

Scale- Invariant Face Recognition Using Triangular Geometrical Model

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Abstract—This work proposed a geometrical model based on multiple triangular features for the purpose of handling the challenge of scale variations that affect the process of face recognition especially in real time applications where the test images are usually taken in random scales that may not be of the same scale as the probe image. Geometrical approaches have proved to be robust to lighting and illumination variation. Furthermore geometrical methods in general do not hold computational complexity and have the benefit of faster processing time, which make them appropriate for real time applications. Fifteen triangle similarity measurement equations were derived and used to build a class of feature vectors for each subject. Ten images in ten different scales were taken for each subject for a total of fifty samples. Classification results show that the proposed model is promising in handling the challenge of scale variations.

Keywords: Geometrical model; Scale variations; Triangular features; Similarity proportion ratios; Class.

I. INTRODUCTION

Biometric authentication technology plays an important role in information security, and many researchers pay attention to it, particularly in recent years. Biometrics involves methods for distinctively recognizing human based on one or more intrinsic physical or behavioural traits, such as fingerprints, face recognition, hand and palm geometry, iris recognition, vein recognition, signature recognition, and speech recognition.

Face recognition is the most appropriate method among these physical traits for biometrics [1]; it reaches the recognition object according to individual facial features of humans.

Since the early 1990's Face Recognition Technology (FRT) became an active research area. A broad statement of the problem of face recognition can be formulated as follows: given still or video images of a scene, identify one or more persons in the scene using a stored database of faces [2]. The answer for such problem involves segmentation of faces from cluttered scenes, extraction of features from face region, identification, and matching. The major approaches for face recognition can be classified into two categories: model based

and appearance based. The model based method tries to extract geometrical parameters measuring the facial parts while the appearance based approach uses the intensity intensity-derived parameters such as Eigen-faces coefficients [3]. Due to changes of lighting conditions, expression, etc., the human face appearance could change considerably, so that our work is based on geometrical shapes of face or, in more scientific words, on model based approach.

Most of ground breaking work in face recognition was done based on the geometric features of a human face [4]. This technique involves computation of a set of geometrical features such as nose length, mouth and eye positions with respect to each other, etc., from the picture of the unknown face to be recognized. This set of features is compared with the features of known individuals and the closest match is then found.

Roy et al. [5] used a triangular approach for face verification and age range estimation. Their system is based on extracting cretin facial features (eye brows, eyes, and mouth), then coordinating the midpoints of each feature to form an isosceles triangle in the middle of the face. They claimed that this triangle is unique for every person which makes it useful for face recognition in general, and face recognition with age variations particularly.

Alom et al. [6] conducted a research study to optimize facial features for age classification. They used a8 focal points that were designated manually on a 150 images of 50 subjects that were collected from the FG-NET face aging dataset. They presented mathematical relationships between 16 feature points' distances to reduce the number of facial features to four distances, and they specified the relationships between face height and width as to be equal to the golden ratio (1.618).

There are also methods proposed based on depth estimation techniques. Depth estimation techniques fall under the 3D shape recovery research [7, 8, 9, and 10]. However, such methods either require specialized hardware and/ or they are not real-time.

In our research study we are aiming to develop a geometric model that is scaled and orientation invariant. In our work we have explored the approach of using a mathematically developed geometrical model for stating the degree of similarity between six triangular features which is employed to participate in addressing the problem of face recognition

under variable scale, orientation, pose, and age variations. The system to be developed is aimed to operate in real time environment such as surveillance systems.

The remainder of this paper is organized as follows: Section 2 introduces the proposed face recognition geometrical model where we define the mathematical relationships between our proposed triangular features, and our trend in building the systems' facial feature vectors. The results and discussion of experiments are presented in Section 3. This is followed by conclusions in Section 4.

II. PROPOSED GEOMETRICAL MODEL

The proposed system decomposed multiple stages. Face detection is the first stage at the beginning of each face recognition system. In our system a commercial version of the conventional Viola and Jones face detector [11] is employed to detect and crop the face area that contains the main features (Eyes, Mouth, Nose, and chin). Viola and Jones detector is robust and effective in real time applications. Fig. 1 illustrates the block diagram of the proposed system.

