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Estimation of Response Transfer Functions of Offshore Structures Using the Time-Varying ARX Model

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ABSTRACT

The purpose of this paper is to propose and investigate a new approach for estimating response transfer function of offshore structures with wave as excitation input. The approach is based on time-varying autoregressive with exogenous input (TVARX) model. This method is virtually unexplored in offshore engineering field, as a number of works have shown that transfer functions such as response amplitude operator are estimated based on discrete Fourier transform (DFT). Here, we outline a practical algorithm for TVARX model which uses expectation-maximization (EM) algorithm based on Kalman smoother to generate the transfer function. The method is then applied to sampled discrete wave as excitation input and the motion responses of offshore structures as output data, generated from simulated field measurements. The proposed approach outlined here has shown the tremendous potential in the estimation of transfer function. The results indicate that TVARX model produces accurate, smooth and less noise TF estimates over DFT method. TVARX model also allows for the creation of time varying transfer function (TVTF).

KEY WORDS: Motion responses; Kalman smoother; Time-varying ARX model; Transfer function.

INTRODUCTION

Generally, the motion responses of platforms and environmental conditions such as wind, wave and current are available in the form of field measurements, experimental and numerical data. The time history of such recorded data can be utilized to generate dimensionless form namely response functions (RFs) either in time or frequency domain, i.e. transfer function (TF) and response amplitude operators. This paper is addressed to generate TF from the available recorded data. This is motivated from the offshore monitoring campaign (Buchner, 2009; Boom, 2005) and physical model limitations (Chakrabarti, 1998) that transfer function can be used for modal analysis, dynamic response prediction, motion control systems design and damage detection of offshore platforms.

There are three transfer functions that can be generated from marine structures (Taghipour, 2008). The first is wave to force TF, second is force to motion TF and the last is wave to motion TF. This paper will work on the estimation of wave to motion TF due to the availability of data. In order to estimate the response transfer functions, DFT has been the most widely used technique. However, due to sea state exhibits nonstationary and nonlinearity, the method is not recommended anymore. It has been shown by prior researches such as (Huang, 1998; Hwang, 2003; Liu, 2000; Schlurmann, 2003). The non-stationarity and nonlinearity may also come from platform motion responses. The use of DFT method may affect the frequency response of offshore structures (Hwang, 2003).

To overcome the aforementioned condition, this paper utilizes the TVARX model to estimate the transfer function. This method is proposed in order to accommodate the non-stationarity and nonlinearity which come from either in waves or platform motion responses. It does not matter where the platforms located, either in shallow or deep water. Example is in deep water the waves can still be described as stationary and ergodic process; however the process will be non-stationary in extreme weather or else the non-stationary can come from platform motions itself. In other word, the proposed method is applicable in any kind of sea state as long as there are non-stationarity and nonlinearity in the given time series. This is because TVARX model is flexible method. If the time series is stationary, then the TVARX model can be switched to ARX model. Besides that, TVARX model also provides TVTF of analyzed system that may have never been carried out by the DFT method.

TIME-VARYING TRANSFER FUNCTION

By assuming that a system is linear and causal, general multivariable system with L inputs and P outputs is expressed in Eq. 1,

$$\begin{Bmatrix} y_1(t) \\ \vdots \\ y_P(t) \end{Bmatrix} = \begin{bmatrix} h_{11}(t) \cdots h_{1L}(t) \\ \vdots \quad \cdots \quad \vdots \\ h_{P1}(t) \cdots h_{PL}(t) \end{bmatrix} \begin{Bmatrix} u_1(t) \\ \vdots \\ u_L(t) \end{Bmatrix}. \tag{1}$$

There exist linear relationship between scalar excitations $u_1,\cdots,u_L$ and scalar responses $y_1,\cdots,y_P$, noted by TF in term of impulse response h_{PL}. Because we are concerned only with wave to motion TFs, Eq. 1 will be single input multi output system where u is ocean wave and $y_1,\cdots,y_P$ are motion responses of offshore structure. Each entry of h_{PL} is related with respective input (wave) and output (surge, heave, sway, roll, pitch and yaw) and can be approached with single-input single-output system. In the point of view of frequency domain, the transfer functions in Eq. 1 may be written as Eq. 2.

$$H_{LP}\left(e^{j\omega}\right) = \frac{S_{LP}\left(e^{j\omega}\right)}{S_L\left(e^{j\omega}\right)}. \tag{2}$$

Notation $H_{LP}\left(e^{j\omega}\right)$ denotes the TF from the input L to the output P signals. Term $S_{LP}\left(e^{j\omega}\right)$ is cross-spectrum of the input L and output P signals (wave and motion responses), while $S_L\left(e^{j\omega}\right)$ is auto-spectrum of the input L (wave) at particular frequency.

