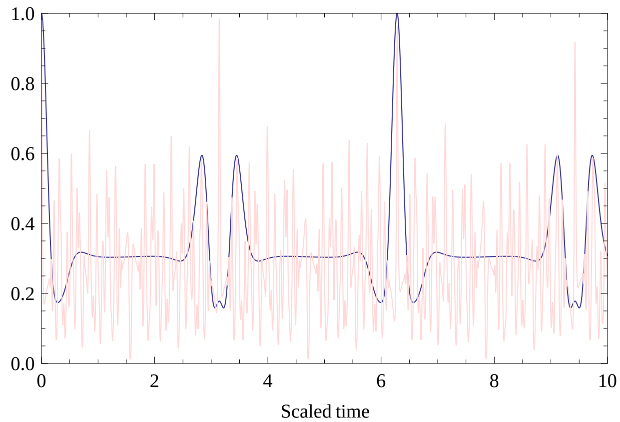

Fig. 4. The purity as a functions of the scaled time with the setting identical to Fig. 3.

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