Exact solution of the unsteady natural convective radiating gas flow in a vertical channel

M. Narahari and Rajashekhar Pendyala

Citation: AIP Conference Proceedings **1557**, 121 (2013); doi: 10.1063/1.4823888
View online: http://dx.doi.org/10.1063/1.4823888
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1557?ver=pdfcov
Published by the AIP Publishing

Exact Solution of the Unsteady Natural Convective Radiating Gas Flow in a Vertical Channel

M. Narahari[1] and Rajashekhar Pendyala[2]

[1]*Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, Tronoh 31750, Perak, Malaysia. E-mail: marneni@petronas.com.my*

[2]*Department of Chemical Engineering, Universiti Teknologi PETRONAS, Tronoh 31750, Perak, Malaysia E-mail: rajashekhar_p@petronas.com.my*

Abstract. An exact solution is presented for the problem of unsteady natural convective radiating flow in an open-ended vertical channel. Laplace transform technique is used to obtain the analytical solutions for the velocity and temperature fields in the optically thin limit case when the channel walls temperature are maintained at different constant temperatures. The effects of radiation parameter and Grashof number on the flow fields are discussed through graphs. It is found that the effect of radiation is to decrease the fluid velocity whereas the effect of Grashof number is to increase the fluid velocity in the vertical channel.

Keywords: Heat transfer, natural convection, thermal radiation, vertical channel, Laplace transform technique.
PACS: 44.40.+*a*, 44.20.+b, 44.05.+e, 44.25.+f, 44.27.+g

INTRODUCTION

Natural convection flows in vertical parallel plate channel attracted the attention of numerous researchers because of its wide applications in engineering systems. The typical applications include cooling of electronic equipment, cooling of nuclear reactors, crystal growth and solar energy collectors. Transient free convective flow in a vertical parallel plate channel heated asymmetrically was investigated analytically by Singh et al. [1]. The transient free convective flow between two long vertical parallel plates with a constant heat flux and constant temperature on the walls was investigated analytically by Narahari et al. [2] using the Laplace transform technique. The closed form solution for transient free convective flow of a viscous and incompressible fluid in a vertical channel due to symmetric heating of channel walls was investigated by Jha et al. [3] using the Laplace transform technique. Singh and Paul [4] presented a closed form solution for the transient free convection flow of a viscous and incompressible fluid between two long vertical walls as a result of asymmetric heating or cooling of the walls using the Laplace transform technique. An exact solution of the unsteady free convective flow of a viscous incompressible fluid in a vertical channel with ramped temperature and a constant temperature on the channel walls was investigated by Narahari and Raghavan [5]. Recently, Narahari and Dutta [6] investigated the unsteady free convection flow and heat transfer in a viscous incompressible fluid between two long vertical parallel plates due to linearly varying wall temperature with time and a constant temperature on the walls. In all the above mentioned studies thermal radiation effect on the free convection flow was neglected.

The heat transfer by thermal radiation is important in high temperature systems and aerospace applications. An exact solution for the fully-developed, radiating, laminar convective flow in a vertical heated channel in the optically thin limit was obtained by Greif et al. [7]. The effects of thermal radiation on the combined free and forced convection flow of an electrically conducting fluid in a vertical channel was investigated by Gupta and Gupta [8] in the optically thin limit case. An exact solution of the transient natural convective Couette flow of a viscous incompressible optically thick fluid between long vertical parallel plates with constant heat flux and constant temperature on walls was presented by Narahari [9] in the presence of thermal radiation. The transient free convective flow between long vertical parallel plates with ramped temperature at one boundary and the other boundary is maintained at constant temperature was investigated by Narahari [10] in the presence of thermal radiation and constant mass diffusion for optically thick fluids. Recently, the effect of thermal radiation on free convective unsteady Couette flow between long vertical parallel plates due to linearly varying wall temperature with time at one boundary was investigated by Narahari and Yahya [11] for optically thick fluids. In the studies [9-11], the radiation heat flux for optically thick fluids was modeled using the Rosseland diffusion approximation.

