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Ensemble Averaging Subspace-based Approach for ERP Extraction

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***Abstract*-- A novel approach based on Subspace methods is proposed for extracting the Event Related Potentials (ERPs) from the background Electroencephalograph (EEG) colored noise. First, the enhancement of SNR to the neighborhood of -2 dB is achieved through the ensemble averaging of the EEG data over a limited number of trials. Then a linear estimator is used to reduce further the amount of the EEG signal in the ERPs. With this estimator the EEG colored noise is first whitened using Cholesky factorization then the eigendecomposition of the covariance matrices of prewhitened data performed and the subspace is decomposed into signal subspace and noise subspace. The components in the noise subspace are nullified and the components in the signal subspace are retained to do the improvement. The proposed algorithm is verified with simulated data and the results shows reliable performance in terms of accuracy and failure rate.**

***Index Terms*-- Visual evoked potentials, subspace filtering, generalized eigendecomposition.**

I. INTRODUCTION

Event related potential (ERP) testing is a term that mainly describes three different types of non-invasive tests conventionally used in clinical diagnosis: 1) the visual evoked potential (VEP) test, 2) the brainstem auditory evoked potential (BAEP) test, 3) the somatosensory evoked potential (SSEP) test [1]. The neuro physicians/neuro researchers used these tests for checking of the nerve signals (vision, hearing etc.) which are transmitted to the brain and spinal cord.

Generally speaking, the conduction of ERP test is done for neuro physicians/neuro surgeons to help in analyzing Alzheimer's [2], multiple sclerosis [3], schizophrenics [4], diabetic retinopathy [5, 6], migraine [7], glaucoma [8], and acute induction to high altitude [9] as well as checking of the integrity of the visual pathways from the retina all the way to the occipital cortex [1].

The steady-state ERPs are usually concealed in the ongoing background electroencephalogram (EEG) generated in the brain. The EEG signal is a colored type of noise usually exists at much higher level than the ERP, with a typical signal-to-noise ratio (SNR) of -5 to -10 dB [10, 11].

This makes the extraction of the ERP signal from the brain background noise a challenging issue. The conventional method of ensemble averaging is efficient but time consuming and require hundreds of trials for good estimate of the ERP. This may cause fatigue to the subject under examination and make the process nonstationary. In order to overcome the problem related to ensemble averaging Nidal *et al*. proposed the single-trial subspace-based approach [12]. This approach represented turning point in the area of ERP extraction in terms of its requirement to only one trial and its efficiency in extracting the ERP signals. However, recent studies conducted using the single-trial subspace-based approach showed relatively high failure rate and less accuracy in estimating the latencies of P_{100}, P_{200}, and P_{300} than previous studies, especially when SNR < -3 dB [13].

In this paper, a novel algorithm based on ensemble averaging and signal subspace is proposed which is named as Ensemble Averaging Subspace-based approach (EASA). The algorithm first improves the SNR of the EEG signal near to the value of -3 dB by using the ensemble averaging of few trials. Second, prewhitening scheme is applied for whitening of Covariance matrix of EEG colored noised which was taken when no stimulation is applied to subject. Finally, a subspace-based linear estimator is used for the estimation of clean VEP Signal.

The proposed algorithm is evaluated in terms of its accuracy in detecting the latencies of P100, P200, and P 300 and its failure rate and is tested with simulated data and validated using the ensemble averaging method.

The paper is organized as follows. In section 2 the proposed method is explained. Section 3 describes the results in simulated and real environments. Section 4 concludes the paper.

For clarity, an attempt has been made to adhere to a standard notational convention. Lower case **boldface** characters will generally refer to vectors. Upper case **BOLDFACE** characters will generally refer to matrices. Vector or matrix transposition will be denoted using $(.)^T$ and $(.)^*$ denotes conjugation for complex valued signals. $\mathbb{R}^{K\times K}$ denotes the real vector space of $K \times K$ dimensions.