Conventionally, spectrum either wave or motion responses is carried out by the DFT to produce the TF. This method has been the standard technique used in frequency domain analysis for offshore engineering. One of the drawbacks of the DFT is that it does not provide any information about the time at which a frequency component occurs. Nonetheless, when the signals are nonstationary, then the DFT is not applicable anymore. If the spectrum either wave or motion response are presented in time-frequency distribution (TFD), then time-varying TF (TVTF) can be generated. It can be calculated by modifying equation (2) as follows:

$$H_{LP}\left(k, e^{j\omega}\right) = \frac{S_{LP}\left(k, e^{j\omega}\right)}{S_L\left(k, e^{j\omega}\right)}. \tag{3}$$

Term k indicates discrete time index and lead to the TVTF. The norm of $H_{LP}\left(k, e^{j\omega}\right)$ is TV gain of the system. Among many tools that can be used for time-frequency analysis, short time Fourier transform (STFT) and Wavelet transform are very popular methods. However, like other nonparametric approaches, STFT and Wavelet has resolution conflict in both frequency and time domain due to Heisenberg uncertainty principle. The best solution is to employ time-varying spectral analysis which is not affected by resolution conflict. Generally, parametric approaches are solution for such a case.

TVARX MODEL

TVARX model is an extended ARX model, but its coefficients are time-variant. As a parametric approach, TVARX model generate TVTF by modeling the signals as a time series. This realization enables to produce the zeros and poles of the system (roots of the characteristic polynomial) through TVARX coefficients. TVARX model in discrete time index k is given by Eq. 4. Notations P and M represent the order of TVARX model while $u(k)$ and $y(k)$ are the input and output signal, respectively.

$$y(k) = \sum_{i=1}^{P} a_i(k)\, y(k-i) + \sum_{\ell=0}^{M} b_\ell(k)\, u(k-\ell) + e(k). \tag{4}$$

Terms $a_i(k)$ and $b_\ell(k)$ are the TVARX coefficients and $e(k)$ is the driving noise which is Gaussian with zero mean and variance σ_e^2. From Eq. 4, it can be seen that identification of $a_i(k)$ and $b_\ell(k)$ is the main task by setting up the model order. Estimation of $a_i(k)$ and $b_\ell(k)$ can be computed through several methods, namely adaptive method and basis function approach (Sodsri, 2003). Until now, criteria for selecting the proper basis function is not available yet and still open research (Sodsri, 2003; Asutkar, 2010; Zhang, 2010), while adaptive method is very popular due to its simplicity, generality and ability of real time processing. Hence, adaptive method is addressed in this paper. To accommodate the use of adaptive method in estimation of $a_i(k)$ and $b_\ell(k)$, Eq. 4 must be converted into a measurement equation in vector notation as expressed in Eq. 5.

$$h(k) = C(k)x(k) + v(k). \tag{5}$$

Notation $C(k) = \left[y(k-1), \cdots, y(k-P), u(k-1), \cdots, u(k-M)\right]$ is the vector of the past measurements; vector $x(k) = \left[a_1(k), \cdots, a_P(k), b_1(k), \cdots, b_M(k)\right]^T$ is the array of TVARX coefficients and $v(k)$ is the measurement noise with covariance matrix R. By simplifying the TVARX coefficients evolve over a time linearly and first-order Gauss-Markov process, and then $x(k)$ can be expressed as state equation in Eq. 6.

$$x(k) = Ax(k-1) + w(k). \tag{6}$$

The term A in Eq. 6 is state transition matrix and $w(k)$ is the state noise with covariance matrix Q. Equations 5-6 represent TVARX models in a state-space form. Both equations contain model parameters which are assumed before the application of adaptive method. These parameters are initial conditions $x_0 \sim N(\mu_0, \Sigma_0)$, A, Q, R and denoted by $\theta = \{A, Q, R, \mu_0, \Sigma_0\}$. If some simplifications are introduced in Eq. 6, then the equation calls for two remarks:

1. If there is no state noise in the state equation and state transition matrix A is constrained to a scaled identity matrix, then Eq. 6 can be written in Eq. 7 where state variable depends on the choice of the forgetting factor λ, expressed as Eq. 7.

$$x(k) = \lambda^{1/2} x(k-1). \tag{7}$$

Equation 7 is called adaptive autoregressive (AAR) model and tuning parameter is λ and can be estimated with least mean square (LMS) or recursive least square (RLS) algorithm.

2. If state equation in Eq. 6 is modeled as a random-walk model, then it can be expressed as Eq. 8.

$$x(k) = x(k-1) + w(k). \tag{8}$$

Noise covariance matrix is constrained to an identity matrix: $Q = I^{pxp}\sigma_w^2$ where σ_w^2 is a noise state variance. The unknown parameter in random-walk model is σ_w^2.

Estimation of $x(k)$ in Eq. 7 using LMS and RLS algorithm had been investigated by (Sodsri, 2003). He revealed that the adaptive method under this class is sensitive to the noise and fails to track the systems with fast or broad frequency. Estimation of Eq. 8 was successfully carried out (Nguyen, 2009) using amplitude demodulation-Kalman smoother (AD-KS). However, both covariance matrix (Q and R) are set up manually. Further, the drawback of both models had been investigated by (Khan, 2007) and they found out that both impose constraints to reduce the number of tuning parameter and might deteriorate the performance of time-varying spectrum estimation. They proposed the use of Kalman smoother with EM algorithm as a solution because of its superiority. This paper addresses the use of Kalman smoother with EM algorithm and applied in TVTF estimation.

