# Model Tests for the Effect of Random Waves on the Hydrodynamic Forces on Rigid Vertical Cylinders

V. J. Kurian[1], A. M. Al-Yacouby[2], M. S. Liew[3]
1. Professor, 2. PhD. Candidate, 3. Associate Professor,
Department of Civil Engineering, Univesiti Teknologi PETONAS
31750 Tronoh, PERAK, Malaysia

*Abstract*— **Evaluation of accurate hydrodynamic forces on slender pipes has great significance in designing offshore structures. For the analysis and design of jacket platforms made up of small diameter tubular members, consultants use Morison's equation, where the wave forces are calculated using drag ($C_D$) and inertia ($C_M$) coefficients. The hydrodynamic coefficients used by consultants are constant values, although $C_D$ and $C_M$ are dynamic coefficients that depend on Keulegan Carpenter number (KC) and Reynolds's number (Re). This paper provides the preliminary results of model tests on rigid vertical cylinders with different pipe diameters subjected to random waves. The experimental data are derived from 1:70 scale model tests in which the hydrodynamic forces acting on individual tubular pipes were investigated in the wave tank. The hydrodynamic forces on the model were measured using aluminium block load sensors. The model pipe diameters used for the preliminary tests were 22 mm, 34 mm and 42 mm, and the effects of random waves on the magnitude of prototype hydrodynamic forces were analyzed in terms of power spectral density.**

*Keywords* — **Dynamic responses, Inertia coefficient, Drag coefficient, Malaysian locations, Morison equation.**

## I. Introduction

The dynamic response analysis of a single pipe and array of circular cylinders subjected to random waves is a very complex phenomena. Some parameters that govern the response include Reynolds number (Re), Kuleugan Carpenter number (KC), cylinder arrangement and incoming flow characteristics [1]. During the last decades, several successful experimental works pertaining to hydrodynamic forces acting on a single cylinder have been conducted by several researchers. The pioneering research on hydrodynamic force acting on slender pipe was first published by [2]. In that paper, the authors conducted experimental investigation to estimate the wave forces on slender pipe. They suggested that the force exerted by unbroken surface wave on a vertical cylinder pile which extends from the bottom through the free surface is composed of drag and inertia forces which can be estimated using the well-known Morison equation. There are several comprehensive reviews of the flow around a single, isolated tubular cylinder in the literature, including articles by [3], [4],[5], [6], [7], and [8]. In addition, Chakrabarti et al.[9] conducted experiment on inclined pipes. The study revealed that the parameters which influence the forces due to waves on a cylinder near a sloped boundary are the local maximum water particles velocity at the cylinder, $U_m$, the wave period, $T$, the cylinder diameter, $D$, the gap between the cylinder and the bottom $\ell$, the boundary slope, $\theta$, the boundary layer thickness, $\tau$, the mass density of the fluid, $\rho$ and the kinematic viscosity, $\nu$. In another study, Troesch and Kim [10] have studied the hydrodynamic forces acting on cylinders oscillating at small amplitude of a circle, rounded square, and square, both theoretically and experimentally. An extensive experimental investigation of hydrodynamic forces acting on production risers consisting either of a single pipe or of a larger pipe surrounded by the ring of smaller pipes was conducted by [11]. The tests were made with models to scale 1:2 exposed to steady currents, flow of constant acceleration, oscillating flow and oscillating flow in combination with a steady current, either parallel or at right angle. Recent studies on hydrodynamic forces acting on circular pipes include[12] and [13]. Regarding the fluid-cylinder interaction of two circular cylinders in tandem arrangements subjected to hydrodynamic loadings, several experimental works have been done over the last years, for instance [14], [15], and [16] for arrays of vertical cylinders. However, there is no literature addressing the issue of determination of accurate hydrodynamic coefficients for Malaysian offshore locations experimentally in the wave tank.

## II. NUMERICAL FORMULATIONS

### *A. Calculation of Hydrodynamic Forces*

The determination of wave forces exerted on structures is very complex. Morison's equation is generally used for estimating the external viscous hydrodynamic forces on tubular cylinders subjected to wave loadings. Two types of fluid forces result from fluid motions i.e. drag and inertia forces. Wave load per unit length is estimated using Equation 1.

$$F = C_M \frac{\rho \pi D^2}{4}\frac{du}{dt} + C_D \frac{\rho D}{2}|u|u \qquad (1)$$

where $C_M$ and $C_D$ are the hydrodynamic inertia and drag coefficients, $\rho$ is the water density, $D$ is the pipe diameter, $u$ is the water particle's velocity, and $\frac{du}{dt}$ is the water particle's acceleration.

