

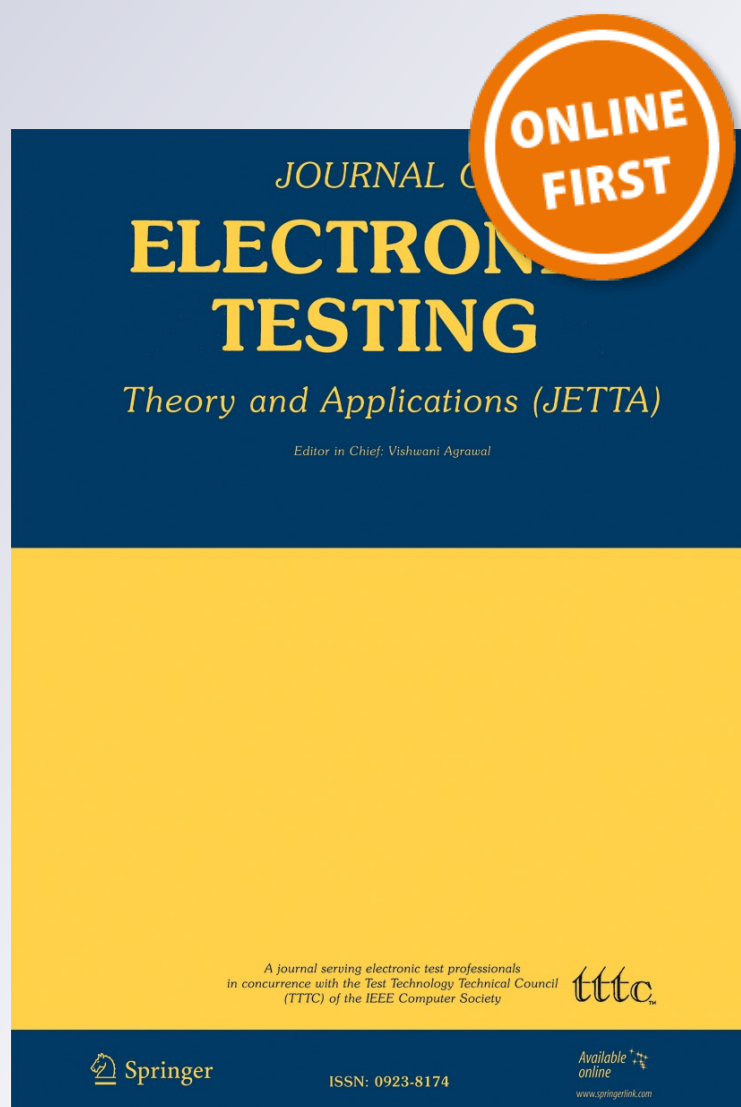
Survey and Evaluation of Automated Model Generation Techniques for High Level Modeling and High Level Fault Modeling

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Survey and Evaluation of Automated Model Generation Techniques for High Level Modeling and High Level Fault Modeling

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Abstract It is known that automated model generation (AMG) techniques for linear systems are sufficiently mature to handle linear systems during high level modeling (HLM). Other AMG techniques have been developed for various levels of nonlinear behavior and to focus on specific issues such as high level fault modeling (HLFM). However, no single nonlinear AMG technique exists which can be confidently adapted for any nonlinear system. In this paper, a survey on AMG techniques over the last two decades is conducted. The techniques are classified into two main areas: system identification (SI) based AMG and model order reduction (MOR) based AMG. Overall, the survey reveals that more advanced research for AMG techniques is required to handle strongly nonlinear systems during HLFM.

Keywords Automated model generation · High level modeling · High level fault modeling · Model order reduction and system identification

1 Introduction

Rapid reduction in the feature size of silicon chips allows designers to encapsulate more complex mixed signal designs into a single chip. Verification and testing prior to fabrication of Integrated Circuits (ICs) becomes more challenging due to their size and complexity. Analogue testing is considered to be time intensive and expensive in the development of analog

and mixed signals (AMS) ICs [1, 3]. For digital circuits mature testing and verification methodologies are already available, but there is still a long way to go for analog or mixed signal circuits and systems. It is necessary to take into account low level details of analog circuits for testing but the generic abstractions available for digital (e.g. stuck-at and delay faults) are not applicable.

An efficient approach for verifying complex circuit is to replace the original with a much simpler ‘model’ that is able to replicate the exact input–output characteristics. The model may be a mathematical description of the original circuit given in the form of Differential Algebraic Equations (DAE) that can be easily translated into Hardware Description Languages (HDL) such as VHDL-AMS or Verilog-AMS, or even in SPICE sub-circuits. Verification with this model-based system can be achieved more rapidly than using the original system.

The model can be obtained either manually or automatically. For the former, extraction of all required parameters and building the model is a tedious job. Therefore, it is desirable to use Automated Model Generation (AMG). These techniques are based on DAEs together with computational methods that automatically generate the equivalent model of the presented circuit.

A fundamental decision in selection of an AMG approach is to choose the right model structure. These model structures are selected according to the type of the system being analyzed. The main system types are [67]: Linear Time Invariant (LTI), Linear Time Varying (LTV), Nonlinear Time Invariant (NLTI) and Nonlinear Time Varying (NLTV). Most electrical circuits fall into two major categories, i.e., LTI and Nonlinear (NL). A class of circuits includes RF mixers, switching capacitors, sampling circuits, etc. fall under the LTV category. Very few techniques are available for direct LTV modeling, however, LTV systems may be converted into nonlinear models and handled using nonlinear techniques [26, 64, 66].

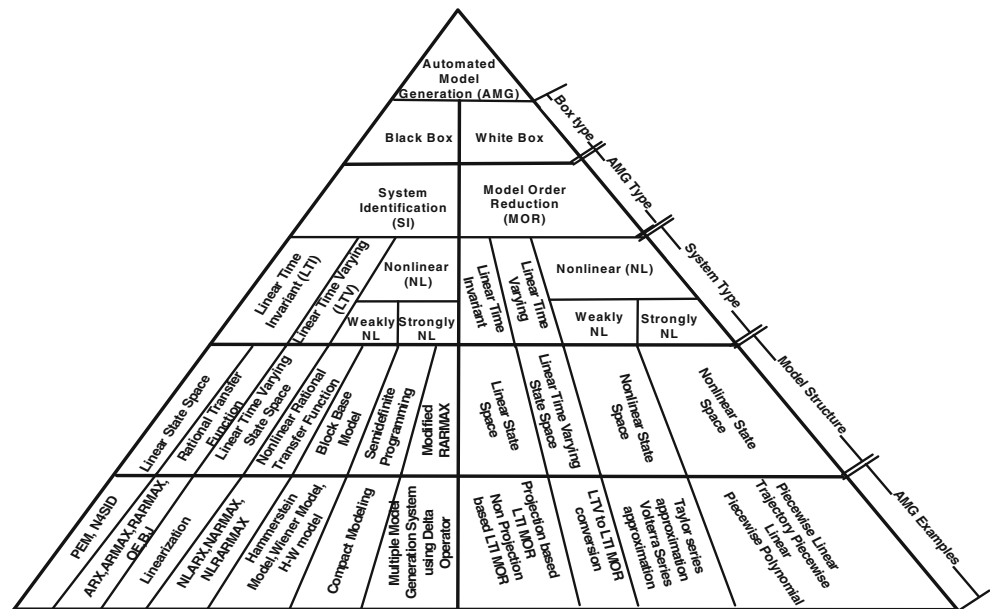
Figure 1 illustrates the taxonomy of various AMG techniques in pyramid form. It is categorized as System

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Fig. 1 Classification of AMG techniques



Identification (SI) and Model Order Reduction (MOR) based AMG techniques.

