



ICCBT2008

## Dynamic Response of Spar Platforms Subjected to Waves and Current

**O. A. A. Montasir**, *Universiti Teknologi PETRONAS, MALAYSIA*  
**V. J. Kurian\***, *Universiti Teknologi PETRONAS, MALAYSIA*  
**S. P. Narayanan**, *Universiti Teknologi PETRONAS, MALAYSIA*  
**M. A. W. Mubarak**, *Universiti Teknologi PETRONAS, MALAYSIA*

### ABSTRACT

---

*As the research for oil and other natural resources have progressed into deeper waters, the traditional fixed type of offshore structures have become unsuitable and new technologies had to be resorted to. Spar is a type of deepwater floating type of platform used in ultradeep waters of depth more than 1500 m. It is worth mentioning here that Malaysia has recently installed its first spar at Kikeh field near Sabah. In this study, dynamic analysis of two typical types of spar platforms such as classic spar and truss spar have been conducted and the motion responses in surge, heave and pitch have been evaluated. The spar platform has been modeled as a rigid body connected to the sea floor by multi-component catenary mooring lines. Unidirectional regular wave and random wave model spectra are used for computing the incident wave kinematics by Chakraborti's Stretching Formula and for computing the wave force by modified Morison Equation. The response analysis has been performed in time domain to solve the dynamic behavior of the moored spar platform as an integrated system using the iterative incremental Newmark Beta approach. The results show that truss spar has better responses when subjected to waves and ambient deep current. Even though the truss spar responses (i.e. surge, heave and pitch) are higher when subjected to waves, this spar is increasingly becoming popular because of lower cost.*

---

**Keywords:** *Spar platform, Dynamic response, Morison equation, Wave kinematics.*

---

\*Correspondence Author: Assoc. Prof. Dr. V.J. Kurian, Universiti Teknologi PETRONAS, Malaysia. Tel: +605-3687345, Fax: +605-3656716. E-mail: [jokurian26@yahoo.com](mailto:jokurian26@yahoo.com)

## 1. INTRODUCTION

As the offshore industry depletes hydrodynamic reservoirs below the sea bed in deep water depths (up to 1500m), it is increasingly required to develop such deposits in considerably high deeper water. The increased water depth makes the use of fixed platforms uneconomic leaving a variety of floating platform types as the only viable options for oil and gas production operations.

One such option is the classic spar platform which is basically a very large floating vertical cylinder structure having draft around 200m draft and diameter around 40m. The deep-draft cylindrical spar has been shown to be an efficient platform for deep water production, drilling, and storage (Glanville et al., [8]). Its deep draft gives it excellent motion characteristics even in most severe sea states, which has been proved through numerical simulations, model tests and field observation. The relevant theory and comparison with experiments for this kind of spar are reported in Ran et al. [3], Mekha et al. [6], Cao and Zhang [10] and Kim et al. [11]. Recently, an alternative shallower-draft truss spar has received considerable attention as a more economical design (Halkyard, [12]), especially in a loop-current environment. The upper part of the truss spar consists of a relatively shallower hard tank, and is connected to a truss structure with a number of heave plates. The multiple heave plates greatly increase the heave added mass and viscous damping, which contributes to minimize the heave motion despite the increase of the heave wave exciting force due to shallower cylinder draft. A series of model tests was conducted for the Amoco Marlin truss spar in the Offshore Technology Research Center's (OTRC) 3-dimensional wave basin at Texas A&M University, and the results showed that the truss spar exhibited excellent motion characteristics.

Research using numerical simulations has utilized the two traditional approaches namely frequency domain and time domain analysis. One such study presented by Weggel et al. [2] uses the frequency domain technique and directly gives the statistical parameters of the spar response at relatively low computation cost. However it may be subject to large errors due to the linearization of some non-linear terms, such as the viscous term, in the equations of motion. There is evidence that this linearization probably overestimates viscous effects (Ran Z et al. [4]). Most researchers prefer, therefore, to simulate spar motion in the time domain and this is the approach adopted in this paper.

Simulation of the motion of a spar buoy requires the definition of the equations of motion and the evaluation of all forces acting on it due to wind, current ocean waves and mooring lines. The conventional approach in offshore engineering is to use the linear form of the equations to describe the motions of rigid bodies. For large motions, the non-linear equations of motion (Chitrapu et al. [5]) should be used but it is only practical if the exciting forces can be calculated without evolving wave diffraction analysis.

A key element of the analysis of a spar buoy is to evaluate the forces and moments on it due to ocean waves and currents. One possibility to obtain these is to perform a numerical analysis of the fully non-linear interaction between the spar and its surrounding fluid. Although it is not impossible, this task require very powerful computer resources and is, therefore, not feasible in practice. An alternative approach is to carry out a diffraction analysis based on second order potential theory (see for example, Ran et al. [4]). The computation cost of this approach is still quite high. Also this method usually generates results in the frequency domain and thereafter a transformation is needed to obtain forces in the time domain.

