# Epileptic Seizure Detection using the Singular Values of EEG Signals

Arslan Shahid, Nidal Kamel, Aamir Saeed Malik and Munsif Ali Jatoi

Center for Intelligent Signal & Imaging Research Department of Electrical and Electronics Engineering Universiti Teknologi PETRONAS, Malaysia arslanshahid@hotmail.com, munsif.jatoi@gmail.com {nidalkamel & aamir saeed}@petronas.com.my

Abstract - A new technique based on Singular Value Decomposition (SVD) for the detection of epileptic seizures is proposed. The SVD is applied sequentially on a sliding window of one second width of EEG data and the r singular values are obtained and used to indicate sudden changes in the signals. EEG recordings of 4-paediatric patients with 20 seizures are used to validate the proposed algorithm and the preliminary results indicates good level of sensitivity by the singular values to the changes in the EEG signals due to epileptic seizure. This sensitivity can be used to develop more reliable seizure detector than the existing techniques.

# Index Terms - Electroencephalography (EEG), Singular Value Decomposition (SVD), Epileptic Seizures.

## I. INTRODUCTION

Electroencephalography (EEG) is a Neural Signaling technique used for the diagnoses of various brain functionalities and disorders [1]. The important information about the brain function is contained in the frequency and the energy content of the EEG signal [2]. Earlier, EEG signal analysis was carried out by the physicians/neuroscientists by looking at the EEG recordings, but now, different automated algorithms have been developed that analyze the EEG signals and prepare a report themselves to help the physicians and researchers in this area to diagnose the behavior of the signal through different aspects. This advancement in the analysis of EEG signals has increased the use of EEG by physicians for analyzing the patients having neural disorders. EEG has high temporal resolution (of the order of millisecond) that is why it is preferred over CT or MRI for research and diagnosis purpose where high resolution is required in time-series analysis [3]. In [4, 5] various non-linear time series methods were applied for the identification of different physiological conditions. In latest trends, EEG is combined with other modalities such as electrocardiogram (ECG) in order to assess the behavior of patient more precisely. In [6], EEG & ECG were combined to correlate EEG scalp potential with the autonomic nervous system. EEG is in use for the detection and cure of several brain disorders. These disorders include epilepsy, tumors, strokes etc. Among them, Epilepsy is most important and common brain disorder as it affects 1% of the total population in the world [7]. Epilepsy is a chronic disorder of the Central Nervous System that predisposes the patient to experience recurrent seizures [8].

The use of the EEG signals in the detection of epileptic seizures is a major point of interest for the researchers in neuroscience field. Many algorithms were proposed over the last two decades with different degrees of success. In [9] a comparison of different patient-independent and patient-specific algorithms is given. Most of these systems are Non-patient specific and their computational burden is quit high.

Gotman [10, 11] developed an algorithm that sequentially searches EEG channels for some rhythmic activity in a dominant frequency range of 3-20 Hz and amplitude greater than at least 3 times that of the background window. The seizure event is declared if the degree of rhythmicity crosses a threshold on at least 2 channels and continues for 4 seconds. The accuracy of this algorithm was 50% with a median detection delay of 4 sec and a False Detection Rate of 0.5/hour.

Saab's algorithm [12] uses wavelet decomposition to extract features from each EEG channel for estimating the probability of a seizure event. This algorithm was a patient independent seizure onset detector with an accuracy of 78%, Median Detection Delay of 9.8 seconds and a False Detection Rate of 0.86/hour.

Qu's [13-16] patient-specific algorithm uses nearestneighbour classification to assign a list of features, or feature vector, to classify the events as seizure or non-seizure. Their algorithm gave 100% accuracy with a Median Detection Delay of 9.35 seconds and a False Detection Delay of 0.03/hour.

Meier [17] divided the seizures into 6 categories depending on the frequency of dominant rhythm and used Support Vector Machine to classify seizures and normal activity. This algorithm was patient independent but seizure specific. Its accuracy was 96% and the median Detection Delay was 1.6 sec and a False Detection Rate of 0.45/hour.

In this paper, a novel technique is introduced for early detection of epileptic seizure. The proposed algorithm uses the *r*-singular values of EEG within a sliding window of one second wide in indicating sudden changes in brain behavior. The algorithm is a patient-specific algorithm and fast enough to be implemented in real-time EEG monitoring.

The paper is organized as follows. In section II the proposed method is explained. Section III describes the results in real environments. Section IV concludes the paper.