This paper surveys AMG techniques for High Level Modeling (HLM) and High Level Fault Modeling (HLFM) and the related issue of Fault Propagation (FP). The basic definition of HLM and HLFM used here is: faulty or fault free (FF) models generated by an AMG and implemented using an HDL for system level simulations and which ideally achieve speedup over full transistor level models. FP for analog circuits is defined as: the phenomenon by which a faulty behavior propagates from a faulty block to and through a FF block of circuit. Sometimes FP forces the FF block into highly nonlinear regions of operation which may mean that the FF model is inadequate to propagate faulty behavior. In such circumstances, the FF model may need to be changed so that it can accurately propagate the faulty behavior.

The remainder of this paper is organized as follows: in Section 2 the types of systems and hierarchy of AMG techniques are introduced. SI based AMG techniques are surveyed in Section 3. Section 4 introduces MOR based AMG techniques. Conclusions are provided in Section 5.

2 System Types and Hierarchy of AMG Techniques

2.1 Linear Time Invariant (LTI) System

LTI systems are widely used in electronics design and the AMG techniques developed for them are mature [28, 69]. The basic structure of a LTI block for mixed signal circuits is illustrated in Fig. 2, where $u(t)$ and $y(t)$ represent inputs, and outputs to the system in the time domain, respectively. Correspondingly, $U(s)$ and $Y(s)$ represent $u(t)$ and $y(t)$ in the Laplace

domain. An LTI system can be characterized by the convolution of the input with an impulse response $h(t)$ in the time-domain, i.e., $y(t)=x(t)*h(t)$, which transforms in the frequency/Laplace domain into a multiplication relationship, i.e., $Y(s)=X(s)H(s)$. The input–output relationship can be expressed by partial differential equations (PDEs) or ordinary differential equations (ODEs). These models can be implemented using HDLs. Examples of LTI systems include RLC interconnects circuits generated by parasitic extractors of digital circuits and linear amplifiers, etc.

2.2 Linear Time Varying (LTV) System

LTV systems are used in practice because most real-world systems are time-varying as a result of system parameters changing as a function of time. LTV systems can be described by impulse responses in the time domain or transfer functions in the frequency/Laplace domain. The main difference between LTV and LTI systems is that, for an LTV system it does not necessarily follow that, if there is a time-shift in the input, the same time-shift also occurs in output, as happens in LTI systems. The basic structure of an LTV system is depicted in Fig. 3, where $u(t)$ and $y(t)$ represent the inputs and outputs of the system in the time domain, respectively. $U(s)$ and $Y(s)$ are the corresponding signals in the Laplace domain.

LTV systems are able to efficiently model variations with time using the state-space (ss) form and thus the modeling of

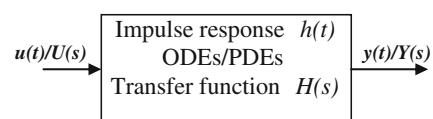


Fig. 2 Linear Time Invariant (LTI) block [67]

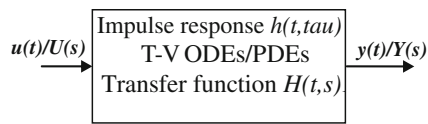


Fig. 3 Linear Time Varying (LTV) block (Nonlinear Systems) [67]

LTV systems can be formulated as problems of nonlinear systems which obey the scalability property, but do not obey the time shift property. More details on nonlinear systems are given in the next subsection. Examples of LTV systems may include RF mixers, switched capacitors, and sampling circuits.

2.3 Nonlinear Systems

Essentially circuits that contain semiconductor devices are nonlinear, most obviously for devices such as diodes and silicon controlled rectifiers where the I-V characteristics change abruptly. Transistors can be modeled as linear devices for small signals, but for large signals they are significantly nonlinear. It follows that active circuits can be designed for close to linear operation within certain limits; however, nonlinearity is specifically required in other situations such as switching functions.

When manually modeling for purely functional circuit simulation, it is possible to take some expected nonlinearities, such as switching, clipping and slewing, into account, creating elements of the model to account for these effects. This approach is established in manual macromodel development, but suffers from the likelihood of missing important nonlinear characteristics which were not predicted by the modeler. AMG has the potential to overcome this issue [67] Significant effort has been put during the last decade for nonlinear AMG, but the majority of these techniques such as [8, 17, 24, 55, 69] are application specific.

In the context of fault modeling, the case for AMG is still stronger because of the modeler is unlikely to have full knowledge of all nonlinearities created by fault conditions and thus faces a near impossible task in hand crafting a model to cover both the nonlinearities in the original circuit and those introduced or modified by faults. However, hand-written behavioral fault models for specific components such as operational amplifiers [41, 63] have been developed. These cover a limited range of faults and operating conditions.

Nonlinear circuits, whether faulty or fault free, can be divided into weakly nonlinear and strongly nonlinear circuits [67]. In fact the idea of dividing nonlinear system models into weakly and strongly nonlinear classes or categories is considered useful in a wide range of disciplines including environmental modeling, fluidics and mechanical engineering. Such classifications tend to be domain and problem specific; there is no single global definition of the boundary between weak and strong nonlinearity. This is directly related to the fact that currently there is not only no comprehensive modeling technique for nonlinear systems in general, but no such singular

approach exists even within the more limited context of electronic circuits. Roychowdhury [67] provides some discussion of this problem with respect to automated macromodel generation.

In the context of circuit modeling we can approach this classification in terms of the expected behavior of circuits; for example amplifiers are expected to be linear, at least until their compression or clipping points, whereas comparators are strongly nonlinear even when considering idealized cases. Further examples of strongly nonlinear circuits include other functions such digital logic gates, switches, analog-to-digital converters (ADC), and digital-to-analog converters (DAC), where rapid switching occurs between two or more states. Complex mixed-signal subsystems such as PLLs also exhibit strong nonlinearities.

Alternatively, the classification can be made in terms of the mathematical techniques required to deliver models of reasonable accuracy. It is generally agreed that if system nonlinear response can be captured through single low order mathematical model, the system can be classified as weakly nonlinear. Example of such methods is linearization techniques or methods model using simple nonlinear approximation methods such as Taylor and Volterra series [67]. On the other hand, strongly nonlinear circuits cannot be modeled based on simple series approaches because the useful information occurs only in low order derivatives, but strong nonlinear effects exist in high order derivatives [69].

3 System Identification Based AMG

SI is the art and science of generating mathematical models from the descriptions of a dynamical system [48]. In general, SI AMG methods generate models following three major steps:

1. Collect data, either from experimentation or simulation of the original system; dividing the data into two parts, one for model generation and the other for model verification.
2. Choose a model set.
3. Pick the 'best' model from the set.

It is quite likely that the first model obtained will not pass model validation tests. In which case the identification process is continued until the model is validated [4, 34, 48]. Successful model generation requires suitable stimulus inputs that can excite all the possible states of the system.

SI AMG approaches are generally categorized as either parametric or non-parametric. The former assumes a model structure *a priori*. This model structure can be an ss model, system difference equation or transfer function model. Non-parametric methods generate impulse or frequency response models directly from the given input/output data. They do not assume a model structure *a priori*. The model structure is

obtained during the identification process. However, “non-parametric” does not indicate that the methods do not involve any parameters. The only difference from parametric methods is that the number of parameters and their characteristics are not known in advance and are adjusted during the identification process. In this paper, only parametric AMG approaches are discussed.

3.1 SI Based AMG Approaches for Linear Systems

A number of successful techniques have been developed to generate models for linear systems in the time or frequency domain, using iterative and non-iterative identification schemes, e.g., [32, 61, 75]. The basis of this success is the ease of exploration of the mathematical structures for linear systems. Generally these models provide enormous insight into a system; however, the assumption that the underlying physical process exhibits qualitatively similar dynamic behavior to the linear model is often only valid close to the operating point at which the model was generated.