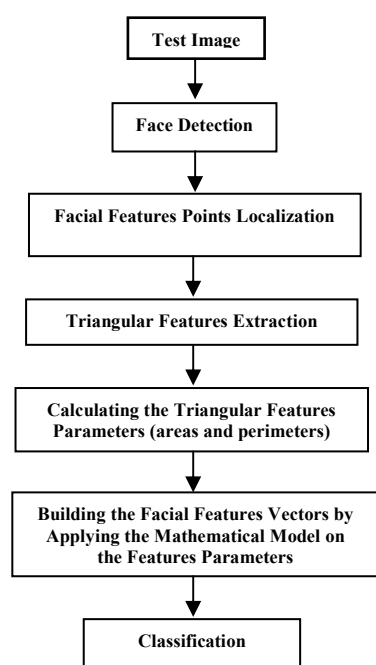


Fig. 1. Block diagram of the proposed system

After detecting the face area twelve facial feature points are to be localized in order to extract six different triangular areas around the main facial features. Following the parameters of the triangular features i.e. (areas and perimeters) are calculated. Then those parameters are passed to a number of equations to create feature vectors for the sample image. In the following those stages are illustrated in details.

A. Facial Features Points Localization And Triangular Features Detection

Craniofacial anthropometry which is the science that involves measurement of the skull and face, citrine landmarks and measurements are known to identify human facial characters and growth pattern. In our study we consider twelve of these landmarks mostly the ones that form the circumscription of the main facial features. Those facial feature points are normally localized using Active Appearance Model (AAM) [12] which designates 64 distinctive focal points. In our model the AAM is reduced to 12 facial feature points using the algorithm proposed in [12]. Craniofacial anthropometry refers to those facial feature points with scientific notation to discriminate between them as follows:

- **En (endocanthion)**: the inner corner of the eye fissure where the eyelids meet. In our model these points are given the numbers 6 and 8.
- **G (glabella)**: the most prominent point in the median sagittal plane between the supraorbital. In our model these points are given the number 9.
- **Ridges. Ex (exocanthion)**: the outer corner of the eye fissure where the eyelids meet. In our model these points are given the numbers 5 and 7.
- **Gun (Jonathan)**: in the midline, the lowest point on the lower border of the chin. In our model this point gives the number 2.

Following six triangles are formed between the focal points and they are given the notation triangle1 through triangle6. as illustrated Fig. 2.a through Fig. 2.c.



Fig. 2.a Triangle1 and Triangle5



Fig. 2.b Triangle3 and Triangle4



Fig. 2.c Triangle2 and Triangle6

B. Calculating the triangular feature parameters

After localizing the facial feature points, the system will gain knowledge of the triangular vertex coordinates. After that Euclidean distances between each triangle coordinates will be calculated, which will enable the system to calculate perimeters and areas of each triangular feature. Those parameters (areas and perimeters) are given the notation A and P for areas and perimeters successively followed by a subscript representing the triangle designation. For example (A_i, p_i) represent the area and perimeter of triangle number one. Finally those parameters are used as inputs for some mathematical equations which will be discussed next, to form the feature vectors for each sample image.

C. Deriving The Mathematical Model

In the geometry science it is known that the triangles are similar if they have the same shape, but not necessarily the same size [13]. This scientific fact inspired us to draw mathematical relationships between the six triangular features extracted during the previous stage. The Human population reached 7 Billion people around the world and thus, it is impractical to use a one-to-one comparison process for the purpose of face recognition using the measurements of our triangular features. As a different approach we were able to make use of the proportional ratio between the different triangles representing the facial features which led to fifteen different mathematical equations representing the degree of similarity between each two triangles. Based on the aforementioned geometrical theory regarding the similarity of triangles, any two triangles are considered similar even if they are of different sizes if the following mathematical relationship represented by "Eq. (1)" is satisfied:

$$\frac{A_i}{A_j} = \frac{p_j^2}{p_i^2} \quad (1)$$

Where A , and p represent triangles areas and perimeters successively, i and j are designations of the two triangles subject of the mathematical relationship.