Another approach, often used in offshore engineering for wave force evaluation, is based on a slender body theory that requires much less computational effort and can be directly implemented in time domain analysis. In this approach, the body is assumed 'thin' and the force (and/or moment) is obtained as the sum of forces on each short segment of the slender body. The force in each segment is decomposed into two parts - an inviscid force and viscous drag force. One typical slender body wave force formulation is the well-known Morison equation, in which the first part is proportional to the relative acceleration and the second part to the product of the relative velocity. Rainey et al. ([13], [14]) has derived an alternative formula for the inviscid force on a slender body. His approach modifies the Morison equation by including axial divergence and centrifugal force terms acting on the spar buoy cross-section and by introducing additional point forces at the two ends of the body. All these forces are nonlinear and don't appear in the normal Morison equation formulation. Several computation studies have been reported in the research literature using the slender body approach-all of them using different methods to calculate the inviscid force. Chitrapu et al. [5] approximated the inviscid force by the sum of a 'Froude-Krylov' force and inertia force. The latter is evaluated in the same way as in the Morison equation but the former is estimated by the integration of the fluid pressure over the spar hull in undisturbed flow. Mekha et al. [7] considered the convective acceleration of the fluid and the axial divergence term given by Rainey et al. ([13], [14]) but showed in their case that the axial divergence term was not very important.

In this work, the wave loads on a structure were computed by integrating forces along the structure centerline from the bottom to the instant free surface at the displaced position. For truss spar, this integration was conducted from the bottom of the hard tank only because the contribution of the truss section in force calculations was much less significant compared with the hard tank. Ambient flow near the structure was calculated using Chakraborti's Stretching Formula. Additional second order contributions from convective accelerations, free surface fluctuation, structure displacement from the mean position and axial divergence were also included in the simulation. The objective of this paper is to investigate the dynamic responses of classic and truss spars in the presence of wave and current. The methodology employed uses the fully non-linear equations of motion with the mooring lines replaced by springs.

## **2. NUMERICAL PROCEDURE**

Considering that the incident waves are long crested and advancing in the x-direction, a typical spar was approximated as a rigid body having three degree of freedom (surge, heave and pitch), deriving its static resistance from support systems (mooring lines, risers) and hydrostatic stiffness. Two coordinate systems were employed in this analysis (see fig.1), the space fixed coordinate system  $oxz$  and two dimensional local coordinate  $G\zeta\eta$  which was fixed on the body with the origin at its center of gravity (CG). The point B marked the center of buoyancy and F, the fairlead point.

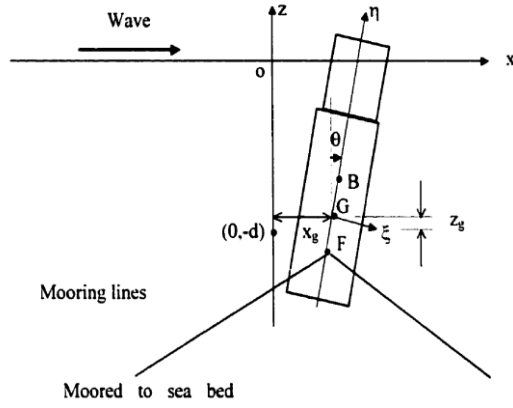


Figure.1 Three-DOF Surge-heave-pitch model of the spar

The dynamic equations of the surge-heave-pitch motions of the spar were:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (1)$$

where:

- ❖  $\{x\}$  was the structural displacement vector with respect to the center of gravity,
- ❖  $\{\dot{x}\}$  was the structural velocity vector with respect to the center of gravity,
- ❖  $\{\ddot{x}\}$  was the structural acceleration vector with respect to the center of gravity,
- ❖  $[M]$  was a mass matrix =  $M^{SPAR} + M^{Added\ Mass}$
- ❖  $[K]$  was stiffness matrix =  $K^{Hydrostat(hy)} + K^{Horizontal\ Spring(hz)}$ ,
- ❖  $[C]$  was structural damping matrix.
- ❖  $[F(t)]$  was the hydrodynamic force vector calculated using modified Morison equation.