A linear system model can be represented in a number of ways, for example, difference equations, ss models or polynomials. Consider a discrete time linear system is defined using the Eq. (1), where, $u(t)$ and $y(t)$ are input and output of the system, respectively, $G(q)$ is the system transfer function obtained by two polynomials $B(q)$ and $F(q)$, $e(t)$ is additive white noise whose dynamics are described by $H(q)$ that is obtained from two polynomials $C(q)$ and $D(q)$ and q is the time-shift operator.

$$y(t) = G(q)u(t) + H(q)e(t) \tag{1}$$

Conventionally discrete time systems are represented using z -transform and delays in difference equations are represented using with z^{-1} operator. However, to keep the notations compact a time-shift q operator is introduced to represent delay or advance in time. A delay, e.g., for input, is represented by $u(t-1)=q^{-1}u(t)$ and is called backward shift operator (q^{-1}). Similarly, an advance in time is represented by $u(t+1)=qu(t)$ and is called forward shift operator (q) [47]. A family of transfer function models can be obtained by varying the polynomial coefficients A, B, C, D and F of Eq. (2) [56].

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \tag{2}$$

These polynomials determine whether the dynamics of the system $G(q)$ and the noise $H(q)$ will have common poles $A(q)$ or separate poles $F(q)$ and $D(q)$, respectively. Examples of model structures from Eq. (2) using different polynomial combinations include ARX, ARMAX, RARMAX, OE (output error) and BJ (Box-Jenkins) models [12, 15, 37, 49]. In

order to estimate the coefficients of these models, different algorithms are used such as the lookup table based approach [89], radial basis functions (RBF) [46], artificial neural networks (ANN) [30] and its derivatives such as fuzzy logic (FL) [77], neuro-fuzzy network (NF) [88], and regression [74]. Regression based approaches can further employ least squares and recursive algorithms such as least square estimator (LSE), weighted least square estimator, recursive least square estimator and recursive maximum likelihood (RML) [47].

These SI based AMG techniques for linear systems are the most mature techniques for obtaining the exact system model for control systems, bio-engineering, system vision and image processing. However, most physical mechanisms of real systems exist in continuous time than in discrete time. This leads to the representation of linear systems in the ss form, in which, relationships between the input, noise, and output are written as a system of first-order differential equations using an auxiliary state vector $x(t)$, depicted in Eq. (3).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t) \\ y(t) &= Cx(t) + v(t) \end{aligned} \tag{3}$$

A and B are $n \times n$ and $n \times m$ matrices of the n -dimensional state and m -dimensional input, C is a $p \times n$ matrix of the p -dimensional output, $w(t)$ and $v(t)$ are assumed to be sequences of independent random variables of zero mean. Matrices A, B and C can be obtained using subspace ss system identification (N4SID) or parametric estimation method (PEM). The modeling is carried out in terms of state variable x that has physical significance (positions, velocities, voltage, currents, etc.), then the measured outputs are the combinations of the states. A major advantage of representing linear systems using Eq. (3) is that physical mechanisms of electrical systems (voltages, currents etc.) can be more easily incorporated than with models based on Eq. (2). Moreover, ss models MIMO systems more efficient than transfer function models [47].

Incorporating more system insight details in an ss model raises the model order, producing computational overhead. It is usually necessary to generate low order ss models for complex larger systems to increase simulation speed. In such situations MOR techniques are employed for linear systems, which are discussed further in Section 4.

Both rational transfer function and ss based SI techniques for AMG are not yet exhaustively tested for electronic circuit blocks. One can generate models using these techniques to increase the simulation speed by performing HLM. A summary of SI based AMG approaches for the linear system is provided in Table 1.

3.2 SI Based AMG Approaches for Nonlinear Systems

These methods are classified into weakly nonlinear systems (WNS) and strongly nonlinear system (SNS) according to the severity of the nonlinear behavior of circuits.

Table 1 SI based AMG approaches for linear systems

Technique	Advantages	Disadvantages
Rational transfer function methods	Mature and stable techniques	Model order increase drastically for larger systems Not available for MIMO systems
ss methods	Efficient for MIMO systems Model provides more insight details of the original system	Computational overhead for larger circuits as order of model (value of internal state variable) increases with circuit size

3.2.1 SI Based AMG for WNS

Transfer function modeling techniques of linear systems can be extended for AMG of WNS. One example is nonlinear ARX (NLARX) [47], whose model structure is shown in Eq. (4).

$$y(t) = F(y(t-1), \dots, y(t-n_a), u(t), u(t-1), \dots, u(t-n_b)) \quad (4)$$

The function F depends on a finite number of previous inputs u and outputs y . n_a, n_b represent the number of delayed output and input values that are used for the prediction of the current output. The model structure is shown in block form in Fig. 4. F is a nonlinear function and the inputs to F are model regressors, composed of delayed input and output values, $u(t), u(t-1), \dots, u(t-n_b), y(t-1), \dots, y(t-n_a)$.

The first block in Fig. 4 is a regressor block that computes the regressor on the basis of the current and past input and output values. The second block is a nonlinearity estimator. It implements two functions: a linear function and a nonlinear function. The outputs from the regressor block are mapped to the model output y using these functions. The combined function $F(x)$ implemented by nonlinearity estimator is shown in Eq. (5).

$$F(x) = L^T(x-r) + d + g(Q(x-r)) \quad (5)$$

where x is a regressor vector, $L^T(x)$ is the output of the linear function block and $g(Q(x-r))$ is the output of nonlinear function block. Examples of nonlinear estimators used in the nonlinear function block include sigmoid networks, tree partitions, wavelet networks and neural networks (NN) [47].

Another approach to decomposing a system into linear and nonlinear functions involves the dynamics of the system being controlled through linear functions, with the nonlinear behavior captured through functions consuming the inputs and outputs of a linear block. The Hammerstein-Wiener ($H-W$) model [35] shown in Fig. 5 is an example of this configuration. Nonlinearity in both input and output is treated in separate blocks with a linear block connecting them.

where

- Block 1 implements nonlinear function (f) on input $u(t)$ and generates output $w(t)$;
- Block 2 implements a linear transfer function between nonlinear input $w(t)$ and linear output $x(t)$;
- Block 3 implements nonlinear function (h) and generates nonlinear output $y(t)$ by consuming inputs $x(t)$ from linear block.

As the block 1 takes care of nonlinearities in input, the function f is called the *input nonlinearity*. Similarly, function h (block 3) is called the *output nonlinearity*. If the model structure contains only block 1 for nonlinearity, i.e., treating nonlinearities only in input, then it is called a *Hammerstein* model. It is a *Wiener* model if the model structure treats nonlinearities only in the output. The combination of both nonlinearities in a single model is called an $H-W$ model. Several algorithms are available for nonlinearity estimators f and h , such as piecewise linear, one layer, sigmoid network, wavelet network, saturation and dead zone [47].

The partitioning of a nonlinear system in to linear and nonlinear blocks in an $H-W$ model generates better output than the NLARX model [11]. Unfortunately, due to lack of feedback,

Fig. 4 NLARX model structure, see Eq. (4) [35]

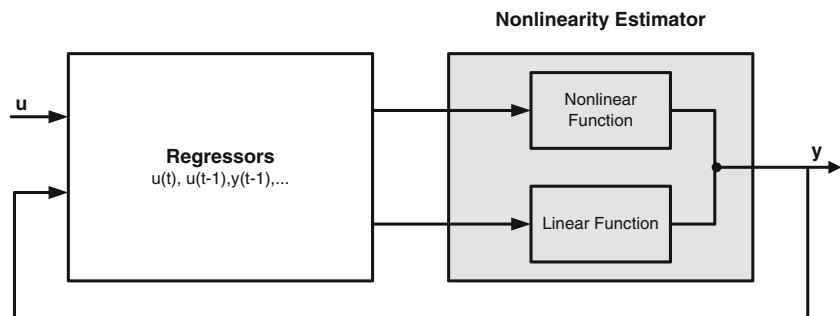


Fig. 5 Hammerstein-Wiener model [35]



the model may become unstable for larger systems. An inherent shortcoming in this model structure is that an *H-W* model considers nonlinearity as a “static” characteristic of a system, which is usually the case with control systems. However, in modern mixed signal electronic circuits, nonlinear behavior is highly dynamic (nonlinearity changes with time), e.g., nonlinear effects generated by cross talk, noise, and intermodulation phenomena. *H-W* model can also be evaluated in the context of HLFM and FP, but similar results to those from NLARX can be expected from the *H-W* model. The *H-W* model is also discrete time and uses the same nonlinear functions, such as sigmoidnet, wavenet etc., for nonlinearity estimation.