II. MULTI-TRIALS SUBSPACE-BASED APPROACH FOR VEP EXTRACTION

The problem taken into consideration in this paper is the extraction of the clean ERP signal $x(n)$ from the degraded ERP signal $y(n)$. A M-dimensional vector of ERP samples $\boldsymbol{x}$ is defined as follows:

$$\bar{x} = [x(0), x(1), ..., x(M-1)]^T \tag{1}$$

where $(.)^T$ denotes transpose operation.

Let $\bar{\boldsymbol{y}}$ denote the corresponding M-dimensional vector of the noisy ERP. Since the EEG noise is assumed to be additive, we have

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{x}} + \bar{\boldsymbol{n}} \tag{2}$$

where $\bar{y} \in R^M, \bar{x} \in R^M, \bar{n} \in R^M$ are the M-dimensional column vectors of noisy ERPs.

Let $\boldsymbol{y}$ denote the ensemble averaging of N noisy VEP vectors, given as:

$$\begin{aligned} y &= \sum_{i=1}^{N} \bar{y}_i \\ &= \sum_{i=1}^{N} \bar{x}_i + \sum_{i=1}^{N} \bar{n}_i \\ &= \boldsymbol{x} + \boldsymbol{n} \end{aligned} \tag{3}$$

Let $\mathbf{H}$ be a $M \times M$ linear estimator of clean VEP vector as follows:

$$\hat{\mathbf{x}} = \mathbf{H} \times \boldsymbol{y} \tag{4}$$

The error signal obtained in this estimation is given by

$$\begin{aligned} \varepsilon &= \hat{\boldsymbol{x}} - \boldsymbol{x} \\ &= (\mathbf{H} - \mathbf{I})\boldsymbol{x} + \mathbf{H} \times \boldsymbol{n} \\ &\overset{\Delta}{=} \varepsilon_x + \varepsilon_n \end{aligned} \tag{5}$$

where ϵ_x represents signal distortion and ϵ_n represents residual noise [14]. The squared signal distortion error ϵ_x^2 and residual noise ϵ_n^2 can then be defined as

$$\begin{aligned} \varepsilon_x^2 &= \mathrm{tr}\left(\varepsilon_x \varepsilon_x^{\mathbf{T}}\right) \\ &= \mathrm{tr}\left((\mathbf{H} - \mathbf{I})\mathbf{R}_x (\mathbf{H} - \mathbf{I})^{\mathrm{T}}\right) \end{aligned} \tag{6}$$

and

$$\begin{aligned} \varepsilon_n^2 &= \mathrm{tr}\left(\varepsilon_n \varepsilon_n^{\mathrm{T}}\right) \\ &= \mathrm{tr}\left(\mathbf{H}\,\mathbf{R}_n\,\mathbf{H}^{\mathrm{T}}\right) \end{aligned} \tag{7}$$

where $\mathbf{R}_x$ and $\mathbf{R}_n$ are the covariance matrices of the clean signal and noise vector, respectively, and $\mathbf{I}$ denotes the identity matrix. Since signal distortion and residual noise can not be simultaneously minimized, the intention is to keep the level of residual noise below some threshold while minimizing the signal distortion. Accordingly, the optimum linear estimator is obtained from the following constraints:

$$\begin{aligned} &\mathbf{H}_{opt} \overset{\Delta}{=} arg\left\{\underset{\mathbf{H}}{min}\ \varepsilon_x^2\right\}, \\ &\textit{Subject to}: \frac{1}{M}\varepsilon_n^2 \le \sigma^2 \end{aligned} \tag{8}$$

where σ^2 is a positive constant. Reducing noise in a given noisy VEP signal causes some distortion in VEP components. In the above optimization, by decreasing noise threshold level σ^2 we can decrease the amount of residual noise and therefore, increase the amount of distortion and vice versa. The optimal distortion in the sense of Eq. (8) can be found using *Kuhn-Tucker* necessary conditions for constrained minimization. Specifically, $\mathbf{H}$ is a stationary feasible point if it satisfies the gradient equation of the Lagrangian :

$$\mathscr{L}(\mathbf{H}, \mu) \overset{\Delta}{=} \varepsilon_x^2 + \mu\left(\varepsilon_n^2 - K\sigma^2\right) \tag{9}$$

and

$$\left(\varepsilon_n^2 - K\sigma^2\right) = 0 \quad \text{for } \mu \ge 0 \tag{10}$$

where μ is the Lagrangian multiplier. From $\nabla_{\mathbf{H}}\mathscr{L}(\mathbf{H}, \mu) = 0$ and Eq. (6) and Eq. (7), we obtain [13]:

$$\mathbf{H}_{opt} = \mathbf{R}_x\left(\mathbf{R}_x + \mu\,\mathbf{R}_n\right)^{-1} \tag{11}$$