*B. Airy Wave Theory*

The surface elevation of an airy wave of amplitude $A$ at any instant of time $t$ and horizontal position $x$ in the direction of travel of wave is denoted by $\eta(x,t)$ and is given by:

$$\eta(x,t) = A\cos(kx - \omega t) \qquad (2)$$

Where wave number $k = 2\pi / L$ in which $L$ represents the wave length, and circular frequency $\omega = 2\pi / T$ in which $T$ represents the period of the wave. The celerity or speed of the wave is given by $L/T$ or $\omega / k$ and the crest to trough wave high is given by $2A$. The Airy wave definition diagram is depicted in Fig. 1.

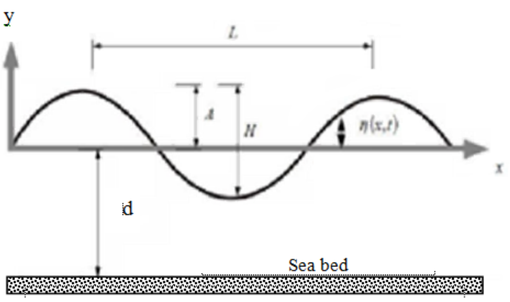


Fig 1: Airy Wave Definition Diagram

The along wave $u(x,t)$ and vertical $v(x,t)$ water particle velocity in an Airy wave at position Y measured from the Mean Water Level in depth of water $d$ are given by:

$$u = \frac{\pi H \cosh ks}{T \sinh kd} \cos\theta \qquad (3)$$

$$v = \frac{\pi H \sinh kx}{T \sinh kd} \sin\theta \qquad (4)$$

$$\dot{u} = \frac{2\pi^2 H \cosh ks}{T^2 \sinh kd} \sin\theta \qquad (5)$$

$$\dot{v} = -\frac{2\pi^2 H \sinh ks}{T^2 \sinh kd} \cos\theta \qquad (6)$$

In which $s = y + d$, $\theta = ks - \omega t$, $y$ is the height of the point of elevation of water particle kinematics, $x$ is point of evaluation of water particle's kinematics from the origin in the horizontal direction, and $t$ is the time instant at which the water particle's kinematics is evaluated, H is the wave height.

## III. Experimental Details

*A. Wave Tank Details*

The physical model experimental study was conducted in the wave tank 22 m long, 10 m wide and of 1m water depth. The wave generator is capable of generating regular, irregular waves and currents. It is also equipped with three movable remote control bridge platforms to support the testing personnel and equipment. The wave tank consists of multi-paddle wave maker capable of generating regular, random, bidirectional and multidirectional waves. In addition, the wave tank is also equipped with a dynamic wave absorber which is mainly designed to minimize the reflection of generated waves. In this study, the experimental model is fixed to the soffit of the bridge. Measurements for wave profile were carried out using wave probes, while aluminum block load sensors which are especially designed by the research group were used to measure the hydrodynamic forces acting on the model. The wave tank facility at University Teknologi PETRONAS is shown in Fig.2.

Fig 2: Wave Tank Facility in UTP

*B. Experimental Model*

During the preliminary experimental investigation of hydrodynamic forces on slender pipes, rigid tubular cylinders were vertically mounted on the overhead bridge. The tubular cylinders used are made of PVC having an outside diameter of $D_1$ = 22 mm, $D_2$ = 34 mm and $D_3$ = 42 mm and a wall thickness of 1.5 mm for 22 mm pipe and 2.5 mm for 34 mm and 42 mm diameter pipes respectively. The immersed length of the model was L=850 mm, giving an aspect ratio of $L/D_1$ = 38.6, $L/D_2$ = 25 and $L/D_3$ = 20.2 and displaced water mass of $m_{D1} = \pi D^2 L / 4$ = 323.1 g, $m_{D2}$ = 323.1 g and $m_{D3}$= 489.6 g.

