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Coupled thermo-hydro-mechanical model with consideration of thermal-osmosis based on modified mixture theory

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ABSTRACT

A coupled formulation based on non-equilibrium thermodynamics, Biot's elasticity is derived to model thermal-osmotic flow in very low permeability rock. Darcy's law has been modified by incorporating thermal effects from the dissipation process by using standard arguments of non-equilibrium thermodynamics. The relationship of chemical potential of water and pore water pressure has been analysed. Helmholtz free energy is used to construct the structure of the mechanics model. The derived coupled equations can be validated by comparing with those from Mechanics approach. Finally, finite elements are used to solve the governing equations. The numerical results show the thermal-osmosis has an important effect on water transport in very low permeability porous media.

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1. Introduction

Shales make up over 75% of drilled formations, cause over 90% of wellbore instability problems and cost the oil industry over \$500–1000 million each year (Steiger & Leung, 1988). Since shales have a very low permeability and can act as membranes, hydraulic transport is not the dominant form of fluid development in complex conditions. In fact, thermal effects which caused by high temperature gradient between the drilling mud and wellbore have a several times larger contribution than hydraulic transport. The water flow caused by thermal effects is known as osmotic flow which can flow directly from warmer to cooler or from cooler to warmer. Dirksen (1969) has observed this phenomenon in experiments (Dirksen, 1969).

Based on the parameters of a temperature gradient of 2 K/m, a hydraulic gradient of 10 J/m and experimental result of Srivastava and Avasthi (1975), Carnahan (1984) has estimated that the thermo-osmotic flow through Kaolinite can be 800 times larger than Darcy's flow (Carnahan, 1983). Ghassemi and Diek (2003) have analysed the mud filtrate invasion due to temperature gradient and found that the thermal osmotic flow can also be several times larger than hydraulic flow (Ghassemi & Diek, 2003). Thus, thermal effects will alter the pore pressure and shale strength in wellbore stability analysis.

For heat transport in shales, Wang and Papamichos (1994) have confirmed that heat conduction dominates the heat transfer process and heat convection can be neglected because of low fluid flow velocity in such rocks (Wang & Papamichos, 1994). In early time, the thermal diffusion into shale formations occurs much more quickly than hydraulic diffusion. Charlez (1997) analysed the thermal diffusion effects on rock stresses between the drilling mud and the formation (Charlez, 1997).

A few researches have been done on coupled thermal-hydro-mechanical coupled model based on mechanics approach and the mixture theory approach. The mechanics approach is originated from hydro-mechanical coupled theory developed

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by Terzaghi (1943) and Biot (1962, 1972). Later, Lewis and Schrefler (1987) have developed a constitutive model for thermo-hydro-mechanical coupling (Lewis & Schrefler, 1987).

Mixture theory is a form of macroscopic approach and has been used to model the biological tissues. The first rigorous theoretical framework of mixture theory was developed by Truesdell in 1957 (Truesdell, 1957a, 1957b), providing a mathematical foundation for interacting continua research. Some review articles have provided more details for the development progress of mixture theory (Bowen, 1976; Atkin & Craine, 1976). Mixture theory has been extended by Bowen (1984, 1980) and most recently by Rajagopal and Tao (1995, 1986, 2002, 2003, 2007). Rajagopal and Tao (1995, 2005) have also re-examined Biot's equations and placed them within the context of the theory of mixtures, and concluded that Biot's approach can be obtained as a special case of mixture theory (Rajagopal & Tao, 1995, 2005).

Mixture theory maintains the individuality of the phases, although constitutive equations for each individual phase have been formulated that take account of phase interaction effects. Other research has shown that the theory can give an accurate description of the deformation and stress fields under a variety of loading configurations (Holmes, 1986; Holmes, Lai, & Mow, 1985).

However, it is difficult to obtain information on the interaction between the phases and there are also a few difficulties associated with applications of mixture theory as discussed by Rajagopal et al. (1986). For example, since balance laws are posited for each constituent, one has problems with the prescription of boundary conditions. In most problems, one only knows what the total stress on the boundary is, or the total displacement or velocity (Rajagopal & Tao, 1995; Rajagopal et al., 1986).

Recently, a single continuum approach to mixtures have been adopted by researchers to avoid the difficulties associated with the application of mixture theory. Heidug and Wong (1996) adopted an approach in isothermal conditions that does not explicitly discriminate between the solid and the fluid phases; rather, it views a fluid-infiltrated rock as a single continuum to analyze the interactions between solid/fluid/chemical (Heidug & Wong, 1996; Baek & Srinivasa, 2004) analyzed large deformation (Baek & Srinivasa, 2004; Humphrey & Rajagopal, 2002) introduced a new constrained mixture theory model for studying growth and remodeling of soft tissues (Humphrey & Rajagopal, 2002).

Heidug and Wang's approach combines Biot's theory and non-equilibrium thermodynamics and Helmolz free energy, combining the advantage of mixture theory and mechanics approach in Geomechanics. They studied the chemical osmosis in a hydro-chemo-mechanical coupled model in Geomechanics. Since the concept is mainly based on mixture theory, it can be referred as modified mixture theory. However, no work has so far been done for non-isothermal conditions based on modified mixture theory.

This paper extended the modified mixture theory to non-isothermal conditions. A fully coupled thermal-hydro-mechanical model has been derived. Thermal osmosis has been included in the coupled formulation. Darcy's law has been modified to include thermal effects and the heat conduction equations to include the pressure effects.

The formulation used Helmholtz free energy to derive the relationships for the solid, fluid and temperature components. The influence of temperature over water chemical potential, and the coupling between the water chemical potential and host rock have been studied. This has resulted in the derivation of an advance coupling between the thermal, water, solid phases. Finally, A simple "experimental" setup has been considered for testing the numerical techniques, with the boundary conditions modified (that is, with respect to usual experimental use) to suit the modelling of thermal osmosis.

2. Balance laws

In mixture theory, a material point \mathbf{X}^α for the α th phase in an arbitrary reference configuration is given. At time t , it's position \mathbf{x}^α can be defined as

$$\mathbf{x}^\alpha = \mathbf{x}^\alpha(\mathbf{X}^\alpha, t) \quad (1)$$

The mass density of the α th phase (which, in mixture theory, can be either solid or fluid) can be defined in two different ways: that is, either based on the volume of the whole mixture as $\rho^\alpha(\mathbf{x}, t)$, or based on the volume that the constituent phase occupies as $\rho_t^\alpha(\mathbf{x}, t)$, in which the subscript t indicates that this is the "true" density. Hence, if the volume fraction of the α th phase is

$$\phi^\alpha = \frac{dV^\alpha}{dV} \quad (2)$$

where V^α and V are the volumes of the phase and mixture, respectively, the relationship between $\rho^\alpha(\mathbf{x}, t)$ and $\rho_t^\alpha(\mathbf{x}, t)$ is given by

$$\rho^\alpha(\mathbf{x}, t) = \phi^\alpha \rho_t^\alpha \quad (3)$$

In a solid-water coupled field, the water component mass density ρ^w is related to the unit volume of the fluid–solid mixture which has been expressed in mixture theory. Thus it is related to the true mass density ρ_t^w through

$$\rho^w = \phi^w \rho_t^w \quad (4)$$

in which ϕ^w is the porosity of the medium.