Now let the eigendecomposition of rank-p covariance matrix $\mathbf{R}_{\mathbf{x}}$ be given by

$$\mathbf{R}_x = \mathbf{V}_x\,\mathbf{D}_x\,\mathbf{V}_x^{\mathrm{T}} \tag{12}$$

Substituting (12) in (11) we have

$$\mathbf{H}_{opt} = \mathbf{V}_x\,\mathbf{D}_x\left(\mathbf{D}_x + \mu\,\mathbf{V}_x^{\mathrm{T}}\,\mathbf{R}_n\,\mathbf{V}_x\right)^{-1}\mathbf{V}_x^{\mathrm{T}} \tag{13}$$

The eigenvectors and eigenvalues of the clean signal covariance matrix, $\mathbf{R}_x$, are unknown but can be estimated from the eigendecomposition of the noisy data covariance matrix, $\mathbf{R}_y$, if the EEG noise covariance matrix is diagonal (white). Thus, since the EEG noise is colored and the covariance matrix $\mathbf{R}_y$ is non-diagonal, we have first to prewhiten the noise vector $\boldsymbol{n}$.

The prewhitening of the observation signal by the use of inverse filtering leads to most accurate solution for the colored noise case before it is arranged in an observation matrix. The common approach for prewhitening schemes of the EEG noise covariance matrix is based on Cholesky factorization. Let Cholesky factorization of matrix $\mathbf{R}_n$ be given as,

$$\mathbf{R}_n = \mathbf{n}^{\mathrm{T}}\,\mathbf{n} = \mathbf{R}^{\mathrm{T}}\,\mathbf{R} \tag{14}$$

where $\mathbf{R} \in \mathbb{R}^{K \times K}$ is an upper triangular matrix.

The prewhitened observation matrix $\boldsymbol{y}$ is then given by

$$\begin{aligned} \tilde{\mathbf{y}} &= \mathbf{y}\,\mathbf{R}^{-1} = \mathbf{x}\,\mathbf{R}^{-1} + \mathbf{n}\,\mathbf{R}^{-1} \\ &= \tilde{\boldsymbol{x}} + \tilde{\boldsymbol{n}} \end{aligned} \tag{15}$$

The additive noise is hereby prewhitened, which can be seen from the calculation of the scaled estimate of the prewhitened observation correlation matrix, i.e.,

Accordingly, the optimum estimator, $\mathbf{H}_{opt}$, in Eq. (11) is given for the prewhitened data in Eq. (13) by :

$$\mathbf{H}_{opt} = \mathbf{R}_{\tilde{x}}\left(\mathbf{R}_{\tilde{x}} + \mu\,\mathbf{I}_K\right)^{-1} \tag{16}$$

The eigendecomposition of the rank-p covariance matrix $\mathbf{R}_{\tilde{x}}$ is given as

$$\mathbf{R}_{\tilde{x}} = \mathbf{V}_{\tilde{x}}\, \mathbf{D}_{\tilde{x}}\, \mathbf{V}_{\tilde{x}}^{\mathrm{T}} = \begin{pmatrix} \mathbf{V}_{\tilde{x}1} & \mathbf{V}_{\tilde{x}2} \end{pmatrix} \begin{pmatrix} \mathbf{D}_{\tilde{x}1} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{V}_{\tilde{v}_{x1}}^{\mathrm{T}} \\ \mathbf{V}_{\tilde{v}_{x2}}^{\mathrm{T}} \end{pmatrix} \tag{17}$$