KALMAN SMOOTHER WITH EM ALGORITHM

Kalman smoother with EM algorithm is smoothed Kalman filter which is optimized with EM algorithm. Basic theory covers Kalman filter, smoothing equations and expectation-maximization of log-likelihood function. Kalman filter calculates the state estimate $(x(k|k), P(k|k))$ in Eq. 8 in two stages: time update equations (predictor) and measurement update equations (corrector). The time update equations project the state variable and state covariance matrix estimates forward from time index $k-1$ to k, written as Eq. 9 & Eq. 10.

$$x(k|k-1) = Ax(k-1|k-1) \tag{9}$$

$$P(k|k-1) = AP(k-1|k-1)A^T + Q. \tag{10}$$

The measurement update equations incorporate a new measurement into the a *priori* estimate to obtain an improved a *posteriori* estimate. The first step in measurement update equations is to compute Kalman gain as expressed in Eq. 11.

$$K(k) = P(k|k-1)C(k)^T\left(R + C(k)P(k|k-1)C(k)^T\right) \tag{11}$$

The next step is to compute an *a posteriori* state estimate $x(k|k)$ as a linear combination of an *a priori* state estimate $x(k|k-1)$, Kalman gain and weight difference between an actual measurement $y(k)$ and a measurement prediction $(C(k)x(k|k-1))$ as expressed in Eq. 12. The difference $(y(k) - C(k)x(k|k-1))$ is *residual.* The residual reflects the discrepancy between the predicted measurement $(C(k)x(k|k-1))$ and the actual measurement $y(k)$.

$$x(k|k) = x(k|k-1) + K(k)(y(k) - C(k)x(k|k-1)) \tag{12}$$

The final step is to obtain an *a posteriori* error covariance estimate via Eq. 13.

$$P(k|k) = \left(I - K(k)C(k)^T\right)P(k|k-1) \tag{13}$$

The use of procedures above will generate lagged response of $x(k|k)$ estimate. Smoothing equations can solve it by reducing delay and decreasing the variance of state estimates (Nguyen, 2009). Combination between Kalman filter and smoothing equations is called Kalman smoother. Because all input-output signals are processed off-line, then fixed-interval smoother is applied in this paper. This method has performance to improve the accuracy of the state estimate (Khan, 2007) and derived in Eqs. 14-16,

$$J(k-1) = P(k-1|k-1)A^T P(k|k-1), \tag{14}$$

$$x(k-1|K) = x(k-1|k-1) + J(k-1)(x(k|K) - Ax(k|k-1)), \tag{15}$$

$$P(k-1|K) = P(k-1|k-1) + J(k-1)(x(k|K) - x(k|k-1))J(k-1)^T. \tag{16}$$

EM algorithm is utilized to tune the model parameters θ based on maximum likelihood of $y(1:K)$ in the presence of hidden variables $x(k|K)$, $k = 1, \cdots, K$. EM algorithm consists of two steps. First step is calculation of the expected complete log-likelihood as a function of θ. The expected log-likelihood is expressed in Eq. 17.

$$F = \mathrm{E}\{log\ p(y(1:K), x(1:K)) | y(1:K)\}, \tag{17}$$

The expected likelihood depends on three quantities as stated in Eqs 18-20.

$$x(k|K) = \mathrm{E}\{x(k)|y(1:K)\}, \tag{18}$$

$$S(k|K) = \mathrm{E}\left\{x(k)x(k)^T | y(1:K)\right\} = P(k|K) + x(k|K)x(k|K)^T, \tag{19}$$

$$\begin{aligned} S(k, k-1|K) &= \mathrm{E}\left\{x(k)x(k-1)^T | y(1:K)\right\} \\ &= P(k, k-1|K) + x(k,|K)x(k-1|K)^T. \end{aligned} \tag{20}$$

Term $P(k, k-1|K)$ in Eq. 20 must be calculated through Eq. 21 while all the quantities in Eqs. 18-19 are calculated using the Kalman smoother equations,

$$P(k, k-1|K) = J(k-1)P(k|K). \tag{21}$$

Second step in EM algorithm is maximization by direct differentiation of F with respect to the θ. These two steps are applied iteratively until convergence achieved. Estimation of model parameters θ from F is explained in detail by (Khan, 2007) and it is not presented here due to the lengthy of the expressions. At the end of EM algorithm, term $x(k|K)$ is TVARX coefficients. Finally, time-varying transfer function (TVTF) by using TVARX coefficients can be expressed in Eq. 22.

$$H\left(k|K, e^{j\omega}\right) = \frac{\sum_{\ell=0}^{M} b_\ell(k|K)e^{-j\omega\ell}}{1 + \sum_{i=1}^{P} a_i(k|K)e^{-j\omega i}}. \tag{22}$$

Terms in Eq. (22) are explained as follows: $a_i(k|K)$ and $b_\ell(k|K)$ are the i^{th}, ℓ^{th} elements of coefficients of TVARX model, respectively and ω is the observed frequency.