2.1. Balance equations for solid and fluid

Let V denote the volume of an arbitrary domain within the rock and S its boundary, which is attached to the solid phase (that is, there is no movement of solids across the domain boundary; only movement of fluid). The balance law for the α th phase is given by

$$\frac{D}{Dt} \int_V \rho^\alpha dV = \int_S \rho^\alpha (\mathbf{v}^\alpha - \mathbf{v}^s) \cdot \mathbf{n} dS \quad (5)$$

where \mathbf{n} is the unit outward normal, \mathbf{v}^α is the velocity of the α th phase, \mathbf{v}^s is the velocity of the solid, and the time derivative following the motion of the solid is

$$\frac{D}{Dt} = \partial_t + \mathbf{v}^s \cdot \nabla \quad (6)$$

where ∂_t is the time derivative and ∇ the gradient.

Thus, the balance equation for the solid mass is

$$\dot{\rho}^s + \rho^s \nabla \cdot \mathbf{v}^s = 0 \quad (7)$$

and for the water it is

$$\dot{\rho}^w + \rho^w \nabla \cdot \mathbf{v}^w + \nabla \cdot \mathbf{I}^w = 0 \quad (8)$$

where \mathbf{I}^w is the mass flux of water defined as

$$\mathbf{I}^w = \rho^w (\mathbf{v}^w - \mathbf{v}^s) \quad (9)$$

and \mathbf{v}^w is the velocity of water.

2.2. Balance equation for heat transport

In the arbitrary domain V , the thermal density can be changed only through the influx and efflux of heat flow across the boundary S . Since the capacities of heat of solid mass and of fluid mass are different, here \mathbf{q}^s denotes the heat density of solid which can be defined as $\mathbf{q}^s = \rho^s C^s T$ while \mathbf{q}^w denotes the heat density of water which can be defined as $\mathbf{q}^w = \rho^w C^w T$. Here, C^s and C^w denote the specific heat capacity of solid and water.

Further, the thermo flow across the boundary S can be separated in two parts: (1) the heat flow contained in the water flow which can be expressed as $h^w \mathbf{I}^w$ in which h^w is the enthalpy of water; (2) reduced heat flow \mathbf{q} (Katachalsky & Curran, 1965). Thus, followed the fundamental balance equation for thermodynamically open systems, the heat flow equation can be derived as:

$$\frac{D}{Dt} \int_V (\mathbf{q}^s + \mathbf{q}^w) dV = \int_S (\mathbf{q} + h^w \mathbf{I}^w) \cdot \mathbf{n} dS \quad (10)$$

From Eq. (6), the above equation can be rewritten as

$$(\dot{\mathbf{q}}^s + \dot{\mathbf{q}}^w) + (\mathbf{q}^s + \mathbf{q}^w) \nabla \cdot \mathbf{v}^s + \nabla \cdot (\mathbf{q} + h^w \mathbf{I}^w) = 0 \quad (11)$$

Further discussion of heat balance equation is in Section 5.3.

2.3. Balance equation for Helmholtz free energy

For a non constant temperature field in the absence of chemical reactions, it is convenient to use Helmholtz free energy (Haase, 1969), which combines both internal energy and entropy.

The balance of internal energy is

$$\frac{D}{Dt} \int_V \varepsilon dV = \int_S (\boldsymbol{\sigma} \mathbf{v}^s - \mathbf{q}) \cdot \mathbf{n} dS - \int_S h^w \mathbf{I}^w \cdot \mathbf{n} dS \quad (12)$$

where ε is the internal energy density, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, \mathbf{q} is heat flux and h^w is the enthalpy density of water.

The balance of entropy density (η) is

$$\frac{D}{Dt} \int_V \eta dV = \int_S \frac{-\mathbf{q}}{T} \cdot \mathbf{n} dS + \int_S -\eta \mathbf{I}^w \cdot \mathbf{n} dS + \int_V \gamma dV \quad (13)$$

where γ is the entropy production per unit volume.

The Helmholtz free energy density is defined as $\psi = \varepsilon - T\eta$ and the chemical potential is defined as $\mu = h^w - T\eta$ (Haase, 1969). Hence, the balance equation for Helmholtz free energy is

$$\frac{D}{Dt} \int_V \psi dV = - \int_S \boldsymbol{\sigma} \mathbf{n} \cdot \mathbf{v}^s dS - \int_S \mu \mathbf{I}^w \cdot \mathbf{n} dS - \int_V T \gamma dV \quad (14)$$

By using Reynold's transport theory, the derivative version of the balance equation for the free energy can be expressed as

$$\dot{\psi} + \psi \nabla \cdot \mathbf{v}^s - \nabla \cdot (\boldsymbol{\sigma} \mathbf{v}^s) + \nabla \cdot (\mu \mathbf{l}^w) = -T\gamma \leq 0 \quad (15)$$

3. Extending Darcy's law and thermal diffusion law

Here, a macroscopic expression for the dissipation generated by the frictional resistance at the solid and fluid interface at non-isothermal conditions can be obtained by using standard arguments of non-equilibrium thermodynamics (Katachalsky & Curran, 1965),

$$T\gamma = \mathbf{q} \cdot \frac{\nabla(-T)}{T} + \mathbf{l}_w \cdot \nabla(-\mu_w) \quad (16)$$

in which μ is the chemical potential of water. Function (16) is different from dissipation function of isothermal condition because of the term $\mathbf{q} \cdot \frac{\nabla(-T)}{T}$.

The relationship between water chemical potential μ and water pressure p can be derived by using the Gibbs–Duhem equation (Moran & Shapiro, 2000) based on the assumption of local isothermal condition as

$$\rho_t^w \nabla \mu = \nabla p \quad (17)$$

in which p is the pore pressure of the fluid. Also, the Darcy velocity is defined as

$$\mathbf{u} = \phi^w (\mathbf{v}^w - \mathbf{v}^s) \quad (18)$$

Hence, Eqs. (15) and (16) can be used to rewrite the dissipation function as

$$0 \leq T\gamma = \mathbf{q} \cdot \frac{\nabla(-T)}{T} - \mathbf{u} \nabla p \quad (19)$$

3.1. Phenomenological equations

Since \mathbf{u} is velocity, the term $\mathbf{u} \nabla p$ should be rewritten as

$$\mathbf{u} \nabla p = (\rho_t^w \mathbf{u}) \left(\frac{\nabla p}{\rho_t^w} \right) \quad (20)$$

Thus, the dissipation function can be written as

$$T\gamma = -(\rho_t^w \mathbf{u}) \left(\frac{\nabla p}{\rho_t^w} \right) - \mathbf{q} \cdot \frac{\nabla T}{T} \quad (21)$$

The relationship between the flow $\bar{\rho} \mathbf{u}$, \mathbf{q} and driving force ∇p , $\frac{\nabla(-T)}{T}$ is obtained with the help of phenomenological equations (Moran & Shapiro, 2000) which express the linear dependence of flow on the corresponding force.