The wave forces were decomposed into the normal force  $F_{EXn}$  and tangential force  $F_{EXt}$

$$\begin{bmatrix} F_{EXn} \\ M_{EX} \end{bmatrix} = \int_{-d_1}^{\zeta} \rho(1+C_m)A(n)a_n \begin{Bmatrix} 1 \\ n \end{Bmatrix} dn + \int_{-d_1}^{\zeta} \frac{1}{2} \rho C_D D |V_n| V_n \begin{Bmatrix} 1 \\ n \end{Bmatrix} dn + \int_{-d_1}^{\zeta} \rho C_m A(n) V_n \tau^T v \tau \begin{Bmatrix} 1 \\ n \end{Bmatrix} dn. \quad (2)$$

where

$$a_n = |a - (a \cdot \vec{\tau}) \vec{\tau}|$$

$$V_n = |V - r_s - ((V - r_s) \cdot \vec{\tau}) \vec{\tau}|$$

$$\tau = \begin{Bmatrix} \sin \theta \\ \cos \theta \end{Bmatrix}$$

$$v = \frac{\partial(u, w)}{\partial(x, z)}$$

$$F_{EXt} = \iint_{S_B} \left( \rho \frac{\partial(\mathcal{G}^1 + \mathcal{G}^2)}{\partial t} + \frac{1}{2} \rho |\nabla \phi|^2 n_t \partial s + \frac{1}{2} \rho C_{Dt} A_t |V_t| V_t \right) \quad (3)$$

$C_m$  was the added mass coefficient,  $C_d$  was the drag coefficient,  $V_n$  the relative normal velocity and  $\vec{\tau}$  was a unit vector along the n-axis.  $a$  and  $V$  were wave particle acceleration and velocity respectively and  $r_s$  was structure velocity. The last term in equation (2) described Rainey's normal axial divergence correction in which  $v$  is the velocity gradient matrix. The tangential force was determined by integrating the hydrodynamic pressure on the bottom surface  $S_B$  and the drag force parallel to the centerline.  $A_t$  was the projected area of the spar in the plane parallel to the centerline and  $C_{Dt}$  was the drag coefficient of the heave.  $V_t$  was the relative velocity at the bottom of the spar with respect to the ambient fluid, which was parallel to the centerline.  $g^1$  and  $g^2$  were the first and second potential of incident waves. Forces  $F_{EXn}$  and  $F_{EXt}$  were transferred into spaced-fixed coordinate system  $oxz$  as:

$$\begin{Bmatrix} F_{EXx} \\ F_{EXz} \end{Bmatrix} = \begin{Bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{Bmatrix} \begin{Bmatrix} F_{EXn} \\ F_{EXt} \end{Bmatrix} \quad (4)$$

In time domain, the equation of motion was solved using numerical integration technique incorporating all the time dependent nonlinearities, stiffness coefficient changes due to mooring line tension with time, added mass from Morison equation, and with evaluation of wave forces at the instantaneous displaced position of the structure. At each step, the force vector was updated to take into account the change in the mooring line tension. The equation of motion was solved by an iterative procedure using unconditionally stable Newmark Beta method and this is programmed using MATLAB.

### 3. APPLICATION

The methodology presented above was applied to determine the motions of a large diameter classic spar (JIP), and a typical truss spar. The particulars of these spars are given in Table 1. Different environmental conditions as outlined in Table 2 were used to determine the responses.

Table 1. Main Particulars of Spars

Spar particulars	Diameter (m)	Draft (m)	Mass (with entrapped water) (Kg)	Moment of Inertia (pitch) (Kg.m <sup>2</sup> )	Center of gravity (from SWL) (m)	Mooring line stiffness (KN/m)
Classic Spar	40.54	198.12	2.592x10 <sup>8</sup>	1.007x10 <sup>12</sup>	- 105.98	191- 406
Truss Spar	32.31	131.062	53.926x10 <sup>6</sup>	1.0854x10 <sup>11</sup>	81.7	191- 406

Table 2. Environmental Conditions

Case	LC1	LC2	LC3
Description	Regular waves, H=6m, T=14s, No current.	Regular waves, H=6m, T=14s, Uniform current, 0.5m/s.	Random waves, Hs=13m, To=14s

#### 4. RESULTS AND DISCUSSION

The responses of the spar platform due to regular waves were determined first. All responses presented in this study were at the C.G. of the spar. Experimental and numerical results for this spar under similar conditions have been presented by Mekha et al. [5], for classic platform motions measured at 55m above SWL. These experimental results were compared with numerical results of this study. Figures. 2, 3 and 4 show the surge, heave and pitch responses in LC1 case. It was shown that truss spar (TS) responses in surge and pitch were only a little higher than those predicted for classic spar (CS). But, the heave response was much higher than that for CS, due to the shallower hard tank of TS.