To deal with dynamical nonlinear behavior, Simeu et al. [74] propose another approach termed as Situation Dependent ARX (SDARX) model, which extends the idea of the general nonlinear ARX (NARX) presented in Eq. (6), by adding situation dependent coefficients in the regressor term $\gamma(t)$ of the model, shown in Eq. (7), where, $\phi_o(\gamma(t))$, $\phi_i^y(\gamma(t))|_{1 \leq i \leq n_y}$ and $\phi_i^u(\gamma(t))|_{1 \leq i \leq n_u}$ are the data dependant coefficients of the model.

$$\begin{aligned}
 y(t) &= f(y(t-1), \dots, y(t-n), u(t-1), \dots, u(t-n_u)) + v(t) \\
 &= f(\gamma(t)) + v(t)
 \end{aligned}
 \tag{6}$$

A Radial Basis Function (RBF) method is used to estimate these situation dependent coefficients. The main idea of the SDARX approach is to divide the parameter search space into two subspaces: linear weight subspace and nonlinear parameter subspace.

$$\begin{aligned}
 y(t) &= \phi_o(\gamma(t)) + \sum_{i=1}^{n_y} \phi_i^y(\gamma(t))y(t-i) + \sum_{i=1}^{n_u} \phi_i^u(\gamma(t))u(t-i) \\
 &+ \varepsilon(t)
 \end{aligned}
 \tag{7}$$

Then a search is made for the best model from these search spaces, by applying optimization techniques to achieve accurate results. To avoid computational overhead, optimization is applied only to the linear subspace. SDARX models weak dynamical nonlinear behavior with good accuracy. Unfortunately, this approach is computationally complex and hence may become unsuitable for HLFM, as it may not be able to increase HLFM speed. In addition the SDARX model is only applicable to SISO systems.

The techniques for WNS discussed above do not guarantee preservation of stability in the reduced model. Megretski et al. [51] propose a *H-W* feedback model and enforced stability constraints during the optimization process. A non-parametric method is used for nonlinear estimation which is implemented in MATLAB. This AMG approach can become computationally expensive while performing HLFM, as model parameters are not known in advance and need to be refined during the identification process. Performing HLFM based on this approach can severely affect the HLFM speed.

All the nonlinear SI techniques mentioned above are categorized as weakly nonlinear AMGs as they are only able to capture less severely nonlinear behaviours. More powerful identification techniques are required that can capture strong nonlinear effects. SI based AMG approaches for WNS discussed in this section are summarized in Table 2.

3.2.2 SI Based AMG for SNS

There are several techniques available for SI based AMG for SNS. Two recently proposed techniques are discussed: Compact Modeling (CM) [11] and Multiple Model Generation System using Delta operator (MMGSD) [87].

CM modeling technique is primarily proposed to overcome the system instability problems faced in MOR based AMG techniques for SNS. In CM, the model identification procedure is based on minimizing the model error over a given training data set subject to an incremental stability constraint, which is formulated as a semidefinite optimization problem [11]. Initially the system of implicit form seen in Eq. (8) is linearized to get a linearized output error upper bound r over the training data set X ,

$$F(v[t], \dots, v[t-m], u[t], \dots, u[t-k]) = 0, G(y[t], v[t]) = 0 \tag{8}$$

where $v[t] \in R^N$ is a vector of internal variables, $y[t] \in R^{N_y}$ is the output, $u[t] \in R^{N_u}$ is the input, $F \in R^N$ is a dynamical relationship between the internal variables and the input, and $G \in R^{N_y}$ is the static relationship between internal variables and output. Then a stability constraint is enforced on the system through the use of a storage function H .

The formulations that enforce both stability and accuracy on the linearized output error upper bound r at the same time are shown in Eq. (9).

Table 2 SI based AMG for WNS

Technique	Advantages	Disadvantages
Nonlinear ARX (NLARX) [47]	Feedback model structure	System may become unstable and generate unbounded output for bounded input. Bad estimator for strong nonlinear systems Unable to perform HLFM and FP accurately
Hammerstein and Wiener (H-W) [47]	Parallel combination of linear and non-linear blocks	Consider nonlinearity as 'static' characteristic of system. Poor estimator of strongly nonlinear systems Discrete time model structure not suitable for HLFM and FP
H-W model with feedback [51]	Very stable W-H model as compared to previous two approaches	Model structure not suitable for implementation at HLM due to nonparametric nonlinear approach
SDARX [74]	Capture weak nonlinear dynamical as well as static nonlinearity	Computationally expensive Poor estimator for strongly nonlinear systems Available only for SISO systems

$$\min_{r, F, G, H} \sum r_t \text{ subject to } r_t + 2\delta_o^T \bar{F}_t(\Delta) + 2\xi^T \bar{G}_t(\delta_o, \xi) - |\xi|^2 + h_{t-1}(\Delta-) - h_t(\Delta+) \geq 0 \quad \forall t, \Delta, \xi \tag{9}$$

$$\begin{aligned} & \text{Minimize} && C^T x \\ & \text{Subject to} && F(x) \geq 0 \end{aligned} \tag{11}$$

where r_t output error upper bound is a function of training data set $\tilde{X}[t] = (\tilde{y}[t], \tilde{V}[t], \tilde{U}[t])$ given by $r_t = r(\tilde{y}[t], \tilde{V}[t], \tilde{U}[t])$, F and G are dynamical and static relationships of input and outputs represented respectively by $\bar{F}_t(\Delta) = \bar{F}(\tilde{V}[t], \tilde{U}[t], \Delta)$, $\bar{G}_t = \bar{G}(\tilde{y}[t], \tilde{v}[t], \delta_o, \xi)$, Δ and ξ represents the incremental variables used for incremental stability. h_t is the storage function that depends on internal state \tilde{V} and incremental variable Δ , given by $h_{t-1}(\Delta-) = h(\tilde{V}^-, \Delta-)$, and $h_t(\Delta+) = h(\tilde{V}^+, \Delta+)$ [11].

All functions F , G , H and r in Eq. (9) are unknowns. Estimation of these unknowns can be treated as an optimization problem and can be converted into semidefinite program (SDP) by allowing the unknown functions F , G , H and r to be chosen as linear combinations of a finite set of basis function \mathcal{O} , given in Eq. (10), where $\mathcal{O}^F, \mathcal{O}^G, \mathcal{O}^H, \mathcal{O}^r \in \mathcal{O}$, and $\alpha^F, \alpha^G, \alpha^H, \alpha^r$ are the free variables solved through SDP.

$$\begin{aligned} F &= \sum_{j \in N_f} \alpha_j^F \phi_j^F(V, U), G = \sum_{j \in N_g} \alpha_j^G \phi_j^G(y, v_o) \\ H &= \sum_{j \in N_h} \alpha_j^H \phi_j^H(V), r = \sum_{j \in N_r} \alpha_j^r \phi_j^r(y, V, U) \end{aligned} \tag{10}$$

A SDP is a problem whose objective function is linear and whose constraints require matrices which are positive semidefinite (PSD) [11]. SDP is a special case of convex optimization concerned with the optimization of a linear objective function subject to the constraint that the affine combination of symmetric matrices is positive semidefinite. Such a constraint is nonlinear, or not smooth, but convex in nature. Hence, SDP is considered to be a special case of convex optimization. A general form of SDP that minimizes a linear function of a variable $x \in R^m$ subject to a matrix inequality is shown in Eq. (11), where $F(x) \cong F_o + \sum_{i=1}^m x_i F_i$

The SDP problem data in Eq. (11) are the vector $C \in R^m$ and $m+1$ symmetric matrices $F_0, \dots, F_m \in R^{n \times n}$. $F(x) \geq 0$ means that $F(x)$ is positive semidefinite, i.e., $z^T F(x) z \geq 0$ for all $z \in R^n$.