NUMERICAL EXAMPLES

In this section, numerical example of single input single output (SISO) system as a verification of the method in estimating frequency response is presented. As a sample, linear time-invariant transfer function (LTIVTF) in z-domain, which has second order denominator, is given by Eq. 23.

$$H(z) = \frac{-2z - 210.5}{1.634z^2 + 0.3634z + 5227}. \tag{23}$$

The LTIVTF in Eq. 24 is excited with chirp input signal that has amplitude and frequency modulation as shown in the top panel of Fig. 1. Input signal has frequency jump that occurs in low frequency from 0.5 Hz into 2 Hz. This is a kind of an attempt in capturing systems having low frequency dynamic. This input signal is designed in such a way that output signal produced from true transfer function in Eq. 24 is nonstationary signal with amplitude and frequency modulation as depicted in the top panel of Fig. 2. Simulation is carried out in 10 seconds with a 200 Hz sampling frequency and no measurement noise is injected. Characterization of input-output signal in time-frequency distribution (TFD) is presented in the bottom panel of Fig.1 and Fig.2.

TFD of input and output signal obtained from TVAR model (TVARX without exogenous input) are solved by least mean square (LMS) and recursive least square (RLS) algorithm based on Eq. 7 and Kalman smoother with EM algorithm (KS with EM) as the proposed method based on Eq. 8 and Eqs. 18-21. Here, two non-parametric methods, namely Hilbert transform and short time Fourier transformation (STFT) are taken as bench mark.

In those figures, Hilbert transform and STFT as non-parametric approach clearly show their drawback in tracking nonstationary signal for the underlying system. Frequency estimate from using Hilbert transform bounces roughly before the jump and after the jump, while STFT and RLS behave similarly. LMS produces lagged response before the jump and still bounces after the jump. Compared to the others, KS with EM has better tracking capability. It is found out that model order of 2 for TVAR model is adequate to estimate spectral contents for this numerical example.

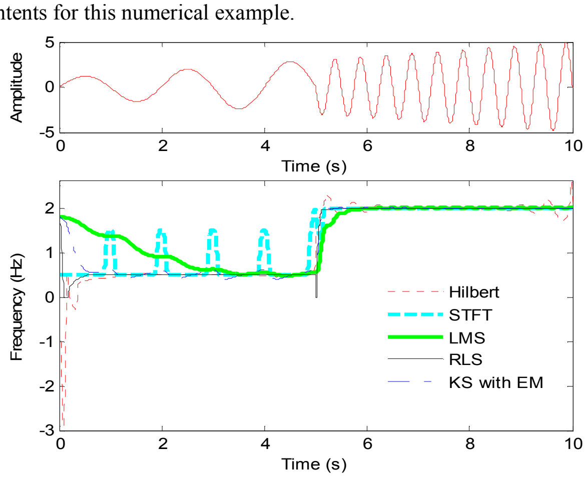


Fig. 1 TFD of input signal under frequency and amplitude modulation without noise

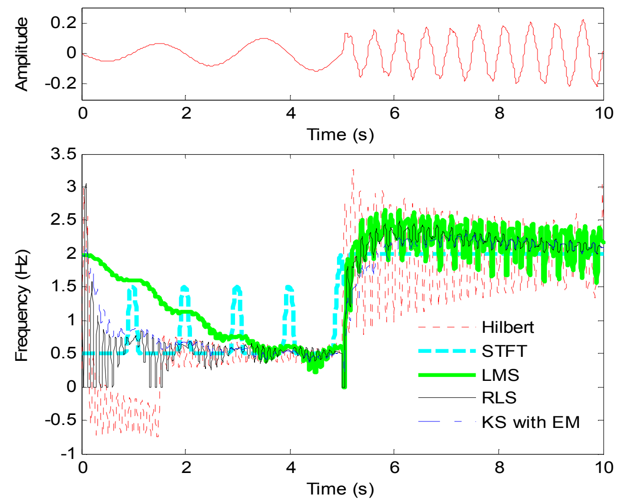


Fig. 2 TFD of output signal under frequency and amplitude modulation without noise

Next step is LTF gain estimation of input-output signal. One non-parametric estimation method and one parametric estimation method have been chosen, namely Welch's averaged periodogram and ARX model, respectively.