*TABLE 1: PROPERTIES OF THE MODEL AND THE PROTOTYPE*

| Model | | | Prototype | | |
|---|---|---|---|---|---|
| Pipe Diameter (m) | Pipe Length (m) | Wall Thickness (m) | Pipe Diameter (m) | Pipe Length (m) | Wall Thickness (m) |
| 0.022 | 1.23 | 0.0032 | 1.54 | 86.1 | 0.224 |
| 0.042 | 1.23 | 0.0025 | 2.94 | 86.1 | 0.175 |
| 0.048 | 1.23 | 0.0025 | 3.36 | 86.1 | 0.175 |

The pipe diameters used during the model test were selected carefully to represent small tubular members for jacket platforms and other structural elements such as conductors and risers which are normally used in offshore oil and gas industry. Fig. 3 shows a typical arrangement of tubular cylinder subjected to waves. Initially, single pipes with different diameters were subjected to random waves, then, to investigate the effects of wave spectrum on the hydrodynamic forces, regular waves with a fixed model wave height of 0.2 m and wave frequencies varying from 0.033 Hz and 1 Hz, were compared with that due random waves generated in the wave tank using Pierson-Moskowitz (P-M) and JONSWAP spectrum were generated in the wave tank, and the scaled up hydrodynamic forces for the prototype were analyzed in terms of power spectral density. The physical properties of the model and the prototype are shown in Table 1.

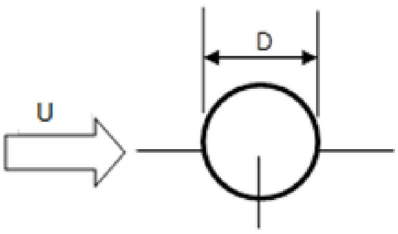


Fig 3: Schematic Diagram for a Tubular Cylinder Subjected To Waves

## C. *Experimental Set up and Procedure*

The model test arrangement consists of single tubular pipes and arrays of circular cylinders fixed on top while the bottom ends of the pipes are free. The top side of pipe is connected to an aluminum block load sensor which was designed and fabricated in the offshore laboratories at UTP. The load sensor is designed in such a way that it measures the resultant hydrodynamic forces on each pipe. A aluminum block of 100 mm (H) x 50 mm (W) x 2 mm thick was fabricated using structural aluminum type 6061with ultimate tensile strength of 310 MPa. In addition, four units 120 Ω general purpose strain gauges, type KFG-3-120- C1-23L1M2R, 3 mm gauge were fixed to the aluminum block to form a load sensor. The connection of the strain gauges to the data acquisition unit was in accordance with Wheatstone full bridge configuration. The aluminum block load sensor along with the strain gauges configuration is shown in Fig. 4. The design of test specimens was in accordance with ASTM E74 standard (ASTM E74, 2006). As the strain resulting from wave forces are dynamic and change over time, dynamic data loggers were used to capture the dynamic characteristics of the model responses. Smart dynamic strain recorder type DC-204R was used for the data acquisition. The data logger has a maximum sampling speed of. 5 μ-sec./channel, upgraded with 10 kHz frequency response and 200 kHz sampling.

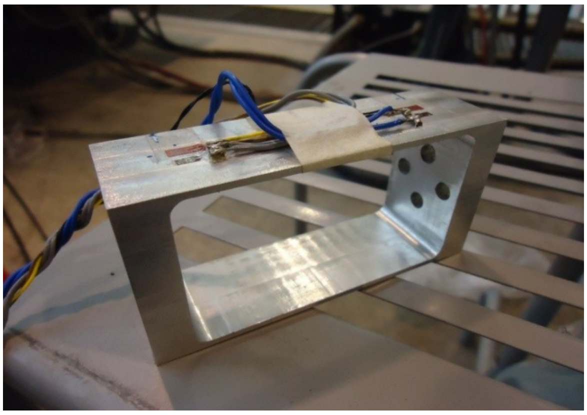

Fig 4: A typical Aluminum Block Load Cell

## D. *Wave Tank Loading Conditions*

The wave tank data shown in Table 2 were programmed and generated. Three different types of sea states that consist of random waves were generated in the wave tank. The model responses were then post-processed and the findings are presented and discussed in section 4.

TABLE 2: DIFFERENT SEA STATES GENERATED IN THE WAVE TANK FOR MODEL TEST

| Model | | | Prototype | | |
|---|---|---|---|---|---|
| Significant Wave Height (m) | Wave Period (s) | Wave Frequency (Hz) | Significant Wave Height (m) | Wave Period (s) | Wave Frequency (Hz) |
| 0.06 | 0.83 | 1.21 | 4.20 | 6.91 | 0.14 |
| 0.04 | 0.78 | 1.29 | 2.80 | 6.49 | 0.15 |
| 0.07 | 1.02 | 0.98 | 4.90 | 8.54 | 0.12 |

### *E. Scaling Law*

The most common dimensionless scaling law for the fluid structures test is the Froude's Law. Using Froude's law and the scale factor as $\lambda$ , the suitable multipliers to be used to obtain the prototype values from the model test are depicted in Table 3[18]. Assuming a model scale of $\lambda$ and geometric similarity, the, Froude's model must satisfy the relationship shown in Equation 7.

$$\frac{u_p^2}{gD_p} = \frac{u_m^2}{gD_m} \tag{7}$$

Where $u$ is the fluid velocity, $D$ is the dimension of the structure, $g$ is the gravitational acceleration, and the subscripts $p$ and $m$ stand for prototype and model respectively.