Eq. (1) Is used to drive what is called triangle similarity proportion, which is a measurement of the degree of similarity between two triangles, and it is represented by "Eq. (2)". TSP represents the triangle similarity proportion relationship.

$$TSP = (A_i * p_j^2 / A_j * p_i^2) \quad (2)$$

The statistical analysis of the data collected in term of triangular features areas and perimeters had shown clearly that there is no significant difference between these measurements of different individuals. As a different approach we were able to make use of the similarity proportional ratio between the different triangles representing the facial features which led to fifteen different mathematical equations representing the degree of similarity between each two triangles. These equations were derived using equation (2) by simply applying the formula between each two triangles, and substituting subscripts i and j by the designations of the two triangles. "Eq. (3)" through, "Eq. (17)" represents the fifteen relationships between the six triangular features as listed below:

$$(T_1, T_2) = \frac{(A_1 * p_2^2)}{(A_2 * p_1^2)} \quad (3)$$

$$(T_1, T_3) = \frac{(A_1 * p_3^2)}{(A_3 * p_1^2)} \quad (4)$$

$$(T_1, T_4) = \frac{(A_1 * p_4^2)}{(A_4 * p_1^2)} \quad (5)$$

$$(T_1, T_5) = \frac{(A_1 * p_5^2)}{(A_5 * p_1^2)} \quad (6)$$

$$(T_1, T_6) = \frac{(A_1 * p_6^2)}{(A_6 * p_1^2)} \quad (7)$$

$$(T_2, T_3) = \frac{(A_2 * p_3^2)}{(A_3 * p_2^2)} \quad (8)$$

$$(T_2, T_4) = \frac{(A_2 * p_4^2)}{(A_4 * p_2^2)} \quad (9)$$

$$(T_3, T_6) = \frac{(A_3 * p_6^2)}{(A_6 * p_3^2)} \quad (10)$$

$$(T_2, T_5) = \frac{(A_2 * p_5^2)}{(A_5 * p_2^2)} \quad (11)$$

$$(T_2, T_6) = \frac{(A_2 * p_6^2)}{(A_6 * p_2^2)} \quad (12)$$

$$(T_3, T_4) = \frac{(A_3 * p_4^2)}{(A_4 * p_3^2)} \quad (13)$$

$$(T_3, T_5) = \frac{(A_3 * p_5^2)}{(A_5 * p_3^2)} \quad (14)$$

$$(T_4, T_5) = \frac{(A_4 * p_5^2)}{(A_5 * p_4^2)} \quad (15)$$

$$(T_4, T_6) = \frac{(A_4 * p_6^2)}{(A_6 * p_4^2)} \quad (16)$$

$$(T_5, T_6) = \frac{(A_5 * p_6^2)}{(A_6 * p_5^2)} \quad (17)$$

For each sample image enrolled in the system those fifteen relationships will be calculated and stored in a vector which will be considered as a feature vector of this specific sample image. When multiple sample images are related to the same subject, the feature vectors of these sample images will be stored in a matrix to form a class for each subject.

As will be illustrated and discussed in the next section, building the feature vectors using the aforementioned proposed mathematical model yielded insufficient classification results. These results imply that using the feature vectors created using the first mathematical model will create classes with low degree of uniqueness. It means that the degree of deference between the classes of subjects is insignificant. As a step to improve system performance and developing the mathematical model, the matrixes representing the subjects' classes were scaled through multiplying each vector elements by a factor to amplify the difference between classes' vectors. Following each class matrix is normalized through dividing each matrix element of the matrix element to keep each matrix elements within a specific range, and aid the classifier in discriminating between the classes.

III. RESULTS AND DISCUSSION OF EXPERIMENTS

Experiments were conducted on 10 subjects of different genders, ages, and ethnicities. For each subject ten sample images were taken at different scales from different distances. Images are JPG of size 300x300. The images were taken under random acquisition environments of variant

illumination and lighting, but they were mainly frontal with some slight misalignments in some cases. The images were taken in indoor spaces as illustrated in Fig. 3.