For clarity, an attempt has been made to adhere to a standard notational convention. Lower case **boldface** characters will generally refer to vectors. Upper case **BOLDFACE** characters will generally refer to matrices. Vector or matrix transposition will be denoted using  $(.)^{T}$ . The notation  $\mathbf{R}^{n}$  is used for a real vector space of *n* dimensions.

### II. THE SINGULAR VALUES-BASED TECHNIQUE FOR SEIZURE DETECTION

The problem considered in this paper is to detect the epileptic seizure for captured EEG signals using *m* channels. The EEG data record is divided into small windows of *n* samples each. Consider  $A_{m\times n}$  represents the EEG data matrix of one window. The singular value decomposition of matrix **A** is given according to the following theorem.

**Theorem 1.** For any real  $m \times n$  matrix **A**, there exists a real factorization:

$$\mathbf{A} = \mathbf{U}_{mxm} \cdot \mathbf{S}_{mxn} \cdot \mathbf{V}_{nxn}^{1}$$
(1)

in which the matrices U and V are real orthonormal, and matrix S is real pseudo-diagonal with nonnegative diagonal elements.

The diagonal entries  $\sigma_i$  of **S** are called the singular values of the matrix **A**. It is assumed that they are sorted in nonincreasing order of magnitude. The set of singular values  $\{\sigma_i\}$ is called the singular spectrum of matrix **A**. The columns  $u_i$ 

and  $v_i$  of U and V are called respectively the left and right singular vectors of matrix A. The space  $S_U^r = \text{span } [u_p, u_2..., u_n]$  is called the *r*-th left principal subspace. In a similar way, the *r*-th right singular subspace is defined.

Proofs of the above classical existence and uniqueness theorems are found in [18].

Define the unit ball UB in  $\mathbf{R}^{m}$  as

$$UB = \left\{ \mathbf{q} \in \mathbf{R}^{\mathbf{M}} \mid \left\| \mathbf{q} \right\|_{2} = 1 \right\}$$
(2)

**Theorem 2.** Consider a sequence of *m*-vectors  $\{a_k\}, k = 1, 2...$ *n* and the associated  $m \times n$  matrix *A* with SVD as defined in Eq. (4) with  $n \ge m$ . Then:

$$E_{\mathbf{u}_i}[\mathbf{A}] = \sigma_i^2 \tag{3}$$

$$\forall \mathbf{q} \in \mathbf{UB}: \text{ if } \mathbf{q} = \sum_{i=1}^{m} \gamma_i \cdot \mathbf{u}_i \text{ , then}$$

$$E_{\mathbf{q}}[\mathbf{A}] = \sum_{i=1}^{m} \gamma_i^2 \cdot \sigma_i^2 \tag{4}$$

**Proof**. Trivial from Theorem 1.

The oriented energy measured in the direction of the *i*-th left singular vector of the matrix A, is equal to the *i*-th singular value square. The energy in an arbitrary direction q is the linear combination of 'orthogonal' oriented energies associated with the left singular vectors. If the matrix A is rank deficient, then there exist directions in  $\mathbb{R}^m$  that contain no energy at all.

With the aid of theorem 2, one can easily obtain, using the SVD, the directions and spaces of extremal energy, as follows [19]:

**Corollary 1**. Under the assumptions of theorem 2:

1. 
$$\max_{\mathbf{q}\in UB} E_{\mathbf{q}}[\mathbf{A}] = E_{\mathbf{u}_{1}}[\mathbf{A}] = \sigma_{1}^{2}$$
2. 
$$\min_{\mathbf{q}\in UB} E_{\mathbf{q}}[\mathbf{A}] = E_{\mathbf{u}_{m}}[\mathbf{A}] = \sigma_{m}^{2}$$
3. 
$$\max_{\mathcal{Q}^{r}\subset \mathbb{R}^{m}} E_{\mathcal{Q}^{r}}[\mathbf{A}] = E_{S_{U}^{r}}[\mathbf{A}] = \sum_{i=1}^{r} \sigma_{i}^{2}$$
4. 
$$\min_{\mathcal{Q}^{r}\subset \mathbb{R}^{m}} E_{\mathcal{Q}^{r}}[\mathbf{A}] = E_{(S_{U}^{m-r})^{\perp}}[\mathbf{A}] = \sum_{i=m-r+1}^{m} \sigma_{i}^{2}$$

where 'max' and 'min' denote operators, maximizing or minimizing overall r-dimensional subspaces Q' of the space  $\mathbf{R}^{m}$ .  $S_{U}^{r}$  is the *r*-dimensional principal subspace of matrix A while  $(S_{U}^{m-r})^{\perp}$  denotes the *r*-dimensional orthogonal complement of  $S_{U}^{m-r}$ .