International Conference on Mathematical Sciences and Statistics 2013 (ICMSS2013)
AIP Conf. Proc. 1557, 121-124 (2013); doi: 10.1063/1.4823888

However, the transient natural convection flow of a radiating gas in a vertical channel has not been studied in the optically thin limit.

Therefore, the aim of the present paper is to study the transient natural convection flow of a radiating gas in a long vertical parallel plate channel when the channel walls are at different constant temperatures. The study is restricted to optically thin fluids and the exact solution is obtained with the Laplace transform technique which could then be useful in the analysis of more general and complex problems.

MATHEMATICAL ANALYSIS

Consider the unsteady viscous incompressible flow inside a vertical channel formed by two parallel plates with distance d apart. The x'-axis is taken along the left plate and the y'-axis is taken normal to the plate. Initially, the temperature of the two plates (left and right plates) and the fluid are assumed to be at the temperature T'_d. At time $t' > 0$, the temperature of the plate at $y' = 0$ (left plate) is raised to a temperature T'_w and the plate at $y' = d$ (right plate) is maintained at the initial temperature T'_d, causing the flow of free convection currents. It is assumed that all the fluid properties are constant except the density variation with temperature in the body force term. Then under the Boussinesq's approximation, the flow can be shown to be governed by the following coupled partial differential equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T'_w - T') + \nu \frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

with the following initial and boundary conditions

$$\left.\begin{array}{l} t' \le 0: \ u' = 0, \ T' = T'_d \quad \text{for} \quad 0 \le y' \le d, \\ t' > 0: \begin{cases} u' = 0, \ T' = T'_w & \text{at} \quad y' = 0, \\ u' = 0, \ T' = T'_d & \text{at} \quad y' = d. \end{cases} \end{array}\right\} \tag{3}$$

where u' is fluid velocity, t' is time, g is acceleration due to gravity, β is thermal expansion coefficient, T' is fluid temperature, ν is kinematic viscosity, ρ is fluid density, C_p is specific heat at constant pressure, k is thermal conductivity and q_r is radiative flux. In the optically thin limit for a non-grey gas near equilibrium, Cogley et al. [12] showed that [7, 8]:

$$\frac{\partial q_r}{\partial y'} = 4(T' - T'_w)\int_0^\infty k_{\lambda w}\left(de_{b\lambda} / dT'\right)_w d\lambda \tag{4}$$

where k_λ is absorption coefficient, $e_{b\lambda}$ is Planck function and the subscript w refers to values at the wall $y' = 0$. Substituting equation (4) into equation (2), gives

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - 4(T' - T'_w)I \tag{5}$$

where

$$I = \int_0^\infty k_{\lambda w}\left(de_{b\lambda} / dT'\right)_w d\lambda \tag{6}$$

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} y = \dfrac{y'}{d}, t = \dfrac{t'\nu}{d^2}, u = \dfrac{u'd}{\nu}, \ \theta = \dfrac{T'_w - T'}{T'_w - T'_d}, \\ Gr = \dfrac{g\beta(T'_w - T'_d)d^3}{\nu^2}, \Pr = \dfrac{\mu C_p}{k}, F = \dfrac{4Id^2}{k}. \end{array}\right\} \tag{7}$$

equations (1), (6) and (3) reduces, respectively, to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta \tag{8}$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - F\theta \tag{9}$$

with the initial and boundary conditions

$$\left.\begin{array}{l} t \le 0: u = 0, \theta = 1 \quad \text{for} \quad 0 \le y \le 1, \\ t > 0: \begin{cases} u = 0, \theta = 0 & \text{at} \quad y = 0, \\ u = 0, \theta = 1 & \text{at} \quad y = 1. \end{cases} \end{array}\right\} \tag{10}$$