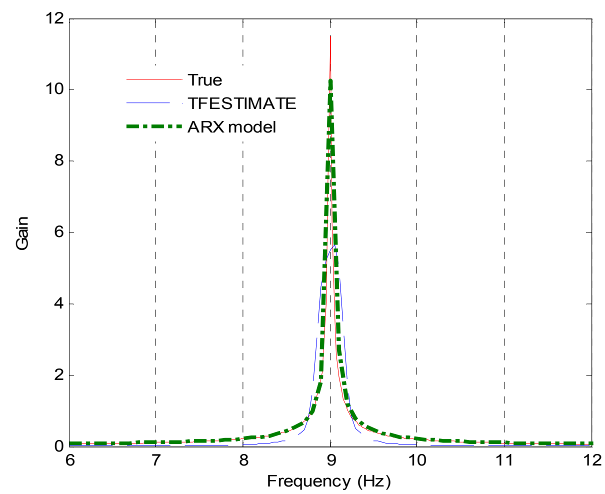


Fig. 3 LTF gain of numerical example by TFESTIMATE and ARX model

The non-parametric method is DFT-based method, where Welch's averaged periodogram is provided by the function `tfestimate` in Matlab's signal processing toolbox. The function `tfestimate` estimates the transfer function by taking the ratio of the Fourier transform of the input-output pair, where the input-output pair is segmented into data blocks to reduce the variance. The best estimate using Welch's method is found by using two data blocks which are overlapped by 50% and filtered with Hanning window. The parametric method is based on ARX model. Solution of ARX model is carried out by using the function `arx` in Matlab's system identification toolbox. It is found that ARX model has model order of (2,2) to fully capture the non-stationarity of the input-output pair. Figure 3 gives comparison between frequency responses obtained from `tfestimate` and ARX model with respect to the true frequency. True frequency of the system is around 9 Hz which is obtained from Bode diagram of Eq. 24. It is clear from the Fig. 3 that both methods can estimate frequency response of the numerical example. However, ARX model produces sharper peak compared to `tfestimate`. The gain is smaller compared to true model. However, both methods only give averaged TF and cannot give information on time localization of LTF.

Further, by applying the TVARX model, TF of system in Eq. 23 can be estimated. The coefficients are depicted in Fig. 4. By putting hose coefficients into Eq. 22, LTF gain can be calculated. LTF gain in Fig. 4 is projection of LTVTF in gain-frequency distribution. From number of points which are evaluated on frequency axis, the obtained LTVTF gain has similar trend with the true TF in Eq. 23. Their peaks are centered on the true frequency. It confirms the capability of TVARX model to estimate the accurate TF gain even it is produced from linear time-invariant system.

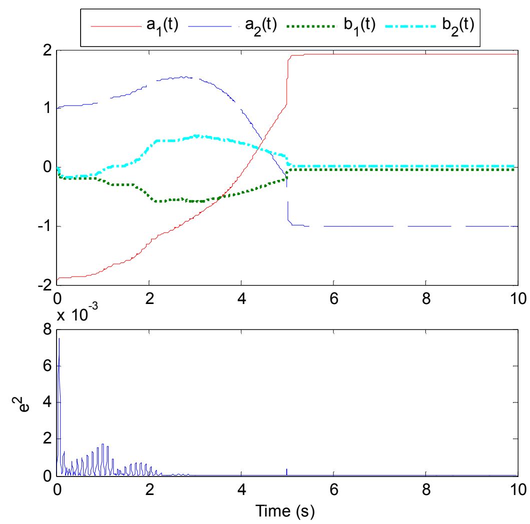


Fig. 4 Coefficients of TVARX model

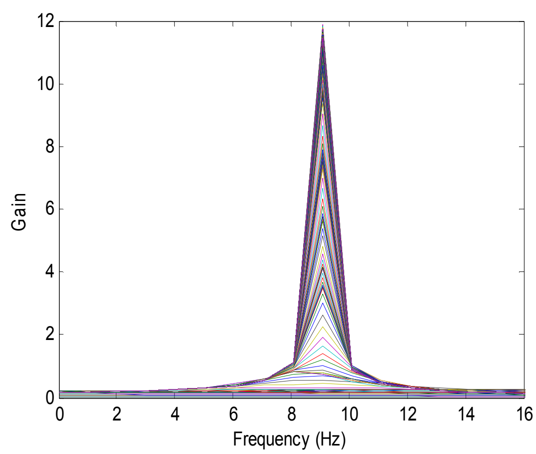


Fig. 5 LTF gain of numerical example by TVARX model

Furthermore, LTVTF gain is projected into time-frequency distribution (in 3D distribution) and depicted in Fig. 6. TVARX model can be estimated successfully by showing that the highest TV gain value is around the true frequency of the system. TVARX coefficients are estimated by Kalman smoother with EM algorithm as explained in the previous section. Here, model order of (2,2) is adequate to estimate the TVTF for this numerical example. As benchmark, STFT-based TVTF is presented in Fig. 7. STFT is carried out with window length 4000. Every window is overlapped by 50% and multiple window procedure is carried out under Hamming window. The result shows that STFT can estimate the TVTF, but the figure confirms the restriction of temporal resolution of STFT due to Heisenberg's uncertainty principle. It should be noted that the STFT-based TVTF is presented here as a quick comparison because of its simple algorithm compared to Hilbert transformation.