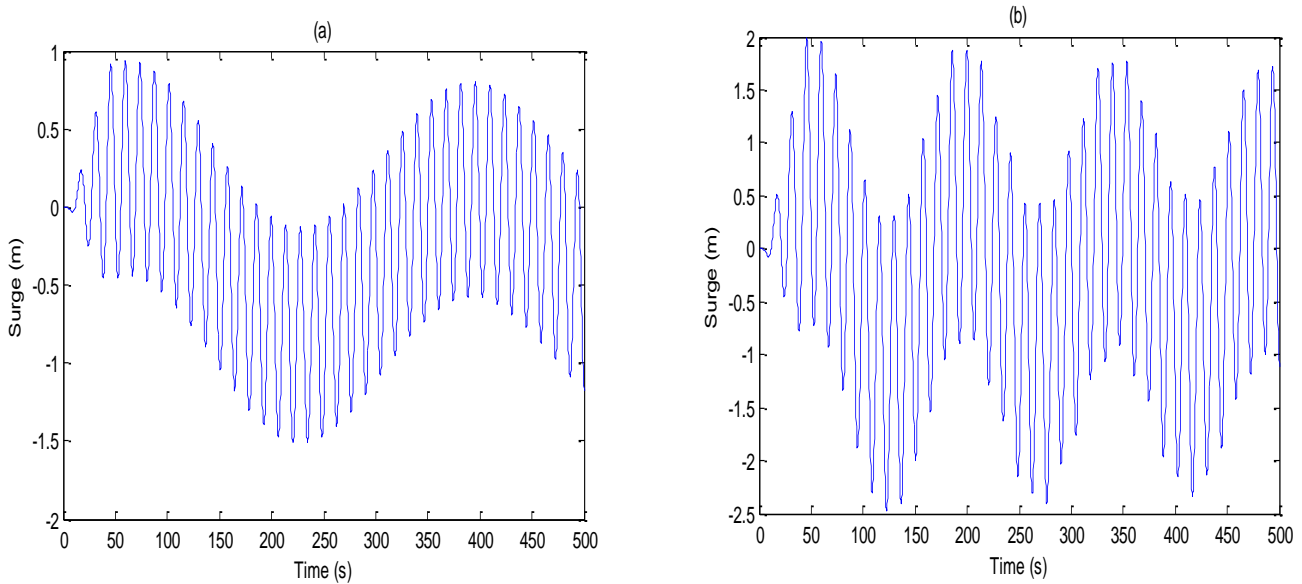


Figure 2. Surge response in regular waves (LC1)

(a) Classic Spar  
(b) Truss Spar

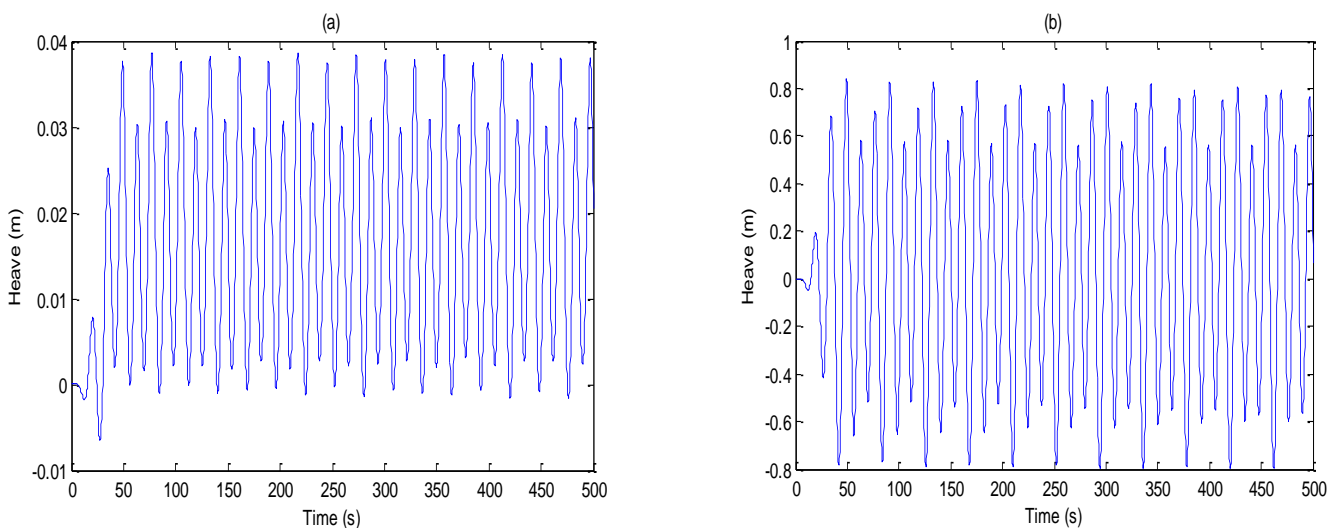


Figure 3. Heave response in regular waves (LC1)

