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Stress Analysis of Reeled Composite Pipelines based on Shallow Shell Theories

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Abstract. The instabilities in reeled pipelines during spooling of (initially straight) or laying of (initially bent) pipeare investigated based on shell theories. A combination of composite materials was studied and number of bending stability cases is discussed. It was demonstrated that composite pipeline exhibits more stability than steel pipes. The bending stability of spooled pipeline is examined in detailed and in particular, the case on the diameter drum of the reel lay system. Moreover, results on the strain energy release in the reel lay system are presented, extending the findings of previous works on controlling the spooling or laying mechanism.

Introduction

Reel pipe lay is a conventional method of installing pipelines in the ocean, when the pipeline is wound on a massive reel mounted on the deck of a pipe lay barge [1]. Pipelines are generally spooled into a reel at an onshore spool-base facility. The first commercial application of reeled pipeline technology was available by *Santa Fe Corporation* in the early 1970s[2-9].

Definition

Assume adouble curved shallow shell element was taken from a pipeline reeled into large dram diameter. The shell element is defined as shallow when it has a small curvature or large radius of curvature.The concept of deepness or shallowness of the shell is adoptedto elaborate thederivation of shell equations, and it is adequate for designingpipe undergo reeling.

Kinematic

According to the first-order transversely shearable shell theory, the following representation of the 3-D displacement field is postulated:

$$\begin{Bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \varepsilon_{\beta\zeta} \\ \varepsilon_{\alpha\zeta} \\ \varepsilon_{\beta\alpha} \\ \varepsilon_{\alpha\beta} \end{Bmatrix} = \begin{bmatrix} \lambda_\alpha & \cdots & & & & 0 \\ & \lambda_\beta & & & & \\ \vdots & & \lambda_\beta & & & \\ & & & \lambda_\alpha & & \vdots \\ & & & & \lambda_\beta & \\ 0 & & & & \cdots & \lambda_\alpha \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{o\alpha} \\ \varepsilon_{o\beta} \\ \varepsilon_{o\beta\zeta} \\ \varepsilon_{o\alpha\zeta} \\ \varepsilon_{o\beta\alpha} \\ \varepsilon_{o\alpha\beta} \end{Bmatrix} + \zeta \begin{Bmatrix} \varepsilon_{1\alpha} \\ \varepsilon_{1\beta} \\ \psi_\beta / R_\beta \\ \psi_\alpha / R_\alpha \\ \varepsilon_{1\beta\alpha} \\ \varepsilon_{1\alpha\beta} \end{Bmatrix} \right) \; and \; \lambda_\alpha = 1/(1+\zeta/\mathrm{R}_\alpha)$$

where,u_o, v_o and $w_o$are the mid-surface displacements and ψ_α and ψ_β are midsurface rotations of the shell. The strains at any point in Codazzi-Gaussshell element can be writtenas follow:

$$\begin{Bmatrix} \varepsilon_{o\alpha} \\ \varepsilon_{o\beta} \\ \varepsilon_{o\beta\zeta} \\ \varepsilon_{o\alpha\zeta} \\ \varepsilon_{o\beta\alpha} \\ \varepsilon_{o\alpha\beta} \end{Bmatrix} = \begin{bmatrix} \frac{1}{A}\frac{\partial}{\partial\alpha} & \frac{1}{AB}\frac{\partial A}{\partial\beta} & \frac{1}{R_\alpha} & 0 & 0 \\ \frac{1}{AB}\frac{\partial B}{\partial\alpha} & \frac{1}{B}\frac{\partial}{\partial\beta} & \frac{1}{R_\beta} & 0 & 0 \\ -\frac{1}{R_{\alpha\beta}} & -\frac{1}{R_\beta} & \frac{1}{B}\frac{\partial}{\partial\beta} & 1 & 0 \\ -\frac{1}{R_\alpha} & -\frac{1}{R_{\alpha\beta}} & \frac{1}{A}\frac{\partial}{\partial\alpha} & 0 & 1 \\ \frac{1}{B}\frac{\partial}{\partial\beta} & -\frac{1}{AB}\frac{\partial B}{\partial\alpha} & \frac{1}{R_{\alpha\beta}} & 0 & 0 \\ -\frac{1}{AB}\frac{\partial A}{\partial\beta} & \frac{1}{A}\frac{\partial}{\partial\alpha} & \frac{1}{R_{\alpha\beta}} & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_o \\ v_o \\ w_o \\ \psi_\beta \\ \psi_\alpha \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_{1\alpha} \\ \varepsilon_{1\beta} \\ \frac{\psi_\beta}{R_\beta} \\ \frac{\psi_\alpha}{R_\alpha} \\ \varepsilon_{1\beta\alpha} \\ \varepsilon_{1\alpha\beta} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{AB}\frac{\partial A}{\partial\beta} & \frac{1}{A}\frac{\partial}{\partial\alpha} \\ 0 & 0 & 0 & \frac{1}{B}\frac{\partial}{\partial\beta} & \frac{1}{AB}\frac{\partial B}{\partial\alpha} \\ \frac{1}{R_{\alpha\beta}} & \frac{1}{R_\beta} & -\frac{1}{B}\frac{\partial}{\partial\beta} & 0 & 0 \\ \frac{1}{R_\alpha} & \frac{1}{R_{\alpha\beta}} & -\frac{1}{A}\frac{\partial}{\partial\alpha} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{AB}\frac{\partial B}{\partial\alpha} & \frac{1}{B}\frac{\partial}{\partial\beta} \\ 0 & 0 & 0 & \frac{1}{A}\frac{\partial}{\partial\alpha} & -\frac{1}{AB}\frac{\partial A}{\partial\beta} \end{bmatrix} \begin{Bmatrix} u_o \\ v_o \\ w_o \\ \psi_\beta \\ \psi_\alpha \end{Bmatrix}$$

The above equation constitutes the field changes relations that requires for the*FOSD theory* that is passable to represent the problem.

Kinetic

Integrating the stresses over the shell thickness,the force and moment resultants will be

$$\begin{Bmatrix} N_\alpha^S \\ N_\beta^S \\ Q_\beta^S \\ Q_\alpha^S \\ N_{\alpha\beta}^S \\ N_{\beta\alpha}^S \\ M_\alpha^S \\ M_\beta^S \\ P_\beta^S \\ P_\alpha^S \\ M_{\alpha\beta}^S \\ M_{\beta\alpha}^S \end{Bmatrix} = \begin{bmatrix} \bar\varsigma_{11}^1 & \varsigma_{12}^1 & \varsigma_{14}^1 & \bar\varsigma_{15}^1 & \bar\varsigma_{16}^1 & \varsigma_{16}^1 & \bar\varsigma_{11}^2 & \varsigma_{12}^2 & \varsigma_{14}^2 & \bar\varsigma_{15}^2 & \bar\varsigma_{16}^2 & \varsigma_{16}^2 \\ & \tilde\varsigma_{22}^1 & \tilde\varsigma_{24}^1 & \varsigma_{25}^1 & \varsigma_{26}^1 & \tilde\varsigma_{26}^1 & \varsigma_{12}^2 & \tilde\varsigma_{22}^2 & \tilde\varsigma_{24}^2 & \varsigma_{25}^2 & \varsigma_{26}^2 & \tilde\varsigma_{26}^2 \\ & & \tilde\varsigma_{44}^1 & \varsigma_{45}^1 & \varsigma_{46}^1 & \tilde\varsigma_{46}^1 & \varsigma_{14}^2 & \tilde\varsigma_{24}^2 & \tilde\varsigma_{44}^2 & \varsigma_{45}^2 & \varsigma_{46}^2 & \tilde\varsigma_{46}^2 \\ & & & \bar\varsigma_{55}^1 & \bar\varsigma_{56}^1 & \varsigma_{56}^1 & \bar\varsigma_{15}^2 & \varsigma_{25}^2 & \varsigma_{45}^2 & \bar\varsigma_{55}^2 & \bar\varsigma_{56}^2 & \varsigma_{56}^2 \\ & & & & \bar\varsigma_{66}^1 & \varsigma_{66}^1 & \bar\varsigma_{16}^2 & \varsigma_{26}^2 & \tilde\varsigma_{46}^2 & \varsigma_{56}^2 & \bar\varsigma_{66}^2 & \varsigma_{66}^2 \\ & & & & & \tilde\varsigma_{66}^1 & \varsigma_{16}^2 & \varsigma_{26}^2 & \tilde\varsigma_{46}^2 & \varsigma_{56}^2 & \varsigma_{66}^2 & \tilde\varsigma_{66}^2 \\ & & & & & & \bar\varsigma_{11}^3 & \varsigma_{12}^3 & \varsigma_{14}^3 & \bar\varsigma_{15}^3 & \bar\varsigma_{16}^3 & \varsigma_{16}^3 \\ & & & & & & & \tilde\varsigma_{22}^3 & \tilde\varsigma_{24}^3 & \varsigma_{25}^3 & \varsigma_{26}^3 & \tilde\varsigma_{26}^3 \\ & & & & & & & & \tilde\varsigma_{44}^3 & \varsigma_{45}^3 & \varsigma_{46}^3 & \tilde\varsigma_{46}^3 \\ & \mathit{Symmetric} & & & & & & & & \bar\varsigma_{55}^3 & \bar\varsigma_{56}^3 & \varsigma_{56}^3 \\ & & & & & & & & & & \bar\varsigma_{66}^3 & \varsigma_{66}^3 \\ & & & & & & & & & & & \tilde\varsigma_{66}^3 \end{bmatrix} \begin{Bmatrix} \varepsilon_{o\alpha} \\ \varepsilon_{o\beta} \\ \varepsilon_{o\beta\zeta} \\ \varepsilon_{o\alpha\zeta} \\ \varepsilon_{o\alpha\beta} \\ \varepsilon_{o\beta\alpha} \\ \varepsilon_{1\alpha} \\ \varepsilon_{1\beta} \\ \varepsilon_{1\beta\zeta} \\ \varepsilon_{1\alpha\zeta} \\ \varepsilon_{1\alpha\beta} \\ \varepsilon_{1\beta\alpha} \end{Bmatrix}$$