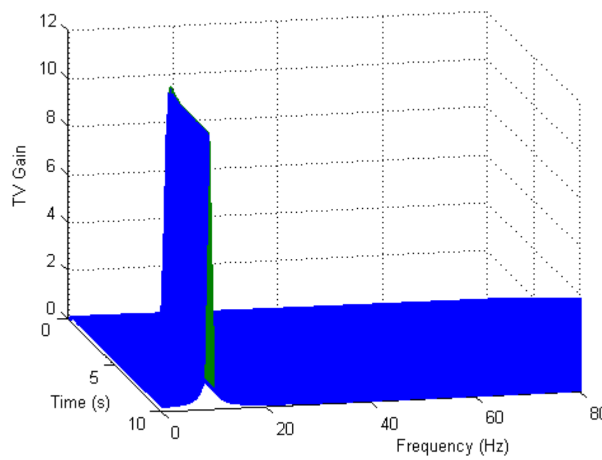


Fig. 6 LTVTF of numerical example by TVARX model

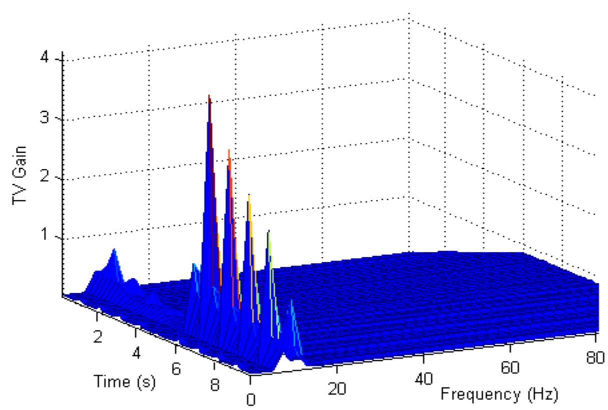


Fig. 7 LTVTF of numerical example by STFT

This is the reason why TVTF is presented in this paper. TVTF provides additional insights of the analyzed system that may have never been revealed using the conventional TIVTF.

APPLICATION TO THE SIMULATED FIELD MEASUREMENTS DATA

In this section, the application of the proposed method to analyze the simulated field measurements data is demonstrated. Sea wave and motion responses of a truss spar platform are depicted in Figs. 8-9. Time series of Fig. 8 reveal that the sea state can be classified as a rough sea where $H_S = 2.5 - 4.0$ m. Sea wave elevation seems to be linear random waves because its amplitudes appear to be symmetrical around its mean value. The behavior of surge motion in terms of time series is displayed in the top panel of Fig. 9. The surge motion has both +ve and –ve amplitudes and oscillates at positive mean value.

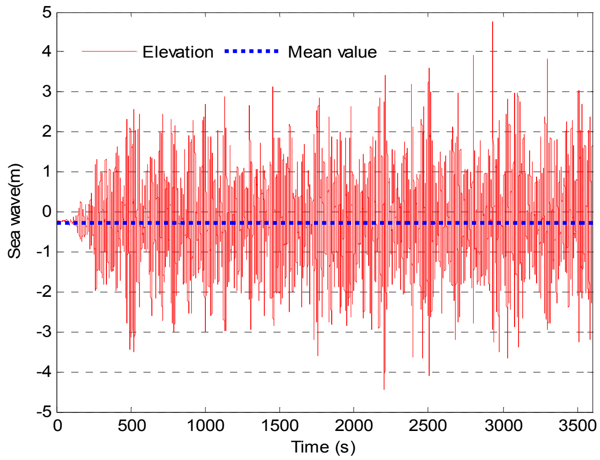
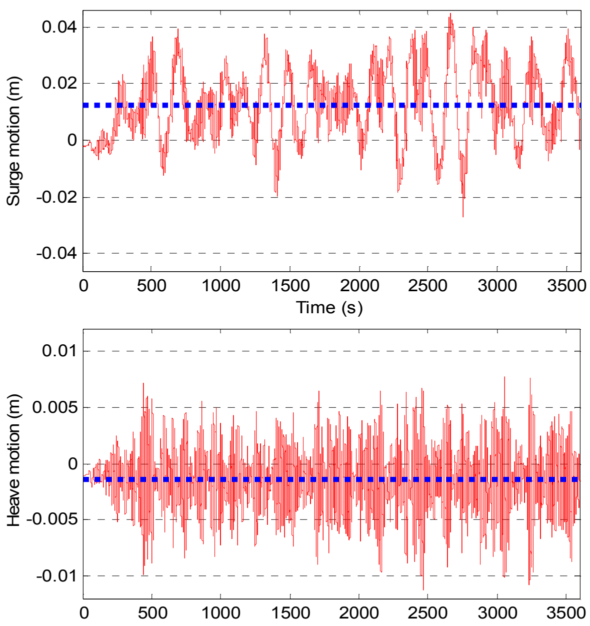


Fig. 8 Simulated field measurement data of sea wave

The response amplitudes appear to be unsymmetrical around its mean value and seem to be shifted upward. It implies that the surge motion appears to be nonlinear, which mean that surge motion in high sea state contains more nonlinear effects. On the other hand, heave and pitch motion have negative mean value. Heave and pitch responses fluctuate about the mean value oscillating from smaller to larger amplitudes and repeating the same trend onwards all through the time series. The fluctuations gradually increase from narrow to broad and after each peak. They appear to be almost symmetrical, meaning that non-linearity is not very sturdy on heave and pitch motion excursion.