The benefit of formulating the optimization problem Eq. (9) as an SDP in Eq. (11) is that it can be solved efficiently using readily available software routines [78, 80, 81]. However, the complexity of the optimization problem in Eq. (9) depends heavily on the choice of the basis function \mathcal{O} for the unknowns F , G , H and r . Therefore, the selection of the basis is critical to obtain a feasible solution [11]. CM utilizes a polynomial basis to efficiently solve the optimization problem. This allows CM to identify a system as a rational model structure. However, only models that are linear in unknown state variables are considered. To achieve compactness in the identified rational model, reduction of states for larger systems are attained through conventional projection methods and further reduction of polynomial basis is obtained through a fitting procedure.

The model generated by CM very closely resembles the original system [10] and guarantees stability in the generated compact model. For example, in [11] a compact model of a Micro Electro-Mechanical Systems (MEMS) device of order 400 is generated. Simulation results for its nonlinear behavior show good accuracy indicating that CM is able to generate stable reduced models for originally high order systems.

An AMG for SNS targeted at HLFM and HLM has been proposed by Xia et al. [87]. The algorithm, termed multiple model generation system (MMGS) consists of two parts: the Automated Model Estimator (AME) and the Automated Model Predictor (AMP). The AME implements the model generation process using Recursive Maximum Likelihood (RML) [47] method and AMP uses these models to predict signals in the simulation [86]. The AME comprises three stages: pre-analysis, estimator and post-analysis. In pre-analysis stage

initial conditions of MMGS are setup e.g., input range and the number of submodels. This step is only performed once in MMGS. The estimator implements RML and provides output responses and error measures. The post-analysis step implements the model generation process.

MMGS produces a new model based on error in different nonlinear regions against input voltage. The model structure used by MMGS for these regions is shown in Eq. (12) [86].

$$y(t) = -a_1y(t-1) - a_2y(t-2) - \dots - a_nay(t-na) + b_1u(t-1) + b_2u(t-2) + \dots + b_nbu(t-nb) \tag{12}$$

The model is calculated using the linear regression in Eq. (13), where θ is the parameter vector shown in Eq. (14), $\varphi(t)$ is the regression vector displayed in Eq. (15).

$$y(t) = \varphi^T(t)\theta \tag{13}$$

$$\theta = [a_1a_2\dots a_nab_1b_2\dots b_nb]^T \tag{14}$$

$$\varphi^T(t) = [-y(t-1)\dots -y(t-na) \quad u(t-1)\dots u(t-nb)] \tag{15}$$

MMGS generates discrete time models that are able to accurately model faults for low frequency circuits; however, these models get contaminated with the effects of aliasing and severe phase shift for high frequency circuits [85].

The shortcomings in MMGS are improved by introducing a *delta operator* (δ) in MMGS model structure that leads to the development of multiple model generation system using delta operator (MMGSD) which handles nonlinearity over time [87]. An attractive property of delta operator is that it produces model coefficients that approximate the discrete time models as continuous time models.

To present the model structure in continuous time, the inverse Laplace transform of the transfer function is shown in Eq. (16) can be used if the sampling interval is sufficiently short [52]. The resulting equation in time domain with delta operator can be presented as Eq. (17) [86].

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0s^n + b_1s^{n-1} + \dots + b_ns^0}{s^m + a_1s^{m-1} + \dots + a_ms^0} \tag{16}$$

$$G(\delta) = \frac{y(t)}{u(t)} = \frac{b_0\delta^n + b_1\delta^{n-1} + \dots + b_n\delta^0}{\delta^m + a_1\delta^{m-1} + \dots + a_m\delta^0} \tag{17}$$

After arranging terms in Eq. (17), the system in Eq. (18) is obtained.

$$y(t)\delta^m = -(a_1\delta^{m-1} + \dots + a_m)y(t) + (b_0\delta^n + \dots + b_n)u(t) \tag{18}$$

It is solved using appropriately modified RML using the delta operator [86]. Unlike in [11, 19, 62], Xia et al. employ single training data to excite all possible states, by superimposing pseudorandom binary sequence (PRBS) signal on a linear waveform.

In addition, HLM and HLFM are performed by converting the MMGSD model into a VHDL-AMS model through a process called multiple model conversion system using a delta operator (MMCSO). MMCSO loads MMGSD model parameters and generates HLM and HLFM implemented in VHDL-AMS. Accurate simulation results are achieved. Unfortunately, simulation speed up in HLFM is not achieved compared with TLFS because of computational overhead in the HDL simulation. This overhead is mainly due to the fact that the MMGSD has to switch between multiple models for a single fault during simulation. Speed can be improved by implementation of fault collapsing, to prevent repetition of effort, and by optimising the number of models generated to prevent unnecessary model switching from consuming computational effort. Intelligent setting of thresholds for model switching during MMGSD may achieve this [21]. Table 3 summarizes AMG techniques for SNS discussed in this subsection.

Table 3 SI based AMG techniques for SNS systems

Technique	Advantages	Disadvantages
Compact Modeling [11]	Preserve stability for high order systems. Faster model generation. Accuracy improved	The projection based MOR has to be used if higher order (e.g., 500) model is used, which will result in instability. Electronic models generated are not realized at HLM. Fault modeling is not implemented
MMGSD [87]	Single training data for model generation Implemented HLFM and HLFS	Simulation speed of HLFM slower than TLFS. Fault collapsing not implemented

4 Model Order Reduction Based AMG

4.1 MOR Based AMG Approaches for Linear Systems

As mentioned earlier in Section 3.1 that the ss form for linear systems, shown again in Eq. (19), suffers primarily from the problem of increase in model order for larger systems.

$$\begin{aligned} E\dot{x} &= Ax(t) + Bu(t) \\ y(t) &= C^T x(t) + Du(t) \end{aligned} \quad (19)$$

where $x(t) \in R^n$ is the internal state, $u(t) \in R^m$ and $y(t) \in R^p$ are the m -input and p -output waveforms. Matrices $A \in R^{n \times n}$, $E \in R^{n \times n}$, $B \in R^{n \times m}$, and $C \in R^{p \times n}$ are constants.

To practically simulate large size models without computational overhead, MOR techniques are employed for the linear systems. However, MOR techniques must produce a model of lower order whose response matches with original system response with good accuracy. Also, the reduced model should maintain the desired properties of the original system with minimum computational overhead, less memory requirements and shortest evaluation time.

There are two types of MOR techniques for linear systems: projection based and non-projection based techniques. The latter comprise methods such as Hankel optimal model reduction [25], singular perturbation method [49], and various optimization-based methods. Whereas, for the former, the most widely used general approaches are Proper Orthogonal Decomposition (POD) methods [7, 42, 43, 84], Krylov-subspace and shifted Krylov-subspace methods [5, 23, 44, 71], and Balancing-based methods [2, 20, 36].

Model reduction using projection methods is implemented by projecting the linear equations into a subspace of a lower dimension. The selection of subspace is based on the approximation of desired properties of original system in the reduced model. Therefore, a major issue in linear reduced models is deciding which properties of the original system should be maintained in reduced model. Several techniques have been developed for MOR such as moment matching and Asymptotic Waveform Evaluation (AWE) [13, 14, 27, 44, 60, 73]. More details on these techniques can be found in [67].