The phenomenological equations can be written as

$$\rho_t^w \mathbf{u} = -L^{11} \left(\frac{\nabla p}{\rho_t^w} \right) - L^{12} \frac{\nabla T}{T} \quad (22)$$

$$\mathbf{q} = -L^{21} \left(\frac{\nabla p}{\rho_t^w} \right) - L^{22} \left(\frac{\nabla T}{T} \right) \quad (23)$$

where L^{11} , L^{12} , L^{21} , L^{22} are phenomenological coefficient.

For the fluid diffusion, the Darcy's law can be derived as

$$\mathbf{u} = -\frac{L^{11}}{(\rho_t^w)^2} (\nabla p) - \frac{L^{12}}{\rho_t^w} \frac{\nabla T}{T} = -\left(\frac{L^{11}}{(\rho_t^w)^2} \right) \left[\nabla p + \left(\frac{L^{12}}{L^{11}} \right) \frac{\rho_t^w \nabla T}{T} \right] \quad (24)$$

in which $\frac{L^{11}}{(\rho_t^w)^2} = \frac{k}{v}$, k is the permeability and v is the fluid's viscosity; $\frac{L^{12}}{L^{11}} = r_f$ can be defined as reflection coefficient, serves as a measure of the efficiency of the osmotic transport.

Consequently, the Darcy's law is changed to

$$\mathbf{u} = -\frac{k}{v} \left[\nabla p + r_q \frac{\rho_t^w \nabla T}{T} \right] \quad (25)$$

in which, the thermal osmosis has been considered.

For thermo diffusion, the thermo flow function can be expressed as

$$\mathbf{q} = -L^{21} \left(\frac{\nabla p}{\rho_t^w} \right) - L^{22} \left(\frac{\nabla T}{T} \right) = -\left(\frac{L^{21} p}{(\rho_t^w)^2} \right) \left(\rho_t^w \frac{\nabla p}{p} \right) - L^{22} \left(\frac{\nabla T}{T} \right) = -L_q \left(\rho_t^w \frac{\nabla p}{p} \right) - \frac{L^{22}}{T} \nabla T \quad (26)$$

where $L_q = \left(\frac{L^{21} p}{(\rho_t^w)^2} \right)$, L_q is pressure diffusion coefficient for thermo; $\frac{L^{22}}{T} = \lambda$ is defined as conduction coefficient.

3.2. The coupling matrix and result analysis

The whole coupling matrix can be written as

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{q} \end{pmatrix} = - \begin{pmatrix} \frac{k}{v} & \frac{kT_q \rho_t^w}{vT} \\ \frac{L_q \rho_t^w}{p} & \lambda \end{pmatrix} \begin{pmatrix} \nabla p \\ \nabla T \end{pmatrix} \quad (27)$$

If without considering the coupling term of heat flow and water flow, the matrix above can be written as the matrix below

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{q} \end{pmatrix} = - \begin{pmatrix} \frac{k}{v} & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} \nabla p \\ \nabla T \end{pmatrix} \quad (28)$$

which is generally used in research of coupled model.

4. Constitutive relations

The constitutive equations for the stress, strain and temperature response are formulated here by considering dissipation function and free energy equation.

4.1. Basic equation of state

It is assumed that the rock maintains mechanical equilibrium, so that $\nabla \cdot \boldsymbol{\sigma} = \mathbf{0}$. By using Eqs. (15) and (16), for Helmholtz free energy density balance and entropy production, the resulting balance equation for $\dot{\psi}$ is derived as

$$\dot{\psi} + \psi \nabla \cdot \mathbf{v}^s - \text{tr}(\boldsymbol{\sigma} \nabla \mathbf{v}^s) + \mu \nabla \cdot \mathbf{I}^w + T \nabla \cdot \left(\frac{\mathbf{q}}{T} \right) = 0 \quad (29)$$

Eq. (29) has been included the thermo part $T \nabla \cdot \left(\frac{\mathbf{q}}{T} \right)$ from dissipation function. Since $\frac{\mathbf{q}}{T} = \mathbf{S}$, thus it can be regarded as entropy flow to simplify Eq. (29). Further, Eq. (16) can also be written as entropy flow as follows:

$$T \gamma = \mathbf{q} \cdot \frac{\nabla(-T)}{T} + \mathbf{I}_w \cdot \nabla(-\mu_w) = \mathbf{S} \cdot \nabla(-T) + \mathbf{I}_w \cdot \nabla(-\mu_w) \quad (30)$$

The difference is that the corresponding force for \mathbf{q} is $\nabla(-T)/T$ while the corresponding force for \mathbf{S} is $\nabla(-T)$. This is explained here to give a right source for next discussion.

The expression of the referential equivalent of Eq. (29) can be derived from classical continuum mechanics, thus the rock's deformation state can be defined. The starting equations in the derivation process are

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}(\mathbf{X}, t), \quad \mathbf{E} = \frac{1}{2}(\mathbf{F}^T \mathbf{F} - \mathbf{1}), \quad J = \frac{dV}{dV_0}, \quad \dot{J} = J \nabla \cdot \mathbf{v}^s \quad (31)$$

where \mathbf{X} is an arbitrary reference configuration, \mathbf{x} is the position at the time t , \mathbf{F} is the gradient, \mathbf{E} is Green strain, J is deformation gradient, dV is the volume of the current configuration and dV_0 is the volume of the reference configuration.

By using Eq. (31), Eq. (29) can be rewritten as

$$\dot{\Psi} = \text{tr}(\mathbf{T}\dot{\mathbf{E}}) + \mu \dot{m} + T \dot{\mathbf{S}}_{\text{mixture}} \quad (32)$$

In which $\Psi = J\psi$ is the free energy in the reference configuration.

$\mathbf{S}_{\text{mixture}}$ is used here to give a clear concept of the entropy flow density per unit referential volume through solid and fluid mixture as $\mathbf{S}_{\text{mixture}} = J \phi^w \mathbf{S}_t^w + J \phi^s \mathbf{S}_t^s$, \mathbf{S}_t^w is the entropy flow density through fluid, while \mathbf{S}_t^s is the entropy flow density through solid; $m = J \rho = J \phi^w \rho_t^w$ is the mass of water per unit referential volume; \mathbf{T} is the second Piola–Kirchhoff stress, $\boldsymbol{\sigma}$ is the Cauchy stress.