where u is dimensionless velocity, t is dimensionless time, y is dimensionless coordinate axis normal to the plates, Gr is Grashof number, θ is dimensionless temperature, Pr is Prandtl number and F is radiation parameter. The coupled linear partial differential equations (8) and (9) subjected to the initial and boundary conditions (10) have been solved using the Laplace transform technique without any restrictions. The solutions for the velocity and temperature fields are as follows:

$$\theta(y,t)=\sum_{n=0}^{\infty}\Big[\exp(-a_0 t)\Big\{F_1\Big(a_1\sqrt{\Pr},0,0,t\Big) - F_1\Big(a_2\sqrt{\Pr},0,0,t\Big) - F_1\Big(a_3\sqrt{\Pr},0,0,t\Big) + F_1\Big(a_4\sqrt{\Pr},0,0,t\Big)\Big\} + \Big\{F_1\Big(a_2\sqrt{\Pr},a_0,0,t\Big) - F_1\Big(a_4\sqrt{\Pr},a_0,0,t\Big)\Big\}\Big] + \exp(-a_0 t) \tag{11}$$

Case I: $\Pr \neq 1$

$$u(y,t)=\sum_{n=0}^{\infty}\Big[a_7\Big\{F_1\Big(a_2\sqrt{\Pr},a_0,a_5,t\Big) - F_1\Big(a_2\sqrt{\Pr},a_0,0,t\Big) - F_1(a_2,0,a_5,t) + F_1(a_2,0,0,t) - F_1\Big(a_4\sqrt{\Pr},a_0,a_5,t\Big) + F_1\Big(a_4\sqrt{\Pr},a_0,0,t\Big) + F_1(a_4,0,a_5,t) - F_1(a_4,0,0,t)\Big\} + a_6\Big\{F_1\Big(a_1\sqrt{\Pr},a_0,a_5,t\Big) - F_1(a_1,0,a_5,t) + F_1(a_1,0,0,t) - F_1\Big(a_2\sqrt{\Pr},a_0,a_5,t\Big) + F_1(a_2,0,a_5,t) - F_1(a_2,0,0,t) - F_1\Big(a_3\sqrt{\Pr},a_0,a_5,t\Big) + F_1(a_3,0,a_5,t) - F_1(a_3,0,0,t) + F_1\Big(a_4\sqrt{\Pr},a_0,a_5,t\Big) - F_1(a_4,0,a_5,t) + F_1(a_4,0,0,t)\Big\} - a_6\exp(-a_0 t)\Big\{F_1\Big(a_1\sqrt{\Pr},0,0,t\Big) - F_1\Big(a_2\sqrt{\Pr},0,0,t\Big) - F_1\Big(a_3\sqrt{\Pr},0,0,t\Big) + F_1\Big(a_4\sqrt{\Pr},0,0,t\Big)\Big\}\Big] + a_6\{1-\exp(-a_0 t)\} \tag{12}$$

Case II: $\Pr = 1$

$$u(y,t)=\sum_{n=0}^{\infty}\Big[a_7\{F_1(a_1,0,0,t) - F_1(a_2,F,0,t) - F_1(a_3,0,0,t) + F_1(a_4,F,0,t)\} - a_7\exp(-Ft)\{F_1(a_1,0,0,t) - F_1(a_2,0,0,t) - F_1(a_3,0,0,t) + F_1(a_4,0,0,t)\}\Big] + a_7\{1-\exp(-Ft)\} \tag{13}$$

where $a_0 = \dfrac{F}{\Pr}, a_1 = 2n+2-y,$

$a_2 = 2n+1-y, a_3 = 2n+y, a_4 = 2n+1+y,$

$a_5 = \dfrac{F}{1-\Pr}, a_6 = \dfrac{Gr\Pr}{F}, a_7 = \dfrac{Gr}{F},$

$$F_1(z_1,z_2,z_3,t)=\frac{\exp(z_3 t)}{2}\times \left[\exp\left(-z_1\sqrt{z_2+z_3}\right)\operatorname{erfc}\left(\frac{z_1}{2\sqrt{t}}-\sqrt{(z_2+z_3)t}\right) + \exp\left(z_1\sqrt{z_2+z_3}\right)\operatorname{erfc}\left(\frac{z_1}{2\sqrt{t}}+\sqrt{(z_2+z_3)t}\right)\right].$$