Krylov subspace based model reduction techniques are considered to be the major milestone in the development of MOR for linear systems. A Krylov subspace $K_m(A, p)$ generated by a matrix A and vector p , of order m , is the space spanned by the set of vectors $\{p, Ap, A^2p, \dots, A^{m-1}p\}$. The basic procedure of projection based reduction using Krylov subspace is as follows.

Select a matrix V whose columns span a 'useful' subspace, and draw an approximation $\hat{x} = Vz$. To obtain the reduced model equations and a residual $r \equiv A\hat{x} + Bu - E d\hat{x}/dt$ is calculated such that r is orthogonal to another space W , i.e. $W^T r = 0$. The ss equations for the reduced model come out to be of the form shown in Eq. (23).

$$\frac{dz}{dt} = \hat{A}z + \hat{B}u, \hat{y} = \hat{C}z + \hat{D}u \quad (20)$$

where

$$\hat{A} = W^T A V, \hat{B} = W^T B, \hat{E} = W^T E V, \hat{C} = C V \quad (21)$$

Nevertheless, the problem in both AWE and Krylov methods is that these methods do not guarantee the preservation of passivity and stability in a reduced model [67]. A system is passive if it cannot generate energy under any conditions and stable if its output remains bounded for bounded input [69].

Passivity is mainly the concern of interconnect networks where multiple nodes are connected to a single node, and stability is an important property that every reduced model needs to guarantee. The techniques discussed next mainly focus on these two goals (and added goals of computational cost and accuracy).

Several authors have developed different model reduction techniques on basis of Krylov methods [10, 40, 73]. However, these techniques either target stability or passivity, but not both at the same time. Similarly, the authors in [55] use Arnoldi-based reduction method with the passivity-retaining properties of the congruence transformation for RLC networks. They propose an algorithm dubbed PRIMA (Passive Reduced Order Interconnect Macromodeling Algorithm). It generates provably passive reduced-order N-port models for RLC interconnect circuits. The modified nodal analysis (MNA) equation is formed using these ports along with sources in time domain as seen in Eq. (22).

$$C\dot{x}_n = -Gx_n + Bu_N, i_N = L^T x_n \quad (22)$$

where vectors i_N and u_N indicate the port currents and voltages respectively; C, G are matrices representing the conductance and susceptance. PRIMA utilizes the Arnoldi algorithm [73] to produce the vectors required for applying congruence transformations to the MNA matrices, i.e., $V=W$. These transformations are used to reduce the order of circuits [40].

The moment-matching properties of Krylov-subspaces enable the reduced model to match original model up to the first q derivatives, where q is the order of the reduced model. Models from PRIMA are able to improve accuracy compared with Arnoldi. The model size is grows with the number of moments (moment is matched by multiplying with the number of ports) and for large number of ports the algorithm leads to impractically large models.

The authors of [16, 76] describe techniques which generate reduced passive models from transfer functions. However, these approaches are only available for single port circuits.

The authors of [29, 31] generate passive models for multiport circuits for stable but non-passive multiport systems. They use a perturbation technique to make the model passive. Unfortunately, these approaches may perform poorly if the original system has significant passivity violations. Also, these perturbations techniques can severely affect the accuracy of model. To overcome this problem Zohaib et al. [50] impose the passivity constraint during the model generation process, unlike conventional perturbation approaches where the models are first generated and then perturbed to make them passive. However, Zohaib et al. only guarantee passivity, not stability.

Another important characteristic of RLC interconnect networks is reciprocity which enables the preservation of block structure of circuit matrices in reduced models. While reciprocity is related to passivity, some of the passivity preserving techniques discussed above are unable to preserve reciprocity in the reduced models. Freund et al. [24] propose a new algorithm termed structure preserving reduced order interconnect macromodeling (SPRIM), which generates provably passive and reciprocal models of *multiport* RLC circuits that match twice as many moments compared to the corresponding model obtained with passivity preserving methods, e.g., PRIMA, with identical computational cost.

Krylov-subspace based model reduction techniques are efficient but they are not optimal in minimizing errors in the reduced models [59]. This can be achieved by using the theory of truncated balanced realization, presented for the first time in [38]. TBR based techniques can be classed as positive-real TBR (PR-TBR), hybrid TBR and bounded-real TBR (BR-TBR) [9]. Phillips et al. [9] present an algorithm based on the input-correlated TBR for parasitic models, which offers advantages like quantifiable error bounds. They claim that the size of the parasitic models from projection-like procedures

can be reduced by exploiting input information such as nominal circuit function. This algorithm can generate guaranteed passive reduced-order models of controllable accuracy for ss systems with an arbitrary internal structure. Kamon et al. [22] combines Krylov subspace techniques with TBR methods so that the size of the TBR is reduced and potentially the computational cost can also be reduced.

All the techniques mentioned above target passivity with less focus on stability. As already mentioned, projection based reduction techniques are not optimal in the minimization of error in reduced models. This error arises due to discretization, and ignoring high-order physical effects may render the system unstable. Thus no projection method is able to reliably generate accurate stable reduced models from originally unstable systems. Bond et al. [65] provides stability-preserving projection framework for efficient reduction of indefinite and mildly unstable systems. They generate guaranteed stable reduced models by formulating a given indefinite and asymmetric system as a semidefinite optimization problem. Unlike the conventional method of first applying a projection method to generate a reduced model and then perturbing the generated model to enforce stability and passivity, they perturb one of the projection matrices (U or V) and search for small ΔU such that the system defined by $\widehat{E} = (U + \Delta U)^T E V$, $\widehat{A} = (U + \Delta U)^T A V$, $\widehat{B} = (U + \Delta U)^T B V$ and $\widehat{C} = V^T C$ is passive. The solution can be found using Eq. (23) [65].

$$\min_{\Delta U} \|\Delta U\| \min^{\Delta U} \text{ Subject to } \widehat{E} \geq 0, \widehat{A} + \widehat{A}^T \leq 0, \widehat{B} = \widehat{C} \quad (23)$$

For example, through the reduction of a MEMS model with an original order $N=1680$ to an order $q=12$, they obtain a

Table 4 MOR base AMG for linear systems

Technique	Advantages	Disadvantages
AWE [14]	Match lower order moments of original with reduced model	Reduced model numerically inaccurate for $N \geq 10$
Moment matching through Krylov subspace method [13, 39]	Numerically accurate reduced model then AWE, able to capture all poles and residues of system	Does not preserve stability and passivity in reduced model
Co-ordinate transformed Arnoldi method [73]	Guarantees stability	Does not guarantee passivity
PRIMA [55]	Preserves passivity	Computationally expensive due to large model size
Rational function fitting approach for passive model generation [29, 31]	Preserves passivity	Available only for single port circuits
Perturbation approach for passive model generation [53, 70]	Preserves passivity for multi-port systems	Does not guarantee accuracy
Semidefinite programming approach [8]	Preserves passivity for multi-port systems and guarantees accuracy	Does not guarantee stability
SPRIM [24]	Passive and reciprocal models of multiport RLC circuits	Does not address stability issue
Truncated Balanced Realization (TBR) approach [9]	Minimizes error in reduced model compared to Krylov subspace based methods and preserves passivity	Computational cost grows cubically with original system's size
Stability preserving projection framework using SDP [65]	Guaranteed passivity and stability preservation Simulation speed up achieved	Model not realised at higher level

stable reduced system with good accuracy. They also show that their technique is up to 15 times faster than conventional projection based methods. Unfortunately, MATLAB models generated by this AMG are not converted into high level descriptions to show speed up for system level simulations. Table 4 provides the summary of MOR based AMG for linear systems in this subsection.