(a) Classic Spar  
(b) Truss Spar

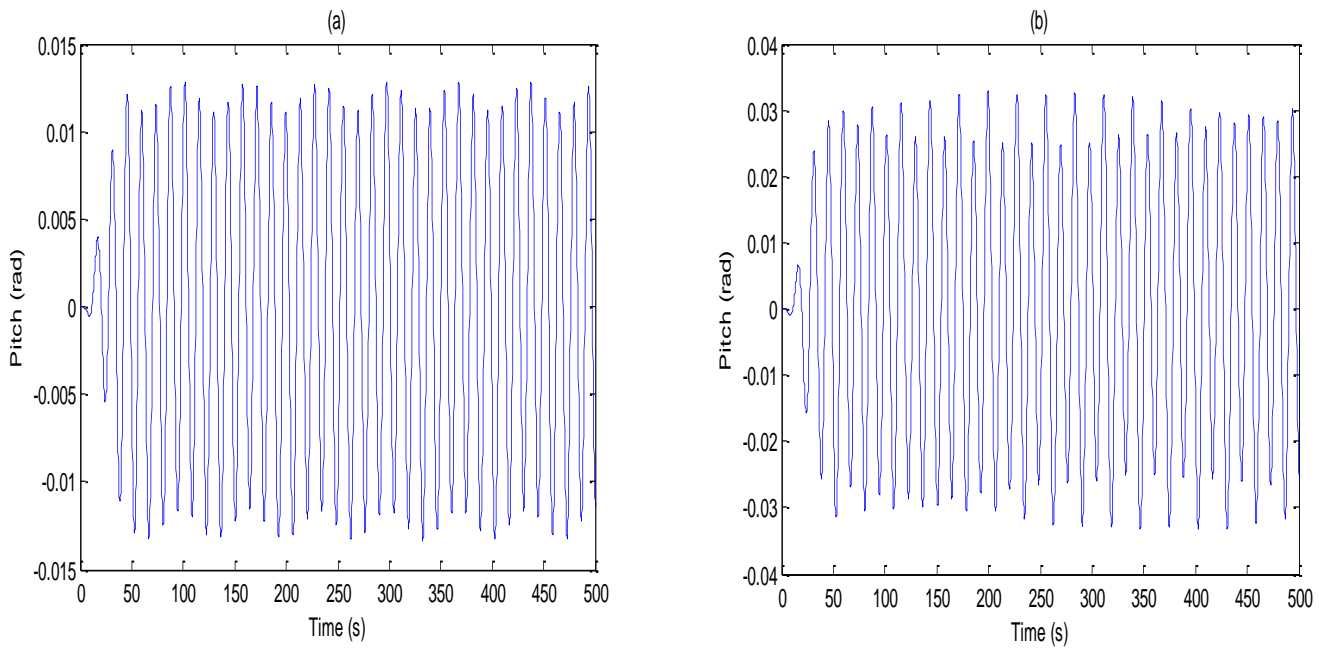


Figure 4. Pitch in regular waves (LC1)  
 (a) Classic Spar  
 (b) Truss Spar

Figures 5, 6 and 7 show the surge, heave and pitch responses due to regular waves and current acting together (LC2). It was shown that the inclusion of current affected surge response only. However, CS was more affected by current than TS.