**Proof.** Properties (1), (2), (3), and (4) follow immediately from the SVD theorem 1 and from theorem 2.

In other words, properties (1) and (2) relate the SVD to the minima and maxima of the oriented energy distribution. In fact, it can be shown that the extrema occur at each left singular direction.

The r-th principal subspace  $S_U^r$  is, among all r-

dimensional subspaces of  $\mathbb{R}^m$ , the one that senses a maximal oriented energy (property 3). Properties (3) and (4) show that the orthogonal decomposition of the energy via the singular value decomposition is canonical in the sense that it allows subspaces of dimension r to be found where the sequence has minimal and maximal energy. This decomposition of the ambient space, as direct sum of a space of maximal and minimal energy for a given vector sequence, leads to very interesting rank consideration [19].

By establishing this link between the oriented energy and SVD, is proved that the first r left singular vectors sensing the maximal energy of matrix A, and thus account for most of the variation in the original data. This means that the first r singular values represent the distribution the energy of matrix into the *m*-Euclidean space. Accordingly, sudden changes in the data will affect the *r*-singular values and redistribute the energy in the *m*-Euclidean space. This characteristic of the singular values will be used to detect sudden changes in EEG signals due to epileptic seizure.

#### III. RESULTS AND DISCUSSION

The EEG data for 4-paediatric patients with 20 seizures used in this study was acquired from the PhysioNet Online EEG database [20]. The data was recorded from pediatric patients at the epilepsy monitoring unit of Children's Hospital Boston. The EEG was captured using 18-channel bipolar montage and sampled at 256 samples/sec.

The EEG data is divided into small windows of one second each. Each matrix contains 18-rows for the 18-channels of EEG and 256 columns for 1 sec of data. The SVD

is calculated for each matrix and the largest *r*-singular values are obtained. The Euclidean distance from the *r*-singular values of a baseline window located 1 hour before the seizure is used to indicate the variation in EEG. The baseline was calculated by taking the  $2^{nd}$  norms of the Singular Values for 10 matrix windows in the normal resting state (without seizure) and averaging them. To summarize, the steps of the operation are given below:

*Step 1:* Calculate the baseline by taking the 2<sup>nd</sup> norm of the singular values from 10 sec of normal (non-epileptic) EEG and averaging them.

*Step 2:* Take a matrix window of dimension 18×256 for 18-channels of EEG and 256 samples for one second of data.

*Step 3:* Apply Singular Value Decomposition and convert the diagonal matrix of Singular Values to a column vector.

*Step 4:* Calculate the 2<sup>nd</sup> norm of Singular Value Column Vector.

Step 5: Take a new 18×256 window for the next second.

Step 6: Repeat Steps 2-5 till the end of seizure.

Step 7: Plot the 2<sup>nd</sup> norms of the Singular Values.

Figures 1-4 shows the distances from the baseline of the *r*-singular values of four patients. The vertical dotted line indicates the start of seizure. The horizontal straight line is the baseline.



Fig. 1 The Euclidean distance of the *r*-singular values from the baseline of Patient no. 1



Fig. 2 The Euclidean distance of the *r*-singular values from the baseline of Patient no. 2



Fig. 3 The Euclidean distance of the *r*-singular values from the baseline of Patient no. 3



Fig. 4 The Euclidean distance of the *r*-singular values from the baseline of Patient no. 4

It can be observed that the singular values deviated notably upward from the baseline at the start of epileptic seizure. This shows that the singular values are very sensitive to the amplitude variations in the EEG and they can be used for the detection of an epileptic seizure. Similar behaviour is observed in other 20 seizures for all the 4 patient's data used in this research.

#### **IV. CONCLUSIONS**

Singular Values are a very useful numerical tool to detect epileptic seizures in an EEG signal. This tool is applied here for seizure detection and has shown reliable results. Moreover, it can be integrated with some machine learning algorithm to classify seizure and non-seizure events. This will lead to a more accurate and robust system for the detection of epileptic seizure and development of an integrated system for real-time analysis of EEG Signals in terms of detection delay and false detection rate for seizure recognition/detection.