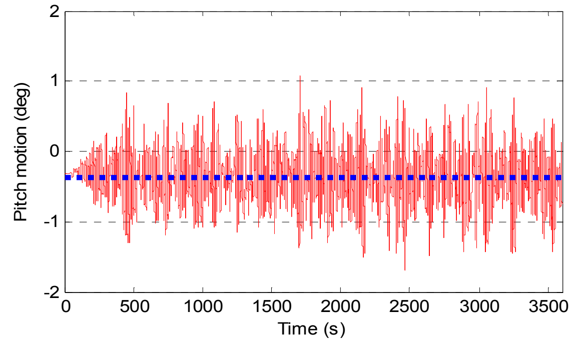


Fig. 9 Simulated field measurement data of truss spar responses

Compared to heave and pitch, surge has low-frequency motion in the given time series

Similar with numerical examples, coefficients of TVARX model must determined before calculating the LTF. Here, the coefficients are reflected in the ability of TVARX model in predicting the true motion response. The comparison of motion response predicted by TVARX model with the true motion response is displayed in the top panel of Fig. 10. The model error between the true motion response and TVARX model is relatively small as shown in the bottom panel of Fig 10. From the figure, it can be seen that the errors for TVARX models are relatively small, meaning that TVARX model can fit the motion responses accurately. Accurate fitting will produce the accurate zeros and poles of the system through the TVARX coefficients. It implies to the accurate TVTF. It is noted that prediction and error of TVARX models for heave and pitch motion are no presented here because they have similar trends.

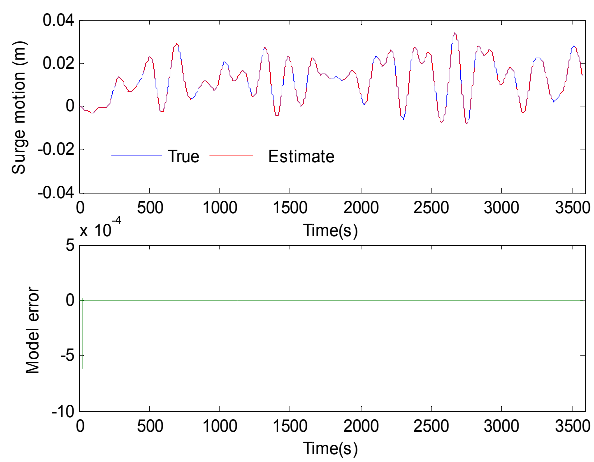
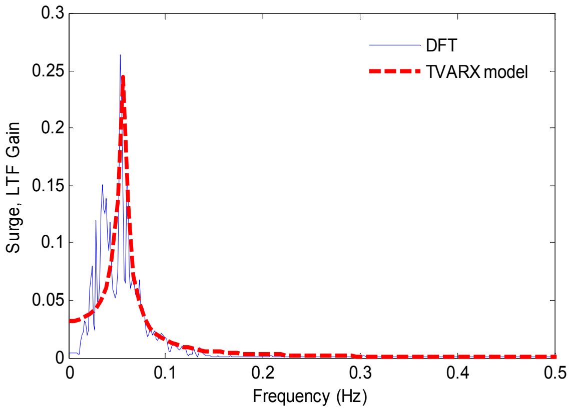
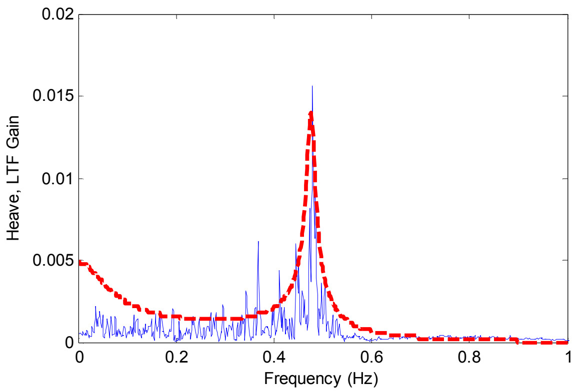


Fig. 10 Surge motion estimation and the model error

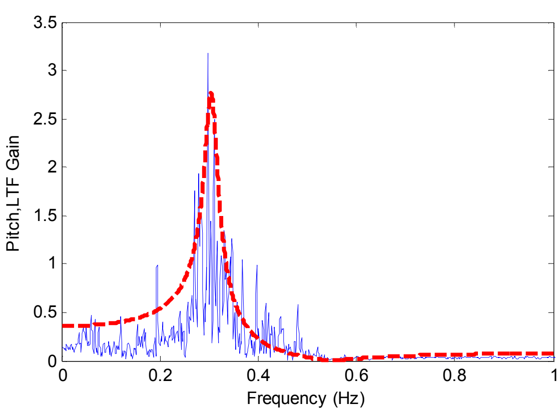
Further, after confirming that TVARX model can produce good fitting, LTF gain estimation can be carried out for surge, heave and pitch motion. DFT based LTF gain is performed as comparison. It should be noted that LTF gain obtained from TVARX model is LTVTF in gain-frequency distribution**.** The results are depicted in Figs. 11(a), 10(b) and (c), respectively. It is clearly observed that TVARX model produces sharper and smoother LTF compared to DFT. It also can be seen that maximum gains is located in frequency 0.05 Hz for surge, 0.5 Hz for heave and 0.3 Hz for pitch.