4.2 MOR Based AMG Approaches for LTV Systems

LTI MOR may not be applicable for mixed signal systems whose behaviour varies with time. Moreover, it is unable to model behaviours such as distortion and clipping in amplifiers. Therefore, LTV MOR is required. The detailed behaviour of the system is described in Eq. (24).

$$\begin{aligned} E(t)\dot{x} &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)^T x(t) + D(t)u(t) \end{aligned} \quad (24)$$

Here A, B, C, D and E are time dependant which enables this model structure to capture variation with time in the system. It is known that LTI MOR methods cannot be directly applied to LTV systems due to the variations of system transfer function with time. However, the authors in [82] demonstrates that the LTV systems can take advantages of LTI techniques if Eq. (24) can be reformulated into the linear model structure shown in Eq. (19). They use extra artificial inputs to capture the variations with time. Then they separate the input and system time variations explicitly using multiple time scales [66] in order to obtain an operator expression for the transfer function $H(t,s)$ shown in Fig. 3 (Section 2). Finally, they apply periodic steady-state methods [40, 73] on the operator expression to obtain a linear system of form Eq. (12). Now this linear system can be reduced using LTI MOR techniques. After the reduction, the linear system is formulated back into a LTV system of form Eq. (27).

4.3 MOR Based AMG Approaches for Nonlinear Systems

4.3.1 MOR Techniques for WNS

Weakly nonlinear techniques are extensions of linear MOR techniques [54, 72]. A standard nonlinear system formation is based on a set of nonlinear differential-algebraic equations (DAEs) shown in Eq. (25), where $x \in R^n$, n is the order of matrices, $x(t)$ represents the state vector, $y(t)$ is the vectors of outputs, $u(t)$ is the input, $q(\cdot)$ and $f(\cdot)$ are nonlinear vector functions, and b and c are the input and output matrices, respectively.

$$\begin{aligned} \dot{q}(x(t)) &= f(x(t)) + bu(t) \\ y(t) &= c^T x(t) \end{aligned} \quad (25)$$

$f(x)$ and $q(x)$ in Eq. (25) can be approximated using Taylor series expansion at the bias point x_0 as shown in Eq. (26),

where $q(x)=x$ (assumed to make notations simpler), \otimes is the Kronecker tensor products operator, and A_i is given as:

$$\begin{aligned} A_i &= \frac{1}{i!} \frac{\partial^i f}{\partial x^i} \Big|_{x=x_0} \in R^{n \times n^i} \\ \frac{d}{dt}(x(t)) &= f(x_0) + A_1(x-x_0) + A_2(x-x_0) \otimes (x-x_0) \\ &\quad + \dots + A_i(x-x_0)^{(i)} + bu(t) \\ y(t) &= c^T x(t) \end{aligned} \quad (26)$$

Volterra series theory [45] and weakly nonlinear perturbation techniques [57] can then be used to justify a relaxation-like approach for this kind of systems. The Volterra series approach is effective for describing nonlinear transfer functions of weakly nonlinear systems. By employing Volterra series, response $x(t)$ in Eq. (26) can be approximated by adding responses at different orders, i.e., $x(t) = \sum_{n=1}^{\infty} x_n(t)$, where x_n is the n th-order response. The linearized first through third order nonlinear responses in Eq. (26) need to be solved recursively using Volterra series as shown in Eqs. (27–29).

$$\frac{d}{dt}(x_1(t)) = A_1 x_1 + bu \quad (27)$$

$$\frac{d}{dt}(x_2(t)) = A_1 x_2 + A_2(x_1 \otimes x_1) - \frac{d}{dt}(x_1 \otimes x_1) \quad (28)$$

$$\begin{aligned} \frac{d}{dt}(x_3(t)) &= A_1 x_3 + 2A_2 \overline{(x_1 \otimes x_2)} + A_3(x_1 \otimes x_1 \otimes x_1) \\ &\quad + \frac{d}{dt}(x_1 \otimes x_1 \otimes x_1) - 2\overline{(x_1 \otimes x_2)} \end{aligned} \quad (29)$$

where $\overline{(x_1 \otimes x_2)} = \frac{1}{2}((x_1 \otimes x_2) + (x_2 \otimes x_1))$

The n^{th} -order response can be calculated using a Volterra kernel of order n , $h_n(\tau_1, \dots, \tau_n)$, as shown in Eq. (30).

$$x_n(t) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_n(\tau_1, \dots, \tau_n) u(t-\tau_1) \dots u(t-\tau_n) d\tau_1 \dots d\tau_n \quad (30)$$

A Volterra kernel can describe efficiently both the nonlinear behaviour and dynamics of system through use of convolution. Volterra kernels are the backbone of any Volterra series. They contain the knowledge of a system's behaviour, and predict the response of the system. Alternatively, moment matching can be done at multiple frequency points using Eq. (31), where $h_n(\tau_1, \dots, \tau_n)$ is transformed into the frequency domain via Laplace transform.

$$\begin{aligned}
 &H_n(s_1, \dots, s_n) \\
 &= \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_n(\tau_1, \dots, \tau_n) e^{-(s_1\tau_1 + \dots + s_n\tau_n)} d\tau_1 \dots d\tau_n
 \end{aligned}
 \tag{31}$$

$H_n(s_1, \dots, s_n)$ is the nonlinear transfer function of order n . The n th-order response, x_n , can also be related to the input using $H_n(s_1, \dots, s_n)$.

Roychowdhury et al. [82] improve the relaxation approach by appropriately modifying each stage of the relaxation process to account for distortion inputs. They apply a separate projection basis at each stage to obtain a reduced model. This achieves accuracy at the cost of increased model size (as the final model is the sum of each stage, a relaxed q order model).

Li et al. [58] combines and extends Volterra and projection approaches using a method termed NORM (Nonlinear Model Order Reduction Method) to achieve reduced model size. NORM calculates the nonlinear transfer function by explicitly performing the moment matching through the use of projection matrices. The first-order transfer function of the linearized system is obtained in Eq. (32).

$$(s-A_1)H_1(s) = b \quad \text{or} \quad H_1(s) = (s-A_1)^{-1}b \tag{32}$$

Without loss of generality, Eq. (32) is expanded at the origin $(0, 0)$ as shown in Eq. (33).

$$H_1(s) = \sum_{k=0}^{\infty} s^k A^k r_1 = \sum_{k=0}^{\infty} s^k M_{1,k} \tag{33}$$

This approach can also be useful to obtain the moments of the second-order or third-order transfer functions. Compared with existing projection based reduction models, such as [6, 79], it provides a significant reduction in model size.

Batra et al. [33] employ NORM to generate reduced-order models of circuits from transistor level netlists. The difference from [58] is that Batra et al. exploit least-mean-square error (LMSE) fitting techniques to find the 3rd order model coefficients instead of using the model equations. Results show that

the models generated achieve considerable reduction in the model size with good accuracy. Unfortunately, speed slow compared with TLS. In addition the models are not converted into HDL to show speedup at system level simulations.

The polynomial based techniques mentioned above attempt to linearize the nonlinear part of a system and then apply model reduction [33, 58, 82]. Other techniques perform the nonlinear model reduction process using different approaches by splitting a system into linear and nonlinear parts, then a reduction technique is applied only to the linear part; after that the linear part is stitched back with the untreated nonlinear part to get the overall reduced nonlinear model [68, 83].

Steinbrecher in [68] decouples a circuit into linear and nonlinear sub-circuits, and then applies passivity-preserving balanced truncation followed by an adequate re-coupling of the unchanged nonlinear sub-circuit and the reduced linear sub-circuit to obtain a nonlinear reduced-order model. He shows the efficiency of his method numerically with initial assumption of small number of nonlinear elements. It indicates that this methodology may not be optimised to circuits containing large number of nonlinear elements. Heinkenschloss et al. [83] follow the same methodology of separating the linear and nonlinear parts of circuits. Again the assumption is that there is small number of nonlinear resistances in the circuit for the overall method to be effective.

The MOR based AMG techniques for WNS discussed in this subsection (summarized in Table 5) are only able to capture nonlinear effects that lie within low order derivatives of a system transfer function. However, strong nonlinear effects often appear in higher order derivatives. Unfortunately MOR based techniques for WNS are extremely poor estimators for capturing strong nonlinear effects that lie in the high order derivatives [18].