#### ACKNOWLEDGMENT

This study was funded by University Research Internal Funding (URIF: 17/2011), Universiti Teknologi PETRONAS, Malaysia.

#### REFERENCES

- S. Nasehi and H. Pourghassem, "Seizure Detection Algorithms Based on Analysis of EEG and ECG Signals: a Survey," Neurophysiology, vol. 44, pp. 174-186, 2012/06/01 2012.
- [2] A. S. Malik, D. A. Osman, A. A. Pauzi, and R. N. H. R. Khairuddin, "Investigating brain activation with respect to playing video games on large screens," In 4th International Conference on Intelligent and Advanced Systems (ICIAS), 2012, pp. 86-90.
- [3] J. W. Phillips, R. M. Leahy, J. C. Mosher, and B. Timsari, "Imaging neural activity using MEG and EEG," Engineering in Medicine and Biology Magazine, IEEE, vol. 16, pp. 34-42, 1997.
- [4] L. Xia, W. Mumtaz, and A. S. Malik, "Evaluation of EEG features as indicators of physiological conditions," Australasian Physical & Engineering Sciences in Medicine, 2012.
- [5] W. Mumtaz, L. Xia, A. Malik, and M. Yasin, "Complexity analysis of EEG data during rest state and visual stimulus," in Neural Information Processing, 2012, pp. 84-91.
- [6] A. R. Subhani, X. Likun, and A. Saeed Malik, "Association of autonomic nervous system and EEG scalp potential during playing 2D grand turismo 5," In Annual International Conference of Engineering in Medicine and Biology Society (EMBC), pp. 3420-3423.
- [7] M. Wilde and C. Haslam, "Living with epilepsy: a qualitative study investigating the experiences of young people attending outpatients clinics in Leicester," Seizure, vol. 5, pp. 63-72, 1996.
- [8] F. Mass, #233, J. Penders, A. Serteyn, M. v. Bussel, and J. Arends, "Miniaturized wireless ECG-monitor for real-time detection of epileptic seizures," presented at the Wireless Health 2010, San Diego, California, 2010.
- [9] A. H. Shoeb, "Application of machine learning to epileptic seizure onset detection and treatment," Massachusetts Institute of Technology, 2009.
- [10] J. Gotman, "Automatic recognition of epileptic seizures in the EEG," Electroencephalography and Clinical Neurophysiology, vol. 54, pp. 530-540, 1982.
- [11] J. Gotman, d. Flanagan, J. Zhang, and b. Rosenblatt, "automatic seizure detection in the newborn: methods and initial evaluation," electroencephalography and clinical neurophysiology, vol. 103, pp. 356-362, 1997.
- [12] M. Saab and J. Gotman, "A system to detect the onset of epileptic seizures in scalp EEG," Clinical Neurophysiology, vol. 116, pp. 427-442, 2005.
- [13] H. Qu and J. Gotman, "Improvement in seizure detection performance by automatic adaptation to the EEG of each patient,"

Electroencephalography and Clinical Neurophysiology, vol. 86, pp. 79-87, 1993.

- [14] H. Qu and J. Gotman, "A seizure warning system for long-term epilepsy monitoring," Neurology, vol. 45, pp. 2250-2254, 1995.
- [15] H. Qu, Self-adapting algorithms for seizure detection during EEG monitoring, 1996.
- [16] H. Qu and J. Gotman, "A patient-specific algorithm for the detection of seizure onset in long-term EEG monitoring: possible use as a warning device," IEEE Transactions on Biomedical Engineering, vol. 44, pp. 115-122, 1997.
- [17] R. Meier, H. Dittrich, A. Schulze-Bonhage, and A. Aertsen, "Detecting epileptic seizures in long-term human EEG: a new approach to automatic online and real-time detection and classification of polymorphic seizure patterns," Journal of Clinical Neurophysiology, vol. 25, pp. 119-131, 2008.
- [18] N. Kamel, M. Z. Yusoff, and A. F. M. Hani, "Single-Trial Subspace-Based Approach for VEP Extraction, "IEEE Transactions on Biomedical Engineering, vol. 58, pp. 1383-1393, 2011.
- [19] N. S. Kamel, S. Sayeed, and G. A. Ellis, "Glove-Based Approach to Online Signature Verification, "IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 30, pp. 1109-1113, 2008.
- [20] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. C. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C. K. Peng, and H. E. Stanley, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals," Circulation, vol. 101, pp. e215-e220, 2000.