4.2. Helmholtz free energy density of the pore water and wetted mineral matrix

If the mass density per unit fluid volume is defined as ρ_t^{pore} and the Helmholtz free energy density of the pore water is ψ_{pore} , then, based on classical thermodynamics, the free energy density can be expressed as

$$\psi_{\text{pore}} = -p + \mu \rho_t^{\text{pore}} + T S_t^{\text{pore}} \quad (33)$$

Also, according to the Gibbs–Duhem equation,

$$\dot{p} - \dot{T} S_t^{\text{pore}} = \rho_t^{\text{pore}} \dot{\mu} \quad (34)$$

From Eq. (33)

$$\dot{\psi}_{\text{pore}} = -\dot{p} + \dot{\mu}\rho_t^{\text{pore}} + \mu\dot{\rho}_t^{\text{pore}} + \dot{T}S_t^{\text{pore}} + TS_t^{\dot{\text{pore}}} \quad (35)$$

and so, by introducing Eq. (34),

$$\dot{\psi}_{\text{pore}} = \mu\dot{\rho}_t^{\text{pore}} + TS_t^{\dot{\text{pore}}} \quad (36)$$

The free energy density of the wetted mineral matrix can be derived by subtracting from the free energy Ψ of the combined rock/fluid system the contribution $J\phi\psi_{\text{pore}}$ due to pore water.

$$(\Psi - J\phi^w\psi_{\text{pore}}) = \text{tr}(\mathbf{T}\dot{\mathbf{E}}) + p\dot{v} + T\dot{S}_{\text{solid}} \quad (37)$$

where $v = J\phi$ is the pore volume per unit referential volume, and where, $\mathbf{S}_{\text{solid}} = \mathbf{S} - J(\phi^w\mathbf{S}_t^{\text{pore}})$ is the referential entropy density of the solid.

4.3. Constitutive equation structure

For reasons of convenience, the dual potential is used instead of directly from $\Psi - J\phi^w\psi_{\text{pore}}$

$$W = (\Psi - J\phi^w\psi_{\text{pore}}) - pv - TS_{\text{solid}} \quad (38)$$

where W is a function of \mathbf{E} , p and T , and thus expressions for $\boldsymbol{\sigma}$, v and S_{solid} may be obtained.

Eq. (37) implies that the time derivative of $W(E, p, T)$ satisfies the relationship

$$\dot{W}(\mathbf{E}, p, T) = \text{tr}(\mathbf{T}\dot{\mathbf{E}}) - \dot{p}v - \dot{T}S_{\text{solid}} \quad (39)$$

Since

$$\dot{W}(\mathbf{E}, p, T) = \left(\frac{\partial W}{\partial \mathbf{E}_{ij}}\right)_{p,T} \dot{\mathbf{E}}_{ij} + \left(\frac{\partial W}{\partial p}\right)_{\mathbf{E}_{ij},T} \dot{p} + \left(\frac{\partial W}{\partial T}\right)_{\mathbf{E}_{ij},p} \dot{T} \quad (40)$$

the following equations are obtained:

$$T_{ij} = \left(\frac{\partial W}{\partial \mathbf{E}_{ij}}\right)_{p,T}, \quad v = -\left(\frac{\partial W}{\partial p}\right)_{\mathbf{E}_{ij},T}, \quad S_{\text{solid}} = -\left(\frac{\partial W}{\partial T}\right)_{\mathbf{E}_{ij},p} \quad (41)$$

If Eq. (41) is differentiated with respect to time, the fundamental constitutive equations for the evolution of stress, pore volume fraction and temperature can be expressed as

$$\dot{T}_{ij} = L_{ijkl}\dot{\mathbf{E}}_{kl} - M_{ij}\dot{p} + S_{ij}\dot{T} \quad (42)$$

$$\dot{v} = M_{ij}\dot{\mathbf{E}}_{ij} + Q\dot{p} + B\dot{T} \quad (43)$$

$$\dot{S}_{\text{solid}} = S_{ij}\dot{\mathbf{E}}_{ij} + B\dot{p} + Z\dot{T} \quad (44)$$

where the parameters L_{ijkl} , M_{ij} , S_{ij} , Z , B and Q are material-dependent constants defined by the following group of equations:

$$\begin{aligned} L_{ijkl} &= \left(\frac{\partial T_{ij}}{\partial \mathbf{E}_{kl}}\right)_{p,T} = \left(\frac{\partial T_{kl}}{\partial \mathbf{E}_{ij}}\right)_{p,T}, \\ M_{ij} &= -\left(\frac{\partial T_{ij}}{\partial p}\right)_{\mathbf{E}_{ij},T} = \left(\frac{\partial v}{\partial \mathbf{E}_{ij}}\right)_{p,T}, \\ S_{ij} &= \left(\frac{\partial T_{ij}}{\partial T}\right)_{\mathbf{E}_{ij},p} = -\left(\frac{\partial S_{\text{solid}}}{\partial \mathbf{E}_{ij}}\right)_{p,T}, \\ Z &= \left(\frac{\partial S_{\text{solid}}}{\partial T}\right)_{\mathbf{E}_{ij},p}, \quad B = \left(\frac{\partial v}{\partial T}\right)_{\mathbf{E}_{ij},p} = \left(\frac{\partial S_{\text{solid}}}{\partial p}\right)_{\mathbf{E}_{ij},T}, \quad Q = \left(\frac{\partial v}{\partial p}\right)_{\mathbf{E}_{ij},\mu} \end{aligned} \quad (45)$$

4.4. Linear isotropic response

The non-linearity of the equations is of a geometrical nature and associated with large deformations. For isotropic materials, the tensors M_{ij} and S_{ij} are diagonal; that is, they can be written in the form of scalars ζ and ω_T , as follows:

$$M_{ij} = \zeta\delta_{ij}, \quad S_{ij} = \omega_T\delta_{ij} \quad (46)$$

and the elastic stiffness L_{ijkl} can be formed as a fourth-order isotropic tensor:

$$L_{ijkl} = G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \left(K - \frac{2G}{3}\right)\delta_{ij}\delta_{kl} \quad (47)$$

where G is the rock shear modulus and K the bulk modulus.