where $\mathbf{D}_{\tilde{x}}$ is a diagonal $M\times M$ matrix contains the eigenvalues of $\mathbf{R}_{\tilde{x}}$ and $\mathbf{D}_{x1} = diag(d_1, d_2, \ldots, d_p)$. Matrix $\mathbf{V}_{\tilde{x}} \in \mathbb{R}^{M\times M}$ represents the M eigenvectors of $\mathbf{R}_{\tilde{x}}$, $\mathbf{V}_{\tilde{x}1} = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_p \end{bmatrix} \in \mathbb{R}^{M\times p}$ and $\mathbf{V}_{\tilde{x}2} = \begin{bmatrix} \boldsymbol{v}_{p+1} & \boldsymbol{v}_{p+2} & \cdots & \boldsymbol{v}_K \end{bmatrix} \in \mathbb{R}^{M\times(M-p)}$

Now since covariance matrix of the whitened noise $\boldsymbol{n}\mathbf{R}^{-1} = \mathbf{I}_M$ has a single degenerate eigenvalue equal to unity with multiplicity of M, so any vector qualifies as the associate eigenvector, i.e., the eigenvectors of $\mathbf{R}_{\tilde{x}}$ are unaffected by the constant diagonal perturbation, and the Eigen decomposition of $\mathbf{R}_{\tilde{y}}$ is obtained as:

$$\mathbf{R}_{\tilde{y}} = \mathbf{V}_{\tilde{y}}\, \mathbf{D}_{\tilde{y}}\, \mathbf{V}_{\tilde{y}}^{\mathrm{T}} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{V}_2 \end{pmatrix} \begin{pmatrix} \mathbf{D}_1 & 0 \\ 0 & \mathbf{D}_2 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1^{\mathrm{T}} \\ \mathbf{V}_2^{\mathrm{T}} \end{pmatrix} \tag{18}$$

where the first p-eigenvectors $\mathbf{V}_1 \in \mathbb{R}^{M\times p} = \mathbf{V}_{\tilde{x}1}$ span the signal subspace and last (M-p) eigenvectors $\mathbf{V}_2 \in \mathbb{R}^{M\times(M\text{-}p)} = \mathbf{V}_{\tilde{x}2}$ span the noise subspace. The diagonal matrix $\mathbf{D}_1 \in \mathbb{R}^{p\times p} = \mathbf{D}_{\tilde{x}1} + \mathbf{I}_p$ and $\mathbf{D}_2 \in \mathbb{R}^{(M\text{-}p)\times(M\text{-}p)} = \mathbf{I}_{M\text{-}p}$.

The diagonal elements of $\mathbf{D}_1$ represent the oriented energy measured along the column vectors of matrix $\mathbf{V}_1$, and the diagonal elements of $\mathbf{D}_2$ represent the measured energy along the column vectors of $\mathbf{V}_2$.

Having decomposed the vector space of the noisy VEP signal into signal subspace spanned by the vectors of matrix $\mathbf{V}_1$ with oriented energy $\mathbf{D}_1$, and noise subspace spanned by $\mathbf{V}_2$ with oriented energy $\mathbf{D}_2$, enhancement in the clean signal estimator $\mathbf{H}_{opt}$ in Eq. (16) is performed by removing the noise subspace and estimating the clean signal from the remaining signal subspace. This results in the following form for the linear estimator:

$$\mathbf{H}_{opt} = \mathbf{V}_1 \mathbf{D}_1 \left(\mathbf{D}_1 + \mu\, \mathbf{I}\right)^{-1} \mathbf{V}_1^{\mathrm{T}} \tag{19}$$

To implement Eq. (14) for $\mathbf{H}_{\mathrm{opt}}$, the rank of the signal subspace p and the optimum value of the Langrage multiplier μ, are considered *a priori known*.

IV. RESULTS AND DISCUSSIONS

In this section, the capabilities of EASA in accurately extracting the clean VEP, is assessed and compared with pure ensemble averaging estimated from 100 trials. The comparison is conducted in the experiment by using artificially generated VEP signals corrupted by colored noise. The capabilities of the two techniques in detecting the P_{100}, P_{200}, and P_{300} and accurately estimating their latencies are used to indicate their performance.