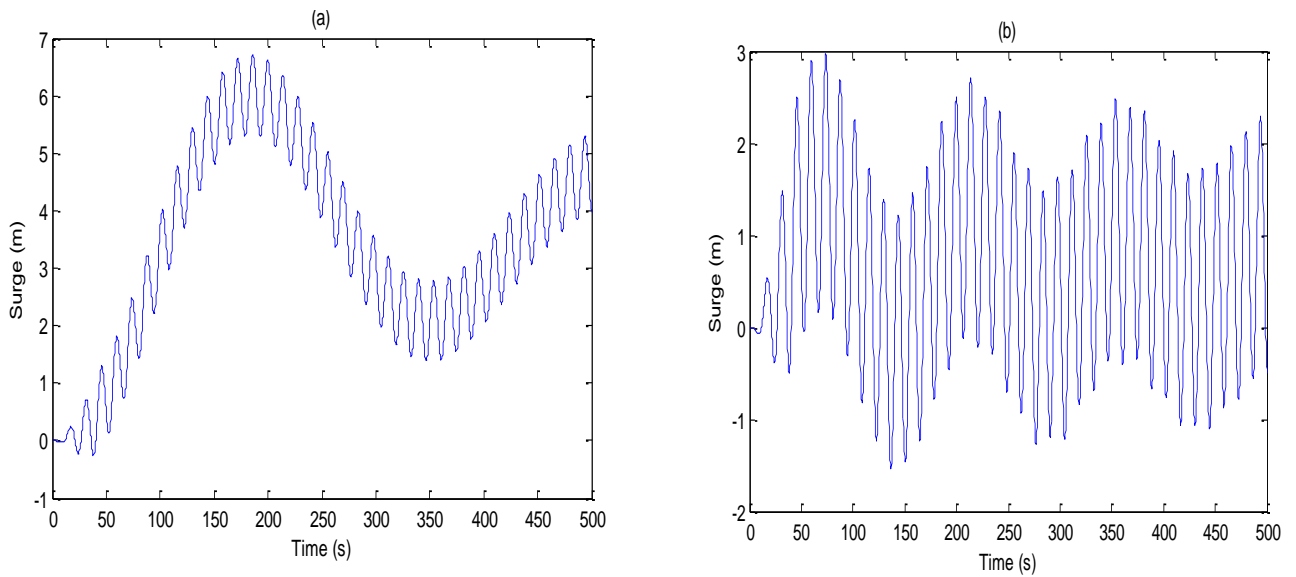


Figure 5. Surge response in regular waves (LC2)  
 (a) Classic Spar  
 (b) Truss Spar

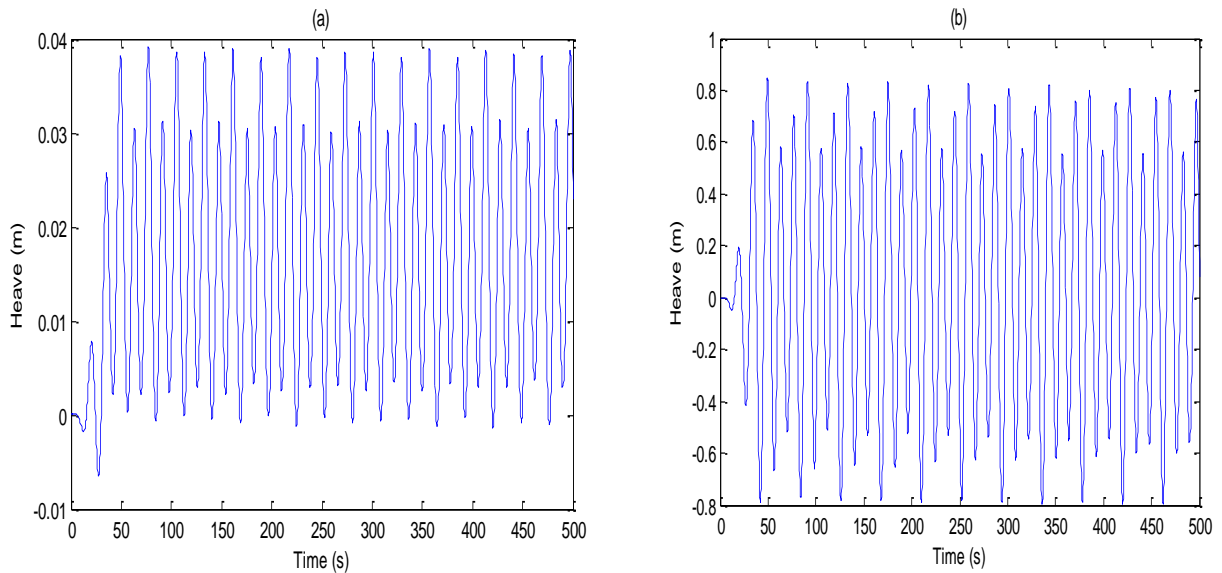


Figure 6. Heave response in regular waves (LC2)