In this study a scale factor of 70 was chosen, as the maximum water depth in the wave tank is 1m, while the study area has actual prototype water depth varying from 70 to 75 m.

TABLE 3: SCALING FACTOR AND SCALING RATIO FOR THE MODEL TEST

| Physical parameters | Scale factor | Scaling Ratio for Scale Factor $\lambda = 70$ |
|---|---|---|
| Length | $\lambda$ | 70 |
| Volume | $\lambda^3$ | 343,000 |
| Time | $\lambda^{1/2}$ | 8.367 |
| Velocity | $\lambda^{1/2}$ | 8.367 |
| Acceleration | $\lambda^0$ | 1 |
| Force | $\lambda^3$ | 343,000 |
| Wave Period | $\lambda^{1/2}$ | 8.367 |

### *F. Wave Profiles Measurement*

Five wave gauges type EQ-009 R4 with active length of 900 mm and wave probe diameter of 6 mm were used to measure the rapidly changing water levels during the physical model test in the wave tank. The wave probes were calibrated regularly throughout the test to ensure that generated waves in. the wave tank are in good agreement with the theoretical values

Before installing the model in the wave tank, five wave probes have been fixed in the wave tank; one was fixed exactly at the virtual neutral axis of the model and two more units were installed at each side of the model i.e. before and after the model. The centre to centre spacing between the probes was set to be 1300 mm, and the dynamic variations of the wave profile were measured for different loading conditions.

### *G. Preliminary Model Test*

The preliminary model test was conducted in the wave tank using a single circular pipe as shown in Fig 5. The water depth in the wave tank was kept constant i.e. 1m, as this is the maximum allowable water depth in the wave tank.

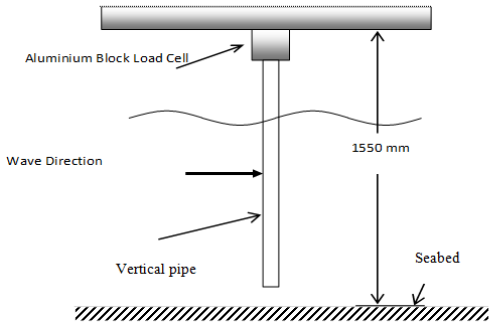


Fig 5: Model Specimen Details

## IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

### *A. Wave Amplitudes*

A typical time domain records for the wave amplitude generated in the wave tank for irregular wave with prototype significant wave height $H_s$ = 4.2 m and frequency, $f$ = 0.145 Hz is shown in Fig. 6, whilst the corresponding wave spectrum for the same wave is presented in Fig. 7.

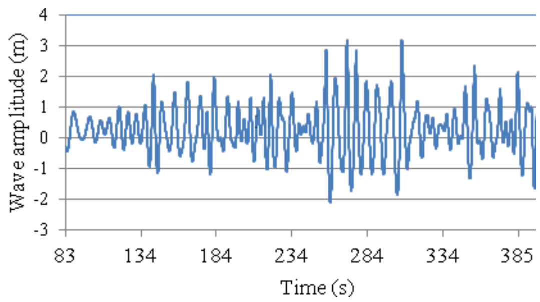


Fig 6: Measured Wave Amplitude

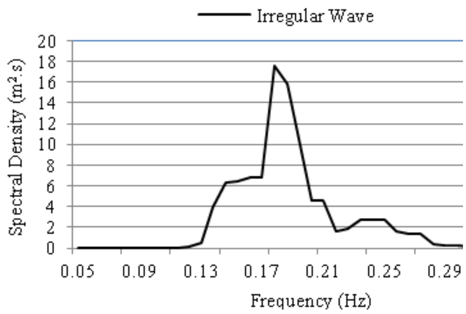


Fig 7: Wave Profile Spectrum

The actual wave profiles generated in the wave tank were measured with and without the model. The measured data was then post-processed to quantify the influence of the model on the wave patterns. However, it could be observed that the effect of the model on the wave profiles was found to be negligible as the ratio between the pipe diameter (D) and the wave length (L) was relatively small i.e. $D/L < 0.2$ .