4.3.2 MOR Techniques for SNS

To overcome the issue above [70], other methods such as piecewise approximation can be used to achieve better solutions. The simplest approach is to represent a nonlinear system

Table 5 MOR based AMG for WNS

Technique	Advantages	Disadvantages
Projection base relaxation approach [82]	Efficiently handles distortion effects	Larger model size
NORM [58]	Reduced model size compared with previous approach	Good for small range of validity, bad global estimator
LMSE with NORM [33]	Significantly reduced model for transistor level circuits	Model simulation speed not compared with TLS Model not implemented using HDL for HLM
Separate linear, non-linear treatment [68, 83]	Preserves passivity with good accuracy	Reduced linear part, non-linear part is untreated Applicable only with few nonlinear elements in circuit

using piecewise linear (PWL) approach. Each small region in the nonlinear response is readily linearized, and a combination of these linear pieces can approximate an overall nonlinear system. However, a major shortcoming is that the number of pieces (regions) increases drastically for strong nonlinear systems in order to achieve sufficient accuracy.

To overcome this problem of explosion of regions, Rewiński et al. [12] developed an approach termed trajectory piecewise-linear (TPWL). Initially, they select multiple points along a trajectory in the ss of a nonlinear system, to generate an equal number of linear approximations. These points are called “centre points” and are generated using different training inputs. A model is generated if the current state point x is ‘close enough’ to the last linearized point x_i , i.e., $\|x-x_i\| < \varepsilon$, which means that x lies within a circle of radius of ε and centred at x_i . Each of the linearized models takes the form shown in Eq. (34), with expansions around states x_0, \dots, x_{s-1} , where $f(\cdot)$ evaluated at states x_i . x_0 is the initial state of the system and A_i are the Jacobians evaluated at x_i .

$$\frac{dx}{dt} = f(x_i) + A_i(x-x_i) + Bu \tag{34}$$

A Krylov subspace projection method is then used to reduce the complexity of the linear model within each piecewise region. Rewiński et al. then combine all s linear models according to a weighting equation in Eq. (35), where $\tilde{w}_i(x)$ are the weights depending on state x .

$$\frac{dx}{dt} = \sum_{i=0}^{s-1} \tilde{w}_i(x) f(x_i) + \sum_{i=0}^{s-1} \tilde{w}_i(x) A_i(x-x_i) + Bu \tag{35}$$

Rewiński et al. state that TPWL is more suitable for circuits that exhibit strong nonlinear effects such as comparators. They also prove that TPWL is more efficient than PWL with respect to the regions explosion problem. Rewiński et al. used specific training inputs for macromodel generation, which implies that macromodel covers only those ss trajectories that are generated through specific inputs. Such macromodels may be unable to generate correct responses for inputs not covered by training stimuli. TPWL macromodels

are able to model strongly nonlinear systems, but their capability to model weak nonlinear systems is limited due to poor approximation properties of linearized models for higher order derivatives. Further, the computational cost involved in generating reduced macromodels for SNS is high. To improve computational cost, Vasilev et al. [83] introduce a two-step hybrid reduction method called TBR-TPWL model reduction. Initially they reduce the ss matrices using a conventional Krylov subspace method and further reduction is achieved through use of TBR projection. Simulation of macromodels generated using TPWL-TBR shows better accuracy than models generated using only the Krylov subspace method. Computational cost is significantly reduced, but unfortunately it does not guarantee stability.

Bond et al. [24] improve the local approximation properties of TPWL by introducing extra parameters in the model structure. In TPWL, a nonlinear system is approximated by the model shown in Eq. (36). A parameter space $\{P_j\}$ is added to the state trajectory space, shown in Eq. (37).

$$\frac{d\hat{x}}{dt} = \sum_{i=0}^{k-1} w_i(\hat{x}, \hat{X}) \left[\hat{A}_i \hat{x}(t) + \hat{k}_i \right] + \hat{B}u(t) \tag{36}$$

$$\frac{d\hat{x}}{dt} = \sum_{i=0}^{k-1} \sum_{j=0}^{P-1} w_i(\hat{x}, \hat{X}) \tilde{P}_j \left[\hat{A}_{ij} \hat{x}(t) + \hat{k}_{ij} + \tilde{B}_j u(t) \right] \tag{37}$$

where $\{P_j\}$ is the additional parameter space in the original ss, j represents the number of linearization points in parameter space and k represents the number of linearization points in full ss.

In TPWL the training inputs generate centre points for linearization along a ss trajectory; here additional trajectories are generated by training the system Eq. (38) at several points in the parameter space $\{\tilde{P}_j\}$.

$$\frac{dx}{dt} = \sum_{j=0}^{P-1} \tilde{P}_j [f_j(x) + B_j u(t)] \tag{38}$$

The additional training produces a linear model in new ss, so that each state is now driven by these parameter variations. However, the model generation process is computationally

Table 6 MOR based AMG for SNS

Technique	Advantages	Disadvantages
PWL [70]	Handles strongly nonlinear systems.	Huge number of linear pieces (regions)
TPWL [12]	Reduced number of regions. Good global estimator	Unable to capture higher-order derivative information due to PWL nature
NLPMOR [24]	Improved accuracy then TPWL for specific cases	Computationally expensive
PWP [18, 19]	Able to capture weakly as well as strongly nonlinear effects by combining polynomial approach with TPWL	Used multiple training data.

expensive. In TPWL, m Krylov vectors are generated from k linear models to construct a projection matrix. Thus the cost of generating a projection matrix is $O(km)$. For a system having P additional parameters, the cost of generating a projection matrix becomes $O(kP^m)$, which is a significant increase.

TPWL and all its variants discussed above can model SNS with good accuracy, but they have poor approximation properties for WNS. Therefore Dong et al. [18, 19] proposed a piecewise polynomial (PWP) extension of TPWL. This is a combination of polynomial model reduction with the trajectory piecewise linear method. It is able to improve TPWL by dividing the nonlinear ss into several regions that are approximated with a polynomial model around the centre expansion point. A training simulation employing DC sweeps can be used to generate these expansion points. However, a combination of several training data is used to excite different operating regions. The resulting model is gradually developed by introducing new regions until the desired accuracy is achieved. Firstly a polynomial function is expanded into many points, and then simplified through approximation of the nonlinear functions in each region to obtain much smaller size models. The resulting models are then combined as a single model. A scalar weight function is used to ensure fast and flat switching from one region to another.

One of the major advantages of PWP is that it can model not only weakly nonlinear effects (such as distortion and intermodulation) but also strongly nonlinear system dynamics (such as clipping and slewing). Moreover, fidelity in large-swing and large-signal analysis can be retained.

PWP is implemented in [17] for extracting broadly applicable general-purpose macromodels from SPICE netlists in which the PWP model is able to model different nonlinear behaviours such as loading effects, simultaneous switching noise (SSN), crosstalk noise and so on. A simulation speed up of eight times is reported [18]. The approach is implemented in the MATLAB environment but it is not established if it can be used to perform HLFM. Moreover, PWP and TPWL are the Taylor polynomial based techniques; their models show their poorer performance (convergence and speedup) as compared with Chebyshev polynomial [21]. MOR techniques for SNS are summarized in Table 6.

5 Conclusion

In this paper, a survey on available linear and nonlinear AMG techniques for HLM and HLFM is conducted. The AMG techniques are categorized into SI based AMG and MOR based AMG. Overall, the techniques for linear systems are mature, whereas for nonlinear systems they are strictly case dependant. In addition, most of these AMG techniques perform well in MATLAB environment, but these AMG models have not all been fully evaluated in a more physical electronics

orientated environment using HDLs such as VHDL-AMS or Verilog-AMS [87]. Therefore, it may become questionable if these MATLAB models can achieve same speedup and accuracy when implemented at higher level.

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