RESULTS AND DISCUSSION

The numerical results for the velocity and temperature fields are computed and displayed in Figures 1 to 5 for various system parameters. The results are computed for fixed value of the Prandtl number at $\Pr = 0.71$ (atmospheric air). The temperature variation with time is shown in Figure 1. It is observed that the temperature decreases with increasing time and approaches to the steady state temperature as the time progresses.

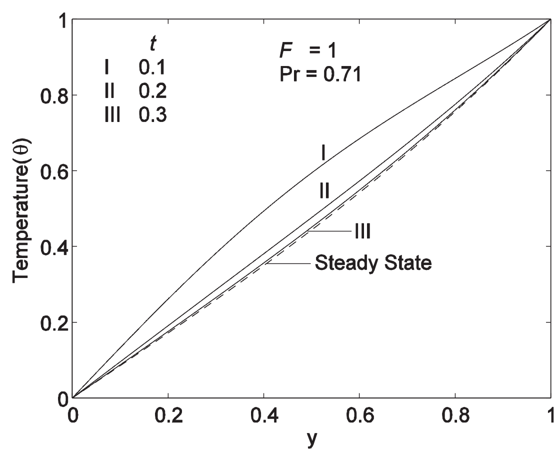

FIGURE 1. Temperature profiles at different t

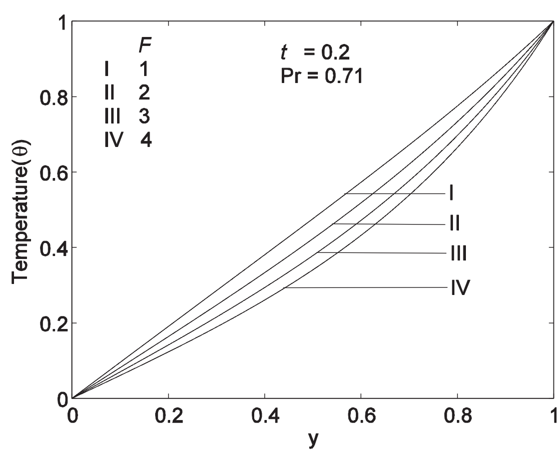

FIGURE 2. Temperature profiles for different F

The temperature variation with radiation parameter is shown in Figure 2. It can be seen that the temperature decreases with the increase in radiation parameter. Therefore, the effect of radiation is to reduce the influence of natural convection.