With the assumption of small strains, the governing stress and pore fraction equations, (42) and (43), can be changed to the form

$$\dot{\sigma}_{ij} = \left(K - \frac{2G}{3} \right) \dot{\varepsilon}_{kk} \delta_{ij} + 2G \dot{\varepsilon}_{ij} - \zeta \dot{p} \delta_{ij} + \omega_T \dot{T} \delta_{kl} \quad (48)$$

$$\dot{v} = \zeta \dot{\varepsilon}_{ii} + Q \dot{p} + B \dot{T} \quad (49)$$

where the void compressibility Q relates to the scalar ζ according to $Q = (1/K_s)(\zeta - \phi) + \phi^w/K_w$, in which K_s is the bulk modulus of the solid matrix and K_w is the bulk modulus of fluid mass; the quantity ζ is related to the bulk moduli, K and K_s , in a poro-elastic manner, that is, $\zeta = 1 - (K/K_s)$; $\omega_T = \beta_s/3$ and β_s is the thermal expansion coefficient of solid; $B = -\phi^w \beta_w - (1 - \phi^w) \beta_s$ in which β_w is the thermal expansion coefficient of water.

Hence, the stress and volume fractions of Eqs. (48) and (49) can be rewritten as

$$\dot{\sigma}_{ij} = \left(K - \frac{2G}{3} \right) \dot{\varepsilon}_{kk} \delta_{ij} + 2G \dot{\varepsilon}_{ij} - \zeta \dot{p} + \omega_T \dot{T} \quad (50)$$

$$\dot{v} = \zeta \dot{\varepsilon}_{ii} + Q \dot{p} + B \dot{T} \quad (51)$$

in which $\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j}$ and $u_i (i = 1, 2, 3)$.

5. Final equations of motion for modified mixture theory

5.1. Solid phase

If the mechanical equilibrium condition

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \quad (52)$$

is introduced into Eq. (48), the final equation is

$$\left(K - \frac{2G}{3} \right) \frac{\partial^2 \dot{u}_k}{\partial x_k \partial x_j} + 2G \frac{\partial^2 \dot{u}_i}{\partial x_k \partial x_j} - \zeta \frac{\partial \dot{p}}{\partial x_i} + \omega_T \frac{\partial \dot{T}}{\partial x_i} = 0 \quad (53)$$

5.2. Fluid phase

From Eqs. (4) and (8), and using the Euler identity, the following equation is derived:

$$(v \rho_t^w) + \nabla \cdot (\rho_t^w \mathbf{u}) = 0 \quad (54)$$

With the further assumption that the fluid is incompressible, Eq. (40) can be rearranged as

$$\dot{v} + \nabla \cdot \mathbf{u} = 0 \quad (55)$$

Thus, the control equation for the fluid phase can be written as

$$\zeta \frac{\partial \dot{u}_i}{\partial x_i} + Q \dot{p} + B \dot{T} - \nabla \cdot \frac{k}{v} \left(\nabla p + \frac{r_q \rho_t^w}{T} \nabla T \right) = 0 \quad (56)$$

5.3. Thermal phase

From Eq. (11), since \mathbf{q}^s and \mathbf{q}^w are related with mass density of ρ^s and ρ^w which are expressed relative to the unit volume of the fluid and solid mixture, the thermo density relative to true mass density can be defined as $\mathbf{q}_t^s \mathbf{q}_t^w$. The relationship between $\mathbf{q}^s \mathbf{q}_t^s$ and $\mathbf{q}^w \mathbf{q}_t^w$ can be defined as

$$\mathbf{q}_t^s = \mathbf{q}^s / \phi^s \quad \text{and} \quad \mathbf{q}_t^w = \mathbf{q}^w / \phi^w \quad (57)$$

By using Euler identity, Eq. (11) can be rewritten as

$$(v \mathbf{q}_t^w + (1 - v) \mathbf{q}_t^s) + \nabla \cdot (\rho_t^w \phi^w (\mathbf{v}^w - \mathbf{v}^s) h^w + \mathbf{q}) = 0 \quad (58)$$

Since, the Darcy velocity is defined through the definition

$$\mathbf{u} = \phi^w (\mathbf{v}^w - \mathbf{v}^s) \quad (59)$$

Eq. (58) can be rewritten as

$$(v \mathbf{q}_t^w + (1 - v) \mathbf{q}_t^s) + \nabla \cdot (\rho_t^w \mathbf{u} h^w + \mathbf{q}) = 0 \quad (60)$$

In which h^w can also be expressed as $h^w = C^w T$.

By neglecting the temperature and pressure dependence of fluid and solid densities and by neglecting the thermo coupling term due to pressure, Eq. (60) can be rearranged as

$$\frac{\partial}{\partial t} \{ [(1 - \phi^w) \rho^s C^s + \phi^w \rho^w C^w] T \} + \rho^w C^w (\nabla^T T) \mathbf{u} - \nabla^T \lambda \nabla T = 0 \quad (61)$$

which is the same function as that derived based on mechanics approach.

Further, in this paper, by neglecting the convection term, the thermo transport equation becomes as

$$\frac{\partial}{\partial t} \{ [(1 - \phi^w) \rho^s C^s + \phi^w \rho^w C^w] T \} - \nabla^T \lambda \nabla T = 0$$

6. Numerical results for thermal osmosis

Thermal osmosis might have an influence on wellbore stability or high level nuclear waste disposal with high temperature gradient. In this paper, a two dimensional axisymmetric model with the width of 50 mm and height of 15 mm is applied to analyze the thermal osmosis.

The boundary and initial conditions below are used to emulate the drilling impact on the wellbore stability:

- (1) Mechanical equilibrium: the mechanical equilibrium has been assumed at the initial time. The openhole boundary is assumed free while the far boundary is assumed fixed in displacement.
- (2) Water transport: the initial formation pressure of 10^6 Pa is assumed. The openhole boundary is raised immediately to 15×10^6 Pa to simulate the water-based mud pressure during drilling. Water will transport from openhole boundary inward due the driving force of water pressure.
- (3) Heat transport: the initial formation temperature is assumed 338 K. The temperature in the openhole boundary is immediately raised to 388 K and maintained at this value throughout to simulate the raising temperature in the mud during drilling.

Three reflection coefficients have been chosen as $r_q = -1, 0, 1$; which represent different kinds of osmosis, such as negative osmosis ($r_q = -1$, water flows from higher temperature to lower temperature if there is no water pressure gradient), no osmosis ($r_q = 0$, the driven force of water flow is only the water pressure gradient), positive osmosis ($r_q = 1$, water flows from lower temperature to higher temperature if there is no water pressure gradient). The direction of the flow depends on the membrane like material such as clay or shale. Such flow behaviour in both directions has been observed in laboratory tests using compacted clays (Gray & O'Neill, 1976). In actual conditions for drilling, the driving force for water flow might be the combination of water pressure gradient and thermal osmosis, apart from formation deformation. The parameters used for the numerical techniques is summarized in Table 1.

The pore water pressure, displacement and temperature have been varied during the simulation to analyse the coupling effects.