where $\varsigma_{ij}^{p} \to p = 1,2,3,4$;is the stiffness properties and the superscript indicateto understand the differences between the extensional, extensional-bending and bending stiffness coefficients, which are defined as follows:

$$\underbrace{\left(\varsigma_{\mathrm{ij}}^1, \varsigma_{\mathrm{ij}}^2, \varsigma_{\mathrm{ij}}^3, \varsigma_{\mathrm{ij}}^4\right)}_{(i,j=1,2,6)} = \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \hat\varsigma_{ij}^{k} (1, \zeta, \zeta^2, \zeta^3)\, d\zeta, \quad \underbrace{\left(\varsigma_{\mathrm{ij}}^1, \varsigma_{\mathrm{ij}}^2, \varsigma_{\mathrm{ij}}^3, \varsigma_{\mathrm{ij}}^4\right)}_{(i,j=4,5)}$$

$$= K_i^2 \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \hat\varsigma_{ij}^{k} (1, \zeta, \zeta^2, \zeta^3)\, d\zeta, \quad \mathit{and} \text{ all other } \varsigma \equiv 0$$

where $[\bar{*}]_{\mathrm{ij}}^{\mathrm{n}} = [*]_{\mathrm{ij}}^{\mathrm{n}} - \mathrm{C_o}[*]_{\mathrm{ij}}^{\mathrm{n+1}}$ and $[\tilde{*}]_{\mathrm{ij}}^{\mathrm{n}} = [*]_{\mathrm{ij}}^{\mathrm{n}} + \mathrm{C_o}[*]_{\mathrm{ij}}^{\mathrm{n+1}}$and $\hat\varsigma_{ij}^{k}$is the transformed properties of orthotropic materials. This method gives a solution for the same choice of deep shell stiffness coefficient, also K_{i} and K_{j} are shear correction coefficients, typically taken at 5/6 (Timoshenko 1921). In addition, h_{k} is the distance from the midsurface to the surface of the k^{th} layer having the farthest ζ-coordinate.

Equation of Motion

For a differential shell element was picked from reeled pipe in reel lay mechanism, see, Fig. 1.