(a)



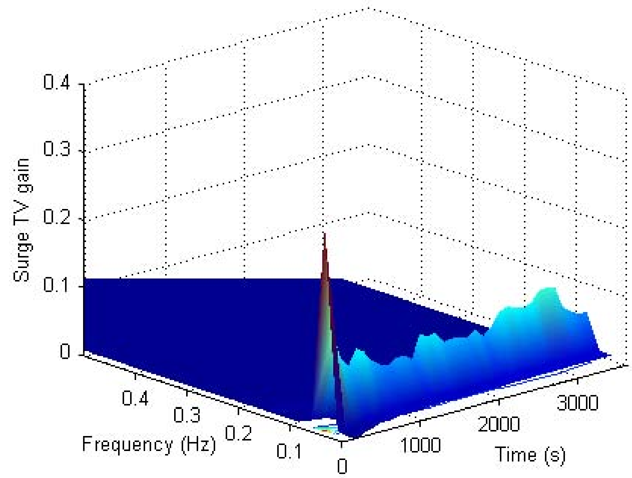
(b)



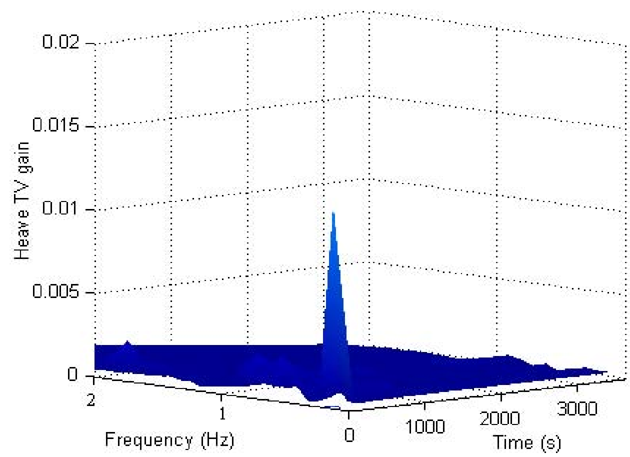
(c)

Fig. 11 LTF gains of surge, heave and pitch motion

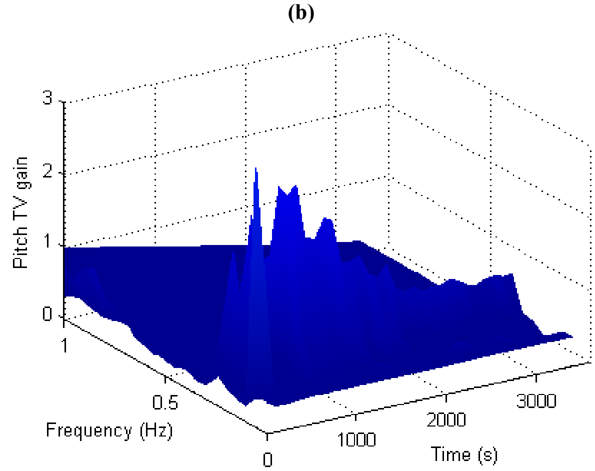
Representation of LTF in gain-time-frequency distribution enables us to observe the behaviour of LTF with respect to time. This is the benefit of using TVARX model. LTVTF of surge, heave and pitch motion to wave are displayed in Figs. 12(a), (b) and (c), respectively. As observed in Fig. 12(a), surge LTVTF provides a good description of characteristic of surge motion response to wave compared to Fig. 11(a).



(a)



(b)



(c)

Fig. 12 Surge, heave and pitch LTVTF gain

Non-stationarity is clearly observed over there. In LTVTF, it can be seen that the TF characteristics of surge motion are varying time-frequency and varying time-gain distributions. TVARX model can extract such informations accurately and produce high resolution with model order of (4, 2).

Gain of LTVTF heave motion reveals that heave motion is more stationary compared to surge or pitch motions as shown in Fig. 12(b). The LTVTF gain for the heave motion is consistent as the time indexes are increased. However, the LTVTF gain of the surge and pitch motion are more time dependent and more complicated than heave motion.

Representation of TVTF gain in time-frequency distribution enable us to observe the behaviors of TFs with respect the time. Recorded signals of wave or motion responses of offshore structures are generally random in nature, contain nonstationarity and nonlinearity. It can be tackled by TVARX model , meaning that it is a promising method in generating time-frequency based TFs for the need of offshore engineering research. However, more investigations are needed to develop this method and make it more reliable and applicable by using more data sets and new measurement data.

CONCLUSIONS

1) Response transfer function using field measurements have been estimated in this paper. The estimation has been carried out by employing the TVARX model, solved with combination of Kalman smoother and EM algorithm. Performance of the TVAR as special form of TVARX model using LMS, RLS and KS with EM compared to Hilbert transformation and STFT has been carried out. It is superior in time-varying spectral analysis for systems which have frequency or amplitude modulation and low frequency signal.
2) TVARX model produces accurate, smooth and less noise TF estimates over DFT method in gain-frequency distribution.
3) The TF characteristics of surge and pitch motion are varying time-frequency and varying time-gain distribution compared to heave motion. The LTVTF gain of the surge and pitch motion are more time dependent and more complicated than heave motion.
4) It is expected that this work will initiate the use of TVTFs in the offshore engineering researches as it provides additional insights of time history of TFs that can not be given by the time-invariant TFs.

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