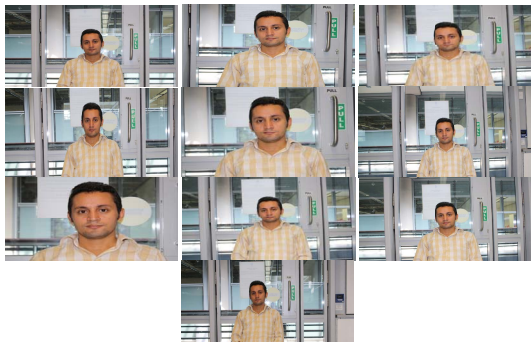


Fig. 3. Samples images for the first subject in 10 different scales

Experiments were conducted in two stages. At the first stage the facial features vectors elements were calculated using the first mathematical model to construct the first dataset, and the developed mathematical model to construct the second dataset. The second stage of the experiments is the classification stage. The data sets were tested on three different classifiers namely: Naïve Bays, Gaussian classifier, and linear Discriminant classifier.

A. First stage (Building the classes)

At this stage fifteen facial features were extracted using the first mathematical model represented by "Eq. (3)" through, "Eq. (18)". For each probe image enrolled in the system, these fifteen features were calculated and stored as a feature vector. Moreover, features vectors of sample images related to the same subject were stored in one metric, which is then considered as a distinct class. A dataset of ten classes (100 samples) was built at the end of the stage. The same procedure was repeated using the developed mathematical model to build the second data sets' classes. The data sets are then stored in the excel files Datasheets, so that they can be easily loaded to the classifier during the classification stage.

B. Z-score matrix normalization ensemble (Amplification between classes vectors)

It is often useful to calculate how far, in standard deviations, a data element was from the mean. This is a very commonly used procedure and this measure has the name z-score. It is also known as a standard score. While several data sets have a fairly normal distribution, it is a very accommodating way to compare data elements from different populations—populations which may very well have

contradictory means and standard deviations [14]. Z-score provide a valuable measurement for comparing data elements from different data sets. The formula used for calculating the z-score is as follows:

$$z = \frac{x - \mu}{s} \quad (18)$$

Where:

Z: z-score; x: data element; μ : mean of the data set; s: standard deviation.

Moreover, matrix normalization is performed through the division of each matrix element by the determinant of the matrix [10].

C. Second stage (Classification)

As mentioned above classification was conducted using three classifiers to study the effect of using each classifier on the classification accuracy. The following is the brief enlightenment of the classifiers considered in the experimental part.

• Naïve Bays classifier [15]:

A Naïve Bays classifier is a simple probabilistic classifier based on applying Bays rule. Bayes' theorem provides a means of calculating the posterior probability, $P(b|x)$, from $P(b)$, $P(x)$, and $P(x|b)$. Naïve Bays classifier assumes that the effect of the value of a predictor (x) on a particular class (c) is independent of the values of other predictors. This hypothesis is called class conditional independence.

$$p(x|b) = \frac{p(x|b)p(b)}{p(x)} \quad (19)$$

$$p(b|x) = p(x_1|b) \times p(x^2|b) \times \dots \times p(x_n|b) \times p(b) \quad (20)$$

- $p(b|x)$ is the posterior probability of class (target) given predictor (attribute).
- $p(b)$ is the prior probability of class.
- $p(x|b)$ is the likelihood which is the probability of predictor given class.
- $p(x)$ is the prior probability of predicting.

- **Gaussian classifier** [16] is based on Gaussian process which is a stochastic process whose realizations consist of random values associated with each point in a range of times (or of space) such that each random variable has a normal distribution. Consider a two-class case: f_i is a measure of the degree of membership of class $C1$:

Let $Y_i = 1$ ($Y_i = -1$) denote that pattern I belong to class $C1$ ($C2$).

$$p(b_1|x_i) \equiv p(y_i = -1|f_i) =$$

$$\sigma(f_i) \equiv \int_{-\infty}^{f_i} e^{-\frac{x^2}{2}} e^{\frac{-x^2}{2\pi}} \quad (21)$$

- **Linear Discriminant classifier** [17]: In the field of machine learning, the aim of statistical classification is to use an object's characteristic to determine which class (or group) it belongs to. The linear Discriminant analysis technique consists of searching, some linear combinations of selected variables, which offer the best separation between the considered classes. These different combinations are called Discriminant functions [5].