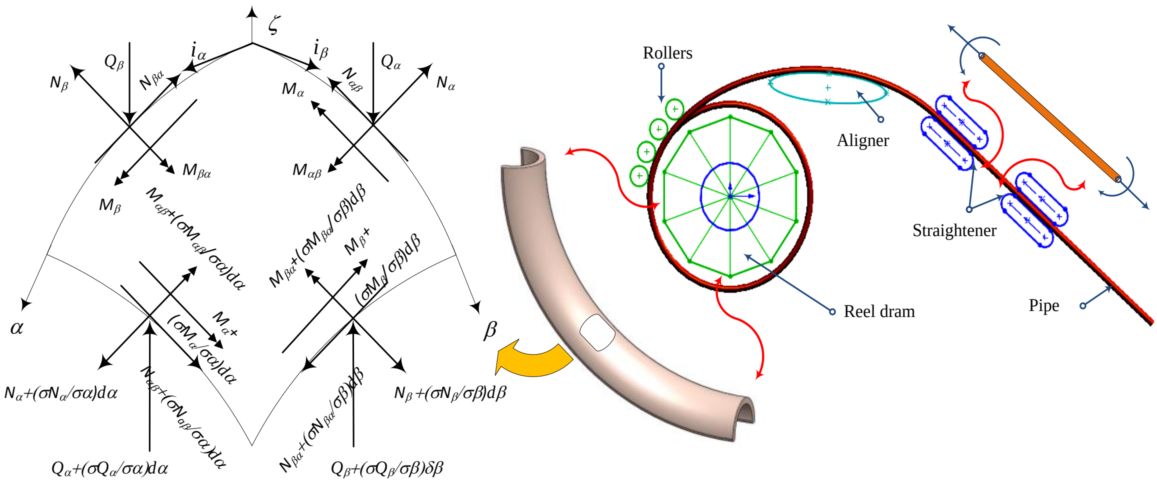

Figure 1. A shallow shell element was taken from reeled pipe.

The equation of motion can be obtained by setting the summations of the externals and internals body forces and moment to zero. Having applied the assumptions of double curved shallow shell theory, the equations of motioncan be written in terms of displacements as $\mathcal{K}_{ij}\{\Delta\} + \mathcal{M}_{ij}\{\ddot{\Delta}\} = \{\mathcal{F}\}$. Where $\{\mathcal{F}\}$ is the forcing function and Δ is:$\{\Delta\}^t = \left\{U_{mn}, V_{mn}, W_{mn}, \Psi^{\alpha}_{mn},\ \Psi^{\beta}_{mn}\right\}$.The forced-type solution can be applied to obtain an exact solution as $\left(\mathcal{K}_{ij} + \lambda^2\mathcal{M}_{ij}\right)\{\Delta\} = \{\mathcal{F}\}$, which is an eigenvalue problem. For a nontrivial solution, the determinant of the matrix in the parenthesis is set to zero. Then, the configuration of the $\bar{\mathcal{K}}_{ij}$ terms for the SS-1, cross-ply and rectangular plane form is listed below[10]

$$\mathcal{K}_{11} = -\tilde{A}^1_{11}\alpha_m^2 - \tilde{A}^1_{66}\beta_n^2,\ \mathcal{K}_{12} = -(A^1_{12} + A^1_{66})\alpha_m\beta_n\ ,\ \mathcal{K}_{22} = -\tilde{A}^1_{22}\beta_n^2 - \bar{A}^1_{66}\alpha_m^2$$

$$\mathcal{K}_{13} = \left(\frac{1}{R_\alpha}\bar{A}^1_{11} + \frac{1}{R_\beta}A^1_{12}\right)\alpha_m,\ \mathcal{K}_{23} = \left(\frac{1}{R_\alpha}A^1_{12} + \frac{1}{R_\beta}\tilde{A}^1_{22}\right)\beta_n\ ,\mathcal{K}_{14} = -\bar{B}^2_{11}\alpha_m^2 - \tilde{B}^2_{66}\beta_n^2,$$