The clean VEP, $x(n) \in \Re^K$, is generated by superimposing J Gaussian functions; each function has different amplitude (A), variance (σ^2) and mean (β) as given by the following equation [14]:

$$x(n) = \sum_{m=1}^{J} v_m(n) \;\; where\; v_m(n) = \frac{A_n}{\sqrt{2\pi\sigma_m{}^2}}\, e^{\frac{-(n-\beta_m)^2}{2\sigma_m{}^2}} \tag{20}$$

The values for A, σ and β are experimentally tweaked to create arbitrary amplitudes with precise peak latencies at 100 *ms*, 200 *ms*, and 300 *ms* simulating the real P_{100}, P_{200} and P_{300}, respectively.

The EEG colored noise is generated by using the following autoregressive (AR) model which is described mathematically:

$$\begin{aligned} g(n) \;=\; & 1.5084g(n\text{-}1) - 0.1587g(n\text{-}2) - 0.3109g(n\text{-}3) \\ & - 0.0510g(n\text{-}4) + u(n) \end{aligned} \tag{21}$$

where $u(n)$ is the input driving noise of the AR filter and $g(n)$ is the filter output [22, 23, 24]. Since the noise is assumed to be additive, the corrupted VEP is given by:

$$y(n) = x(n) + g(n) \tag{22}$$

In the first experiment, the VEP signal is generated according to Eq. (22) and corrupted by additive colored noise at SNR values ranging from -5 dB to -10 dB. Then the proposed technique and the EA algorithms are used to extract the clean VEP. The proposed technique is run with 15 trials whereas the EA is run with 100 trials.

The algorithms are run 200 times at each SNR value and the average errors in detecting the latencies of P_{100}, P_{200}, and P_{300} are calculated.

In addition to the average error, the failure rate is also found and used to indicate the performances of the different algorithms. A trial is noted as a failure with respect to certain peak, if the peak does not show up in the extracted VEP signal or if the error in detecting it exceeds 10 ms.

The average errors and the failure rates of the proposed technique and EA algorithms at P_{100}, P_{200}, and P_{300}, are illustrated in Tables I and II.

Table I. The average error of the proposed technique as a function of SNR.

SNR	e_{100}	e_{200}	e_{300}	Failure Rate
-5 dB	1.61	1.86	2.12	2.60%
-6 dB	1.63	1.88	2.17	2.64%
-7 dB	1.71	1.92	2.33	3.40 %
-8 dB	2.01	2.10	2.55	4.30%
-9 dB	2.40	2.77	3.10	6.23%
-10 dB	2.47	3.02	3.80	7.28%

Table II. The average error of EA as a function of SNR.

SNR	e_{100}	e_{200}	e_{300}	Failure rate
-5 dB	1.54	1.78	1.89	2.20%
-6 dB	1.43	1.82	2.01	2.24%
-7 dB	1.70	1.88	2.13	3.02 %
-8 dB	1.95	2.05	2.27	3.40%
-9 dB	2.17	2.54	2.98	4.02%
-10 dB	2.32	2.87	3.36	6.23%

The results in Tables I and II verify the capability of the subspace technique conduced on averaged signal with limited number of trials (< 15) in approaching the performance of the ensemble averaging with 100 trials, in terms of accuracy and failure rates. The number of trials can be increased for improved accuracy and less failure rate.

V. CONCLUSION

A novel signal processing technique based on subspace methodology is applied for extraction of ERP signals degraded by colored EEG noise. The idea is implemented on averaged EEG signal for limited number of trials. The algorithm utilizes optimization technique to minimize the distortion in the extracted ERP signal and uses subspace transformation for the elimination of noise in an effective way and improve the signal. Cholesky factorization is used to pre-whiten the EEG noise. The performance of the proposed technique is compared with ensemble averaging, for artificially generated ERP corrupted by colored noise, at different SNR values. The two algorithms are used to extract the latencies of P_{100}, P_{200}, and P_{300}. The results show a reasonably better performance by the proposed subspace technique conducted with only 15 trials compared to EA run with 100 trials.

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