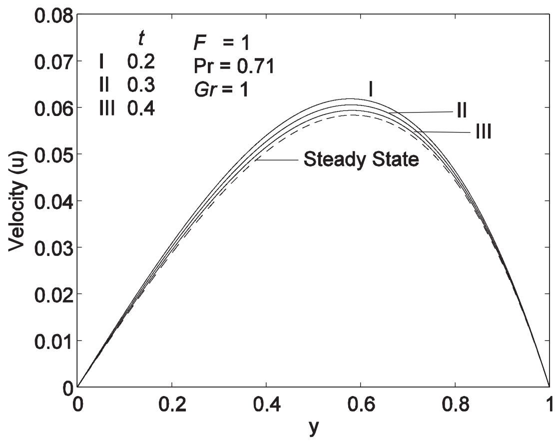

FIGURE 3. Velocity profiles at different *t*

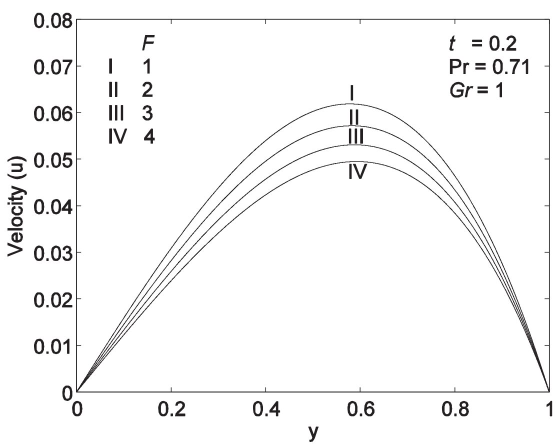

FIGURE 4. Velocity profiles for different *F*

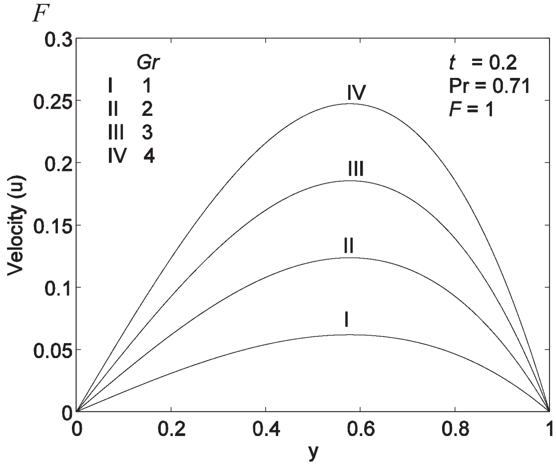

FIGURE 5. Velocity profiles for different *Gr*

The velocity variation with time is shown in Figure 3. From this figure it is clear that the velocity decreases with increasing time and approaches to the steady state velocity as the time progresses. This is due to the decrease in temperature with time and thereby decreases the velocity. The velocity variation is shown in Figure 4 for different values of the radiation parameter. It is clear that the velocity decreases with the increase in radiation parameter. This is due to reduced natural convection effect with the radiation parameter. The velocity variation with Grashof number is shown in Figure 5. It is observed that the velocity increases with increasing Grashof number. This is due to increasing buoyancy force with increasing Grashof number.

CONCLUSIONS

The unsteady natural convection flow of a radiating gas in a vertical channel is investigated analytically for the optically thin limit case. Closed form solutions for the velocity and temperature fields were obtained using the Laplace transform technique. It is found that the radiation effect is to decrease the fluid velocity and temperature in the vertical channel.

ACKNOWLEDGMENTS

The authors would like to thank Universiti Teknologi PETRONAS for financial support.

REFERENCES

1. A. K. Singh, H. R. Gholami and V. M. Soundalgekar, *Wärme-und Stoffübertragung* **31**, 329-331 (1996).
2. M. Narahari, S. Sreenadh and V. M. Soundalgekar, *Thermophys. Aeromech*. **9**(2), 287-293 (2002).
3. B. K. Jha, A. K. Singh and H. S. Takhar, *Int. J. Appl. Mech. Eng*. **8**(3), 497-502 (2003).
4. A. K. Singh and T. Paul, *Int. J. Appl. Mech. Eng*. **11**(1), 143-154 (2006).
5. N. Marneni and V. R. Raghavan, Proceedings of the ASME Summer Heat Transfer Conference 2009, HT2009, **2**, 73-80 (2009).
6. M. Narahari and B. K. Dutta, *Acta Technica* **56**, 103-113 (2011).
7. R. Greif, I. S. Habib and J. C. Lin, *J. Fluid Mech*. **46**(3), 513-520 (1971).
8. P. S. Gupta and A. S. Gupta, *Int. J. Heat Mass Transfer*. **17**, 1437-1442 (1974).
9. M. Narahari, *WSEAS Transactions on Heat and Mass Transfer* **5**(1), 21-30 (2010).
10. M. Narahari, *Meccanica* **47**, 1961-1976 (2012).
11. M. Narahari and Noorhana Yahya, *Applied Mechanics and Materials* **249-250**, 15-21 (2013).
12. A. C. Cogley, W. C. Vincenti and S. E. Gilles, *AIAA J*. **6**, 551-553 (1968).