6.1. Analysis: The effects of thermal osmosis

In order to set the stage for discussion and comparison, the case with no thermal osmosis will be used as the base case. When there is no thermal osmosis ($r_q = 0$), water flows from higher pore pressure to lower pore pressure. This is illustrated by a monotonic decreasing pressure profile in Fig. 1, typical of a diffusion curve, along the distance. At steady state ($t = 3600$ s), the formation pressure equilibrates the prescribed pressure at the boundary, as expected. When the reflection coefficient is set to negative, the thermal osmotic effect comes into play and causes influx of water into the porous medium. This increased transport of water into the formation is driven entirely by the thermal gradient. This thermal-related osmosis causes the pore pressure within the porous medium to rise. This increase in the pore pressure is proportional to the temperature gradient in the porous medium. At steady state, the entire porous medium is at a higher pore pressure than the prescribed boundary pressure. When the reflection coefficient is set to unity, in other words, water flows from lower

Table 1
Parameters used for the numerical experiment (Ghassemi & Diek, 2003).

Young's modulus	2.6×10^4 MPa
Poisson's ratio	0.20
Thermo-osmotic coefficient	6.0×10^{-11} m ² /s
Thermal diffusivity	1.6×10^{-6} m ² /s
Porosity	0.4
Permeability	7.66×10^{-15} m ² /s
Biot coefficient	1
Thermal coefficient of heat expansion, solid	0.9e-6
Thermal reflection coefficient	1, 0, -1

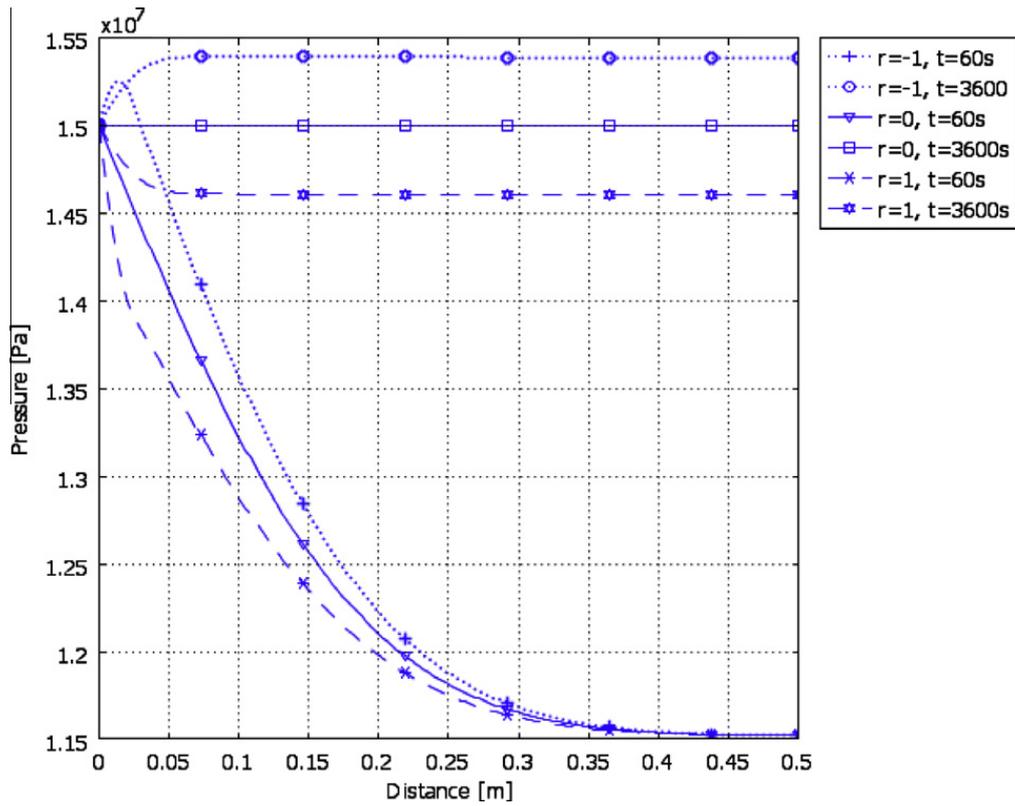


Fig. 1. Pore pressure distribution.

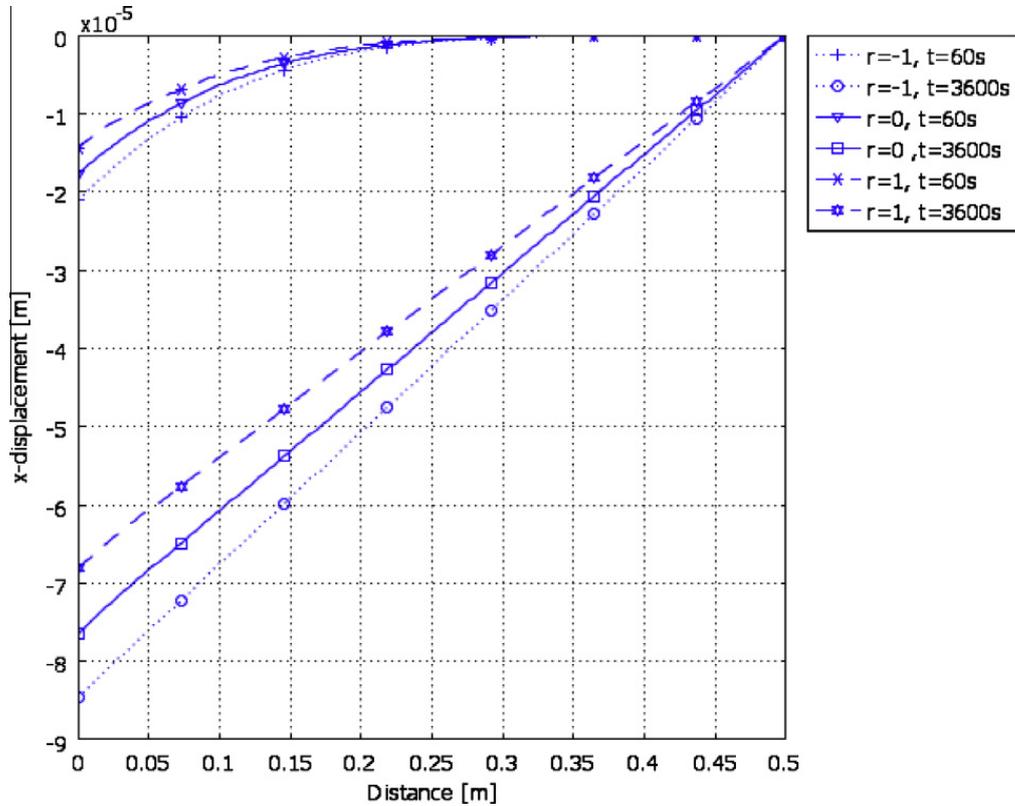


Fig. 2. Displacement distribution.