(a) Classic Spar

(b) Truss Spar

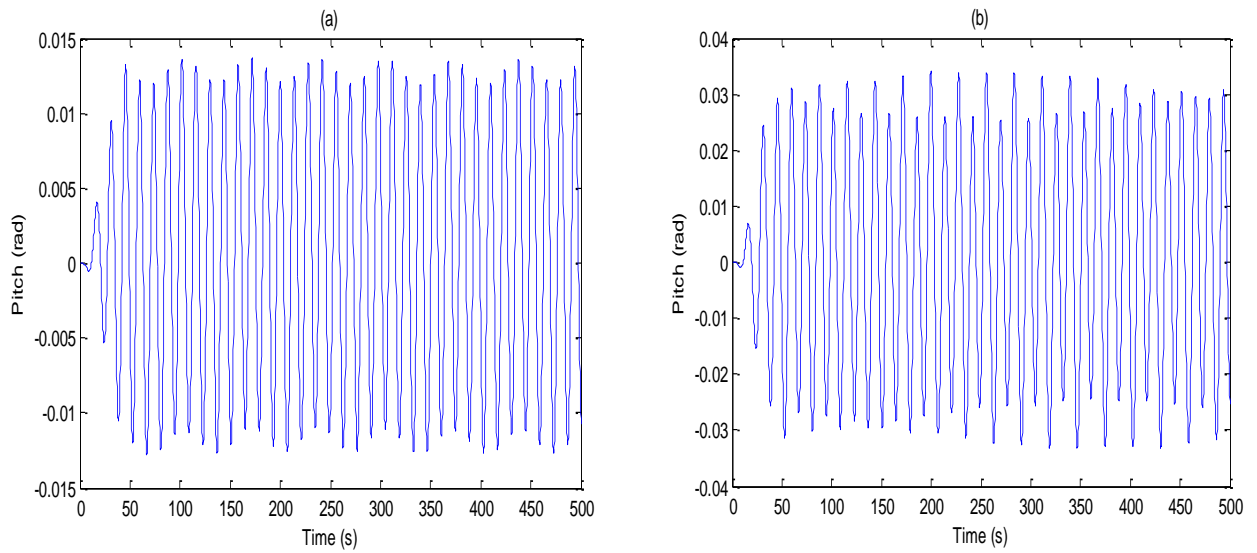


Figure 7. Pitch in regular waves (LC2)

(a) Classic Spar

(b) Truss Spar

JONSWAP spectrum was used for wave simulation in the case of random waves for LC3 because it was more versatile and represented the spectral peaks better than PM spectrum. RAO for surge, heave and pitch (LC3) for CS and TS are shown in figures.8, 9 and 10. It was shown that TS responses were greater than those for CS.



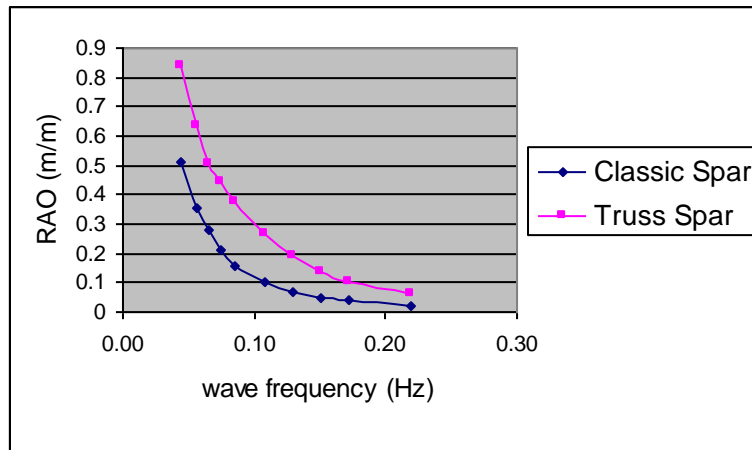


Figure 8. Surge response in random waves (LC3)

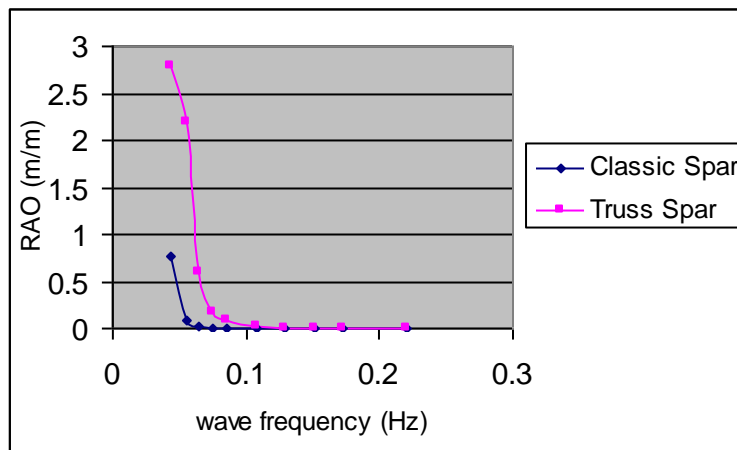


Figure 9. Heave response in random waves (LC3)

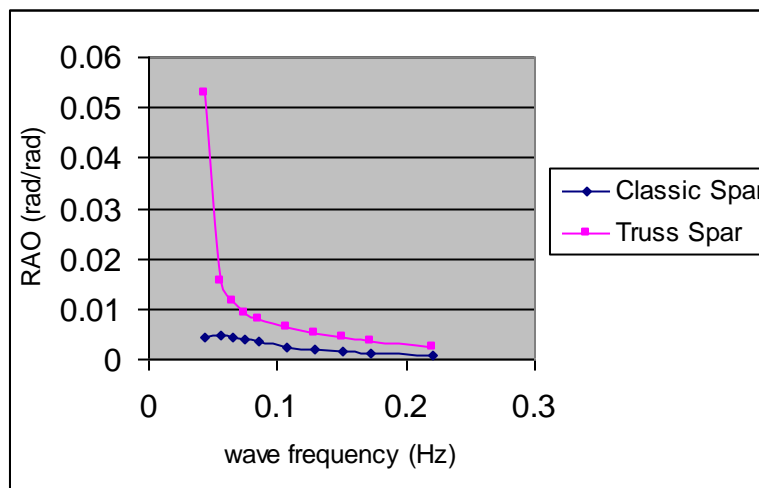


Figure 10. Pitch response in random waves (LC3)