## *B. Effects of Pipe Diameters on Hydrodynamic Forces*

Fig. 8 to 10 show examples of time series of measured hydrodynamic forces for model pipes with outer diameter of 22 mm, 34 mm and 42 mm. The corresponding pipe diameter for the prototype are 1.54 m, 2.94 m and 3.36 m respectively.

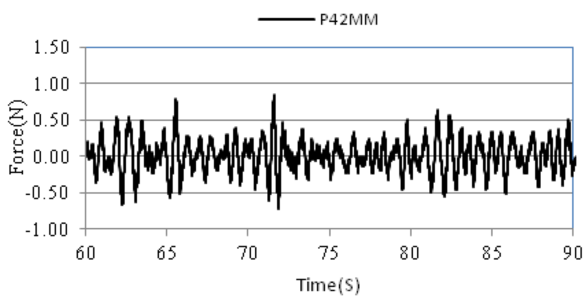


Fig 8: Measured Time Series of Hydrodynamic Forces for 42 mm Diameter Model Pipe

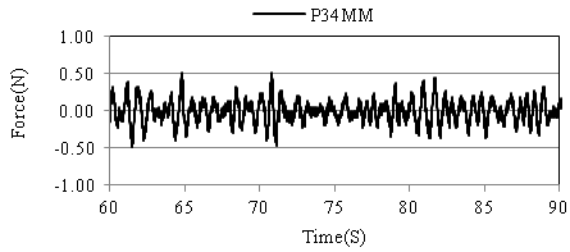


Fig 9: Measured Time Series of Hydrodynamic Forces for 34 mm Diameter Model Pipe

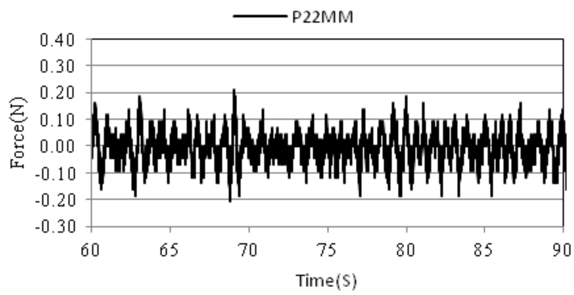


Fig 10: Measured Time Series of Hydrodynamic Forces for 22 mm Diameter Model Pipe

Fig. 11 shows comparison of power spectral density for measured hydrodynamic forces on prototype tubular cylinders of 1.54 m, 2.94 m and 3.36 m diameters subjected to random waves, with significant wave height Hs = 4.2 m and zero crossing frequency, $f_z = 0.144$ Hz. The waves were generated in the wave tank using a scale factor of 1:70. Here, it can be observed that the peak values of energy density spectrum for the different pipe diameters were recorded between wave frequencies of 0.17Hz and 0.21Hz.

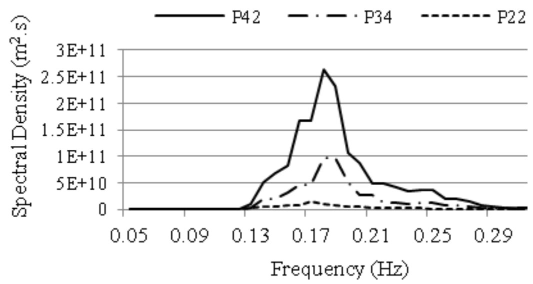


Fig 11: Power Spectral Density for Hydrodynamic Forces on Prototype Tubular Pipes with Different Diameters subjected to Random Waves

Generally, the power spectral density values are proportional to the pipe diameters. Larger pipe diameters resulted in higher peak values of power spectral density, which is in good agreement with Morison equations hydrodynamic forces on tubular pipes. Between wave frequency 0.17 and 0.21 the peak values for power spectral density of 42 mm pipe (3.36 m ) was higher than that of 34 mm pipe (2.94 m) and 22 mm pipe (1.54 m) by 60% and 98% respectively.

Using the measured hydrodynamic forces on tubular cylinders and the theoretical values for hydrodynamic forces estimated using Morison equation, accurate hydrodynamic coefficients $C_M$ and $C_D$ values can be determined for the Malaysian offshore locations. However, due to space limitations, only the effects of random waves on hydrodynamic forces in term of power spectral density have been discussed in this paper. $C_M$ and $C_D$ values will be published later in a separate report.