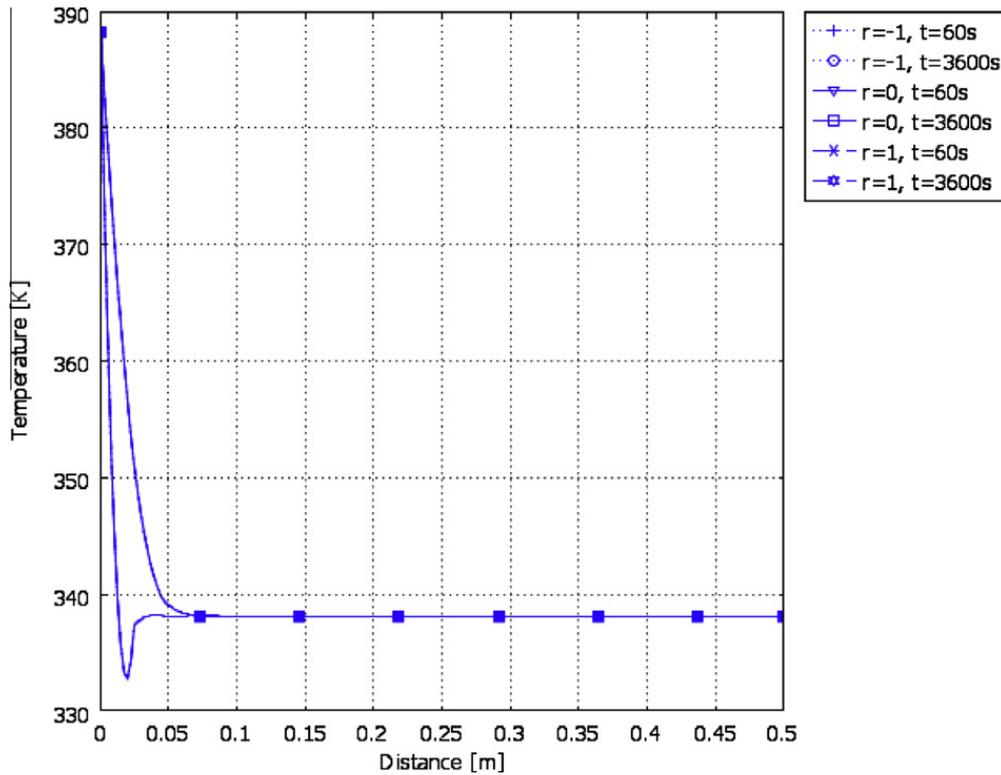


Fig. 3. Temperature distribution.

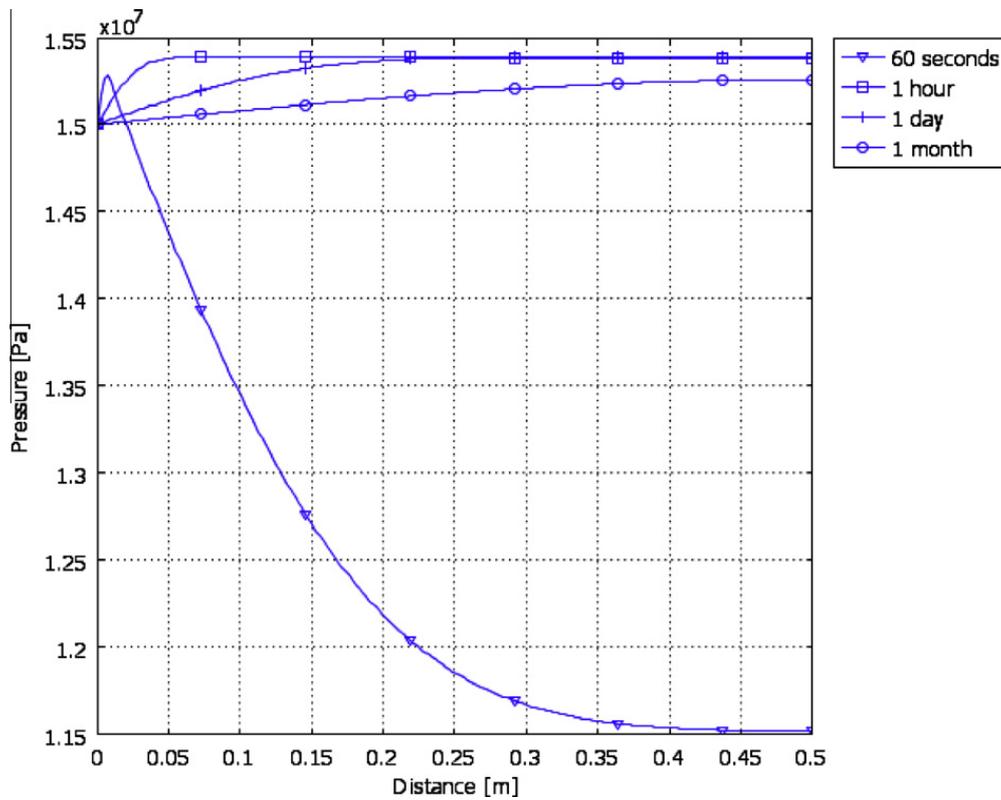


Fig. 4. Pore pressure distribution with time.

temperature to higher temperature, the porous medium experiences an efflux of water. Again, this efflux is driven entirely by a negative temperature gradient. Consequently, the pore pressure in the formation decreases very fast in the beginning of the simulation. At steady state, the entire porous medium is at a lower pressure than the prescribed boundary pressure.