## 5. CONCLUSION

1. The nonlinear responses of a spar platform (classic and truss spars) under different environmental conditions such as regular, random waves and current have been determined using a time-domain simulation model. The model can consider several non-linear effects and the complete non-linear rigid body equations of motion have been solved in the time domain. Hydrodynamic forces and moments were computed using modified Morison equation combined with Chakraborti's Stretching Formula to predict the wave particle kinematics. The program has been able to obtain results having trends comparable with literature results.
2. Classic spar gave lower responses compared to truss spar when subjected to regular waves alone. But, when the waves were combined with deep ambient current, truss spar gave lower response in surge, which is a very important criterion for design. In such cases, a truss spar is an attractive alternative.
3. When subjected to random waves, the truss spar gave higher responses compared to classic spar. However, as the truss spar is of much lower cost and as the responses are within the allowable limits for the platform design, truss spar is increasingly becoming popular nowadays.

## Acknowledgement

The authors would like to gratefully acknowledge the management and Civil Engineering Department of Universiti Teknologi PETRONAS (UTP), for their constant support and encouragement.

## REFERENCES

- [1] Vardeman RD, Richardson S, McCandless CR. Neptune Project: Overview and Project management. OTC 8381, Offshore Technology Conference, Houston, TX, USA. Vol. 2, 1997. p. 227-35.
- [2] Weggel DC, Rosset JM. Second order dynamic response of a large spar platform: numerical predictions versus experimental results. OMAE'96, Florence, Italy. Vol. 1, Part A, 1996. p. 489-96.
- [3] Ran Z, Kim MH, Niedzwecki, JM, and Johnson, RP. Responses of a spar platform in random waves and currents (Experimental vs. Theory). Int. J Offshore and Polar Eng., ISOPE, Vol. 6, No 1, 1996. pp 27-34.
- [4] Ran Z, Kim MH, Zheng W. Coupled dynamic analysis of a moored spar in random waves and currents (time-domain vs. Frequency-domain analysis). OMAE'98, Lisbon, Portugal. Vol. 1, 1998.
- [5] Chitrapu AS, Saha S, Salpekar VY. Time-domain simulation spar platform response in random waves and current. OMAE'98, Lisbon, Portugal. Vol. 1, 1998.
- [6] Mekha, B.B., Johnson, Roesset, J.M. Nonlinear response of spar in deep water: Different hydrodynamic and structural models. In: Proceeding of the Fifth International Offshore and Polar Engineering Conference Vol.3, 1995. Pp.462-469.
- [7] Mekha BB, Weggel D, Johnson CP, Roesset JM. Effects of second order diffraction forces on the global response of spars. ISOPE'96, Los Angeles, USA. Vol. 1, 1996. p. 273-80.

- [8] Manners W, Rainey RCT. Hydrodynamic-forces on fixed submerged cylinders. Proc R Soc London Ser A – Math Phys Engng Sci 1992; 436(1896):13-32.
- [9] Glanville, S, Pauling, JR, and Halkyard, JE. Analysis of the Spar Floating Drilling, Production and Storage Structure. Proc. Offshore Tech. Conf., Houston, 1991. OTC 6701.
- [10] Cao, P, and Zhang, J. Slow motion responses of compliant offshore structures. Proc 6<sup>th</sup> Int. Offshore and Polar Eng. Conf. Los Angeles. ISOPE, Vol.1. 1996. pp 296-303.
- [11] Kim, MH, Roesset, JM and Zhang, J. Nonlinear dynamic analysis methods for spar platforms. Proc. SNAME Conf. (Gulf Section), Houston 1997.
- [12] Halkyard, JE. Status of spar platforms for deep water production systems. Proc. 6<sup>th</sup> Int Offshore and Polar Eng. Conf, Los Angeles, ISOPE, Vol. 1. 1996. pp 262-272.
- [13] Rainey RCT. New equation for calculating wave loads on offshore structures. J Fluid Mech. 1989; 204:295-324.
- [14] Rainey RCT. Slender-body expression for the wave load on offshore structures. Proc. R Soc. London Ser A – Math Phys Sci 1995; 450 (1939):391-416.