## V. CONCLUSIONS

Experimental investigation was conducted in the wave tank to determine the effects of random waves on hydrodynamic forces of tubular pipes. From the model tests, the flowing conclusions can be drawn for the prototype:

1. During the investigation of the effect of random waves on the hydrodynamic forces on rigid vertical cylinders it was observed that between wave frequency 0.17Hz and 0.21Hz, the peak values for power spectral density of 42 mm pipe (3.36 m ) was higher than that of 34 mm pipe (2.94 m) and 22 mm pipe (1.54 m) by 60% and 98% respectively.

2. Generally, the effect of pipe diameters on hydrodynamic forces are in good agreement with Morison equations as larger pipe diameters produced comparatively higher power spectral density.

## ACKNOWLEDGMENT

The authors would like to gratefully acknowledge their gratitude to Universiti Teknologi PETRONAS for support and encouragement.

## REFERENCES

[1] Prastianto, R.W., K. Otsuka, and Y. Ikeda. Hydrodynamic Forces on Multiple Hanging-off Circular Cylinders in Uniform Flows. in Proc. of the 18th International Offshore and Polar Engineering Conference (ISOPE), Vancouver, British Columbia, Canada. 2008.

[2] Morison, J.R., J. Johnson, and S. Schaaf, The force exerted by surface waves on piles. Journal of Petroleum Technology, 1950. 2(5): p. 149-154.

[3] Gerrard, J., The wakes of cylindrical bluff bodies at low Reynolds number. Philosophical Transactions for the Royal Society of London. Series A, Mathematical and Physical Sciences, 1978: p. 351-382.

[4] Sarpkaya, T. and F. Rajabi. Hydrodynamic drag on bottom-mounted smooth and rough cylinders in periodic flow. in Offshore Technology Conference. 1980.

[5] Loken, A., et al., Aspects of hydrodynamic loading in design of production risers. Journal of Petroleum Technology, 1980. 32(5): p. 881-890.

[6] Sabuncu, T. and S. Calisal, Hydrodynamic coefficients for vertical circular cylinders at finite depth. Ocean Engineering, 1981. 8(1): p. 25-63.

[7] Coutanceau, M. and J.-R. Defaye, Circular cylinder wake configurations: A flow visualization survey. Applied Mechanics Reviews, 1991. 44: p. 255.

[8] Williamson, C., Vortex dynamics in the cylinder wake. Annual review of fluid mechanics, 1996. 28(1): p. 477-539.

[9] Chakrabarti, S., K and D. Cotter, C. Hydrodynamic coefficients for a sloped Cylider in Cross Waves. in 19th Annual OTC 1987. Houston, Texas: Offshore Technology Conference-OTC.

[10] Troesch, A.W. and S. Kim, Hydrodynamic forces acting on cylinders oscillating at small amplitudes. Journal of Fluids and Structures, 1991. 5(1): p. 113-126.

[11] N-E. Ottsen, H., V. Jacobsen, and H. Lundgren. Hydrodynamic Forces on Composite Risers and Individual Cylinders. in Offshore Technology Conference. 1979. Houston, Texas: lnst. of Hydrodynamics and Hydraulic Engineering.

[12] Kinoshita, T., B. Weiguang, and S. Shunj, The hydrodynamic forces acting on a cylinder array oscillating in waves and current. Journal OF marine sicence and Technology(2), 1997: p. 135-147.

[13] Spyros, A., M, et al. Experimental And Numerical Investigation of The Hydrodynamic Loads And Wave Elevation on Concentric Vertical Cylinders. in The HYDRALAB III Joint User Meeting,. 2010. Hannover: The HYDRALAB III Joint User Meeting,.

14. Brika, D. and A. Laneville, Wake interference between two circular cylinders. Journal of wind engineering and industrial aerodynamics, 1997. 72: p. 61-70.

[15] Brika, D. and A. Laneville, The flow interaction between a stationary cylinder and a downstream flexible cylinder. Journal of Fluids and Structures, 1999. 13(5): p. 579-606.

[16] Kudeih, M., A. Cornett, and I. Nistor, An Experimental Study of Wave and Current-Induced Forces on a Compact Linear Array of Vertical Cylinders In Shallow Water. Coastal Engineering Proceedings, 2011. 1(32): p. waves. 54.

[17] Haritos, N., Introduction to the Analysis and Design of Offshore Structures - An Overview, 2006, The University of Melbourne: Melbourne, Austalia.

[18] Chakrabarti, S., K, Offshore Structure Modeling. 1994, London: World Scientific Publishing Co. Pte. Ltd.