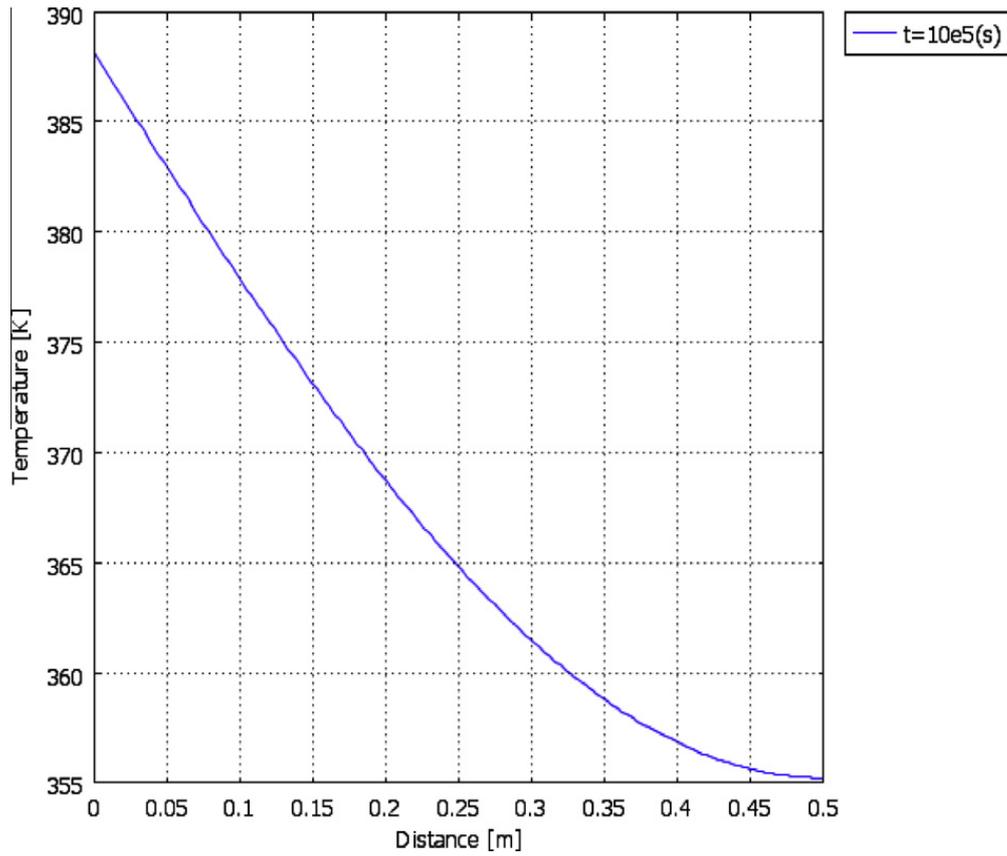


Fig. 5. Temperature distribution.

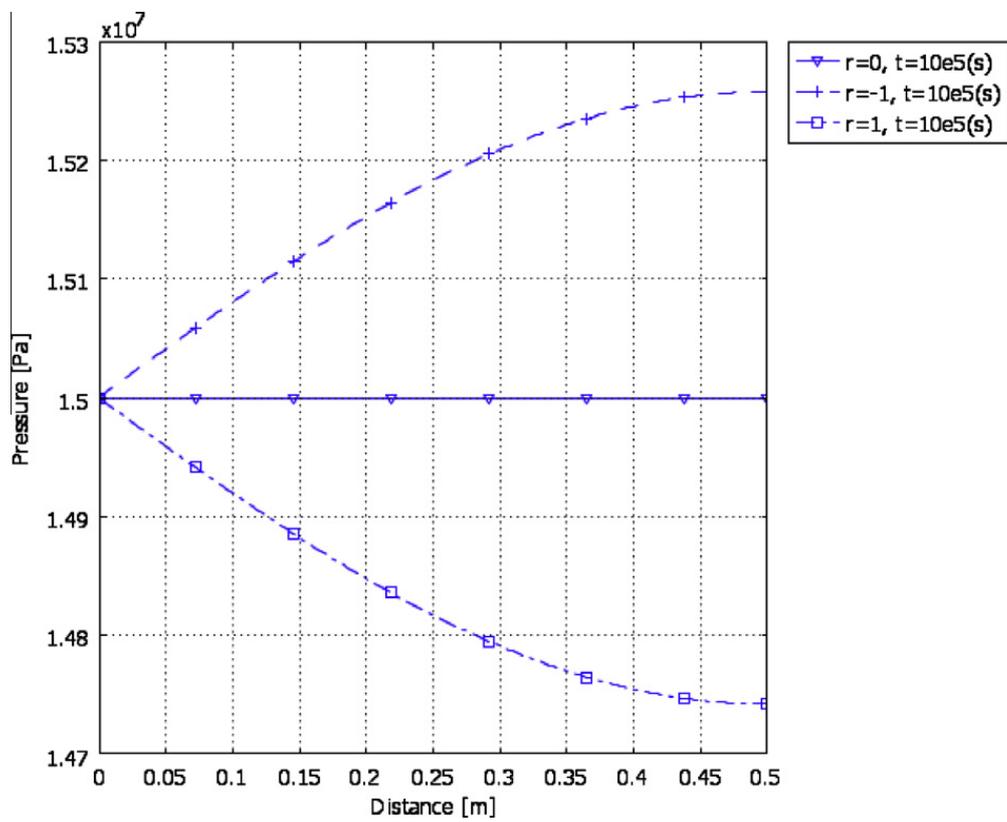


Fig. 6. Pore pressure distribution.

Fig. 2 shows the axial displacement along the distance. The case with negative reflection coefficient experiences the largest displacement due to the efflux of water. This is expected because the coupling nature of the formulation requires that the efflux of water is compensated by the shrinkage of the porous medium. On the other hand, the case with positive thermal-osmotic coupling experiences the least displacement due to the influx of water.

Fig. 3 shows the thermal distribution in the rock sample. It is observed that there is a sudden jump of temperature at the early stage of simulation. In this paper, the focus is on the influence of thermal osmosis coupling, it is thus assumed that thermal conduction is not affected by any other process such as mechanical deformation and water transport. The temperature distribution for different coefficient of reflections is expected to be the same. The temperature jump is hence interpreted here as numerical oscillation which could be due to a sudden increase of prescribed temperature at the boundary with very high thermal gradient. Attempts were made to use smaller time step sizes to no avail.

Different selected time stations at $t = 60$ s, 1 h, 1 day and 1 month have been chosen to illustrate the osmosis effects. Fig. 4 shows the negative thermal osmosis helps raise the driving force pore water pressure gradient for water flow. If the temperature gradient goes down, the thermal osmosis contribution in the driving force goes down and effect the water pressure drops down from 1 day to 1 month. It can be predicted that if the temperature difference disappears, the only driving force for water flow will just be water pressure.

The results for temperature and pore pressure distribution at $t = 10e5$ s have been shown in Figs. 5 and 6, respectively. Since there is a constant gradient of temperature distribution against the distance (Fig. 5), there might be correspondingly constant constant gradient for pore pressure due to thermal osmosis for $r_q = -1$ and $+1$, as shown in Fig. 6. However, if $r_q = 0$, the coupling between thermal osmosis and pore pressure disappears from Eq. (56), thus the pore pressure equals zero and has not be affected by the temperature gradient.

7. Conclusion

In the paper, a new coupled formulation including thermal osmosis based on modified mixture theory has been obtained. Modified mixture theory is based on mixture theory, non-equilibrium thermodynamics and Biot elasticity. Darcy's law has been extended by including thermal osmosis.

A simple numerical example has been chosen to analyze the osmosis effects. Negative thermal osmosis, positive thermal osmosis and non osmosis have been analyzed. It has been predicted that thermal osmosis might have a strong influence on water transport and then mechanical deformation.

Further experiment might need for refine the osmosis coefficient r_q because it depends on the rock or clay itself.

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