Classification test was performed in four rounds, where in each round different of features was considered to study the effect of using different number of features on the classification accuracy for each of the three classifiers as follows:

- First round: All of the fifteen features were taken under consideration.
- Second round: Six features were selected which were features number 1, 2, 3, 6, 13, and 14.
- Third round: Five features were considered, feature number 1,2,3,6, and 14.
- Fourth round: Only four features were selected, which are 1, 2, 3, and 6.

Principle Component Analysis algorithm was used for feature ranking and selection methods to select a subset of the most significant features among the fifteenth extracted features for each round.

The results from each classification round for each classifier were recorded to study the influence of using different feature sets on the overall performance of the classifiers and discussed in the following.

IV. RESULTS DISCUSSION

In the first experiment 100 images with different scale conditions are used to build the dataset using the first mathematical model. The system was coded using Matlab 2011, and was run on an Intel Pentium system with 2 GHZ CPU, and 200 GB RAM. TABLE I illustrates the classification results of the first experiment for each of the three classifiers for each classification round. It can be noticed from the table (1) that the highest classification accuracy was achieved by the Naïve Bays classifier during the second round of classification. The lowest accuracies were reported by the Linear Discriminant classifier.

TABLE II: CLASSIFICATION RESULTS OF THE FIRST PROPOSED MODEL

Features selected	Classification Accuracy (%)		
	Naïve Bays classifier	Gaussian classifier	Linear Discriminant classifier
1 through 15	28.50	12.71	22.309
1,2,5,6,13, 14	45.97	7.71	22.9
1,2,5,6,13	40.97	33.92	25.4
1,2,6,13	35.59	37.26	25

With the Gaussian and Linear Discriminant classifiers the classification accuracy increases as the number of features decreases at each round. The overall performance of the system during the first stage of the classification was low and all of the accuracies were below 50%. At this stage of the experimental work the need for a more sophisticated model that is used to build the feature vectors arises. The same set of sample images was used in the second experiment, but the feature vectors were built using the development version of the mathematical model. TABLE II shows the classification results in term of classification accuracies of the second experimental part for the three classifiers.

It can be seen clearly that the Naïve Bays again outperforms the other classifiers with a maximum accuracy of 100% during the first and second rounds. The Gaussian classifier achieved a maximum accuracy of more than 93 in the fourth round of the classification stage. It can be seen from the table that developing the mathematical model has no influence in improving the performance of the Linear Discriminant classifier, its classification accuracy was still below 50%.

TABLE II: CLASSIFICATION RESULTS OF THE SECOND PROPOSED MODEL

Features selected	Classification Accuracy (%)		
	Naïve Bays classifier	Gaussian classifier	Linear Discriminant classifier
1 through 15	100	17.45	22.57
1,2,5,6,13, 14	100	30.21	34.42
1,2,5,6,13	96.9	69.23	31.92
1,2,6,13	95.14	93.21	27.07

In general the performance of the system experienced noticeable improvement after developing the mathematical model which motivates us to extend the experimental work to involve other challenges such as rotation, pose, and aging variations. Moreover the number of samples to be considered will be increased to study the influence of building the system data set with a large number of clusters in the classification accuracy. Also different classifiers will be used at the classification stage including WEKA classifiers and compare the performance of the classifiers with different set of features and different number of sample images.

V. CONCLUSION AND FUTURE WORK

This research study proposed new geometrical features that are formed by connecting some of the facial feature points defined in the anthropometric scientific. The main goal was to develop mathematical relationships among triangular features to accommodate for the scale variations conditions that may affect any face recognition system. The highest classification accuracy was achieved at the second round by the Naïve Bays classifier which was 100%. The proposed system is

promising toward handling face recognition challenges specially in real time applications due to its simplicity and the ability of working under different lighting and illumination conditions. But the system experiences difficulties when sample images were not taken in frontal pose. Thus, there is a need for poses normalization technique to be applied as a preprocessing step. Moreover, the system needs to be tested in a larger set of images to validate its performance.

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