$$\mathcal{K}_{33} = -\tilde{A}^1_{44}\beta_n^2 - \bar{A}^1_{55}\alpha_m^2 - \left(\frac{1}{R_\alpha^2}\bar{A}^1_{11} + \frac{2}{R_\alpha R_\beta}A^1_{12} + \frac{1}{R_\beta^2}\tilde{A}^1_{22}\right),\ \ \mathcal{K}_{45} = -(D^3_{12} + D^3_{66})\alpha_m\beta_n$$

$$\mathcal{K}_{24} = -(B^2_{12} + B^2_{66})\alpha_m\beta_n\ ,\ \mathcal{K}_{34} = \left(-\bar{A}^1_{55} + \frac{1}{R_\alpha}\bar{B}^2_{11} + \frac{1}{R_\beta}B^2_{12}\right)\alpha_m,$$

$$\mathcal{K}_{44} = -\bar{A}^1_{55} - \bar{D}^3_{11}\alpha_m^2 - \tilde{D}^3_{66}\beta_n^2,\ \mathcal{K}_{15} = -(B^2_{12} + B^2_{66})\alpha_m\beta_n\ ,\ \mathcal{K}_{25} = -\tilde{B}^2_{22}\beta_n^2 - \bar{B}^2_{66}\alpha_m^2$$

$$\mathcal{K}_{35} = \left(-\tilde{A}^1_{44} + \frac{1}{R_\alpha}B^2_{12} + \frac{1}{R_\beta}\tilde{B}^2_{22}\right)\beta_n\ ,\ \mathcal{K}_{55} = -\tilde{A}^1_{44} - \bar{D}^3_{66}\alpha_m^2 - \tilde{D}^3_{22}\beta_n^2$$

Results and Discussion

The analysis of stresses and deformationsin a double curved shallow shell element of areeled pipe made of common composite materials has beenaccomplished. Table 1 displays the center of deflection, in-plane and inter-laminar stresses, for the pipeline to be bend elastically. The table also reveals that composite materialCFRP appears more flexible and applicable to be reeled into a drum of small diameter. However, composite materials Gl-Ep and Gr-Epwere found to exhibit rigid behavior.The nominated composite materials are relevant to the application of compliant "reel" pipelines.It was expected that the composite materialBr-Ep exhibitsa rabid release of strain energy during laying installations. Composite pipeline could be laid without strain energy relaxation and this will benefit the insulation process better.

Table 1.Frequencies ω and center deflections w for some of common composite materials

Material	$\bar{\omega}_{11}^2 = \omega a^2\sqrt{\frac{\rho_{max}}{\varsigma_{max}h^2}}$	$\bar{w}_c = 10^2 \times \frac{w_o}{h}$	$\mathcal{S}_\alpha$	$\mathcal{S}_\beta$	$\mathcal{S}_{\alpha\beta}$	$\mathcal{S}_{\alpha\zeta}^{k=1,3}$	$\mathcal{S}_{\beta\zeta}^{k=2,4}$
			The load is dimensionalized as $= \frac{P_o\alpha^4}{E_2h^4}$				
Gl-Ep	2.0526	2.8739	0.1557	0.0681	0.0430	0.1171	0.0655
Gr-Ep	1.1283	5.6146	0.2712	0.0332	0.0273	0.1760	0.0357
Br-Ep	4.4430	1.7714	0.3856	0.0563	-0.0250	0.2440	0.0483
CFRP	0.9917	7.3194	0.3401	0.0318	0.0240	0.2148	0.0329

† $(N = 4,\ a/h = 0.1,\ a/R = 0.5,\ a/b = 1,\ m = n = 1,\ P_o = 1N)$, while materials properties could be found in [11].

Conclusions

In this paper, the stressanalysis for composite reeledpipeline wasanalysedtheoretically. The accurate treatment of strain energy “the plastic-deformation” that induced into an element selected from the reeled pipeline yields rather sophisticated displacement and stress resultant. A supplement digression on the implication of the effects of pipeline parametersand composite properties has been investigated. The introduced modelis considered of particular interest in designing of composite reel pipeline, or may serve as a reference in developing the wounding pipe. The proposed simulated framework can improve the benchmark solutions for judging the existence of imprecise models and other